BKK the EZ way

Ric Colacito, Max Croce, Steven Ho, Philip Howard
BKK the EZ way
Backus, Kehoe, and Kydland the Epstein and Zin way

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Motivation

- Our goal: characterizing role of long-term risks in international macroeconomics
  - Current events: public concern about long-term world-wide growth prospects
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- Novel BKK-based model with (1) recursive risk-sharing and (2) low investment home-bias
  - positive short-run productivity shocks $\rightarrow$ inflow of investment goods ($NX_I \downarrow$ and $NX \downarrow$) standard
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- Novel empirical evidence G7 countries support our model
International allocation of resources depends on two channels:

1. **Productivity**: resources are invested in the high-productivity country
2. **Risk-sharing**: resources go to the low-productivity country
Insight

- International allocation of resources depends on two channels:
  1. **Productivity**: resources are invested in the high-productivity country
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- Add Epstein-Zin (EZ) to Backus-Kehoe-Kydland (BKK):
  1. **Short-run shocks** → productivity channel dominates
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- Add Epstein-Zin (EZ) to Backus-Kehoe-Kydland (BKK):
  1. **Short-run shocks** → productivity channel dominates
  2. **Long-run shocks** → risk-sharing channel dominates

- Convincing quantitative results:
  1. **Low investment home-bias** → international quantities ✓
  2. **Vintage capital** → international asset prices ✓
Resolved Puzzles

1. Quantity anomaly
   - Cross country correlations of GDP are higher than cross country correlations of consumption
Resolved Puzzles

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   - Excess return on equities over risk-free bonds is high from viewpoint of a large class of traditional models
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5 Forward premium anomaly
   - High interest rate currencies continue to appreciate despite uncovered interest rate parity predictions
   - Our model will produce endogenous time-varying currency risk premia, however the overall amount is not sufficient to fully reconcile the anomaly
Agenda

- Economic Model
- Theoretical Predictions
- Empirical Results
- Summary
Preferences

\[ C_t = \left( \lambda X_t^{1-\frac{1}{3}} + (1-\lambda) Y_t^{1-\frac{1}{3}} \right) \frac{1}{1-\frac{1}{3}} \]
Preferences

\[ C_t = \left[ \lambda X_t^{1-\frac{1}{3}} + (1 - \lambda) Y_t^{1-\frac{1}{3}} \right]^{\frac{1}{1-\frac{1}{3}}} \]

\[ \tilde{C}_t = C_t - \varphi N_t^{1+\frac{1}{7}} A_{t-1} \quad \text{(Raffo)} \]
Preferences

\[ C_t = \left[ \lambda X_t^{1-\frac{1}{\psi}} + (1-\lambda) Y_t^{1-\frac{1}{\psi}} \right]^{\frac{1}{1-\frac{1}{\psi}}} \]

\[ \tilde{C}_t = C_t - \varphi N_t^{1+\frac{1}{\psi}} A_{t-1} \] (Raffo)

\[ U_t = \frac{1 - \beta}{1 - \frac{1}{\psi}} \tilde{C}_t^{1-\frac{1}{\psi}} + \beta E_t \left[ U_t^{1-\frac{1}{\psi}} \right]^{1-\frac{1}{\psi}} \] (EZ)
EZ Risk-Sharing Motive

\[ U_t \approx (1 - \delta) \frac{\tilde{C}_t^{1 - \frac{1}{\psi}}}{1 - \frac{1}{\psi}} + \delta E_t[U_{t+1}] - \left( \gamma - \frac{1}{\psi} \right) \text{Var}_t[U_{t+1}] \kappa_t \]

CRRA Preferences

Utility Variance
EZ Risk-Sharing Motive

\[ U_t \approx (1 - \delta) \frac{\tilde{C}_t^{1 - \frac{1}{\psi}}}{1 - \frac{1}{\psi}} + \delta E_t[U_{t+1}] - (\gamma - \frac{1}{\psi}) \text{Var}_t[U_{t+1}] \kappa_t \]

\[ M_{t+1} = \beta \left( \frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{-\frac{1}{\psi}} \left( \frac{U_{t+1}}{E_t \left[ U_{t+1}^{1 - \gamma} \right]^{\frac{1}{1 - \gamma}}} \right)^{\frac{1}{\psi} - \gamma} \]
**EZ Risk-Sharing Motive**

\[
U_t \approx (1 - \delta) \frac{\tilde{C}_t^{1 - \frac{1}{\psi}}}{1 - \frac{1}{\psi}} + \delta E_t[U_{t+1}] - (\gamma - \frac{1}{\psi}) \text{Var}_t[U_{t+1}] \kappa_t
\]

\[
M_{t+1} = \beta \left( \frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{-\frac{1}{\psi}} \left( \frac{U_{t+1}}{E_t \left[ U_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi} - \gamma}
\]

**CRRA Preferences**

**Utility Variance**

\[S_t\text{ is the relative distribution of wealth}\]

- \[\frac{S_t}{S_{t-1}} = \frac{\Delta C_t}{\Delta C^*} \frac{M_t}{M^*}\]

- Good long-run news ⇒ marginal utility ↓ ⇒ \(S_t\) ↓
Productivity growth

Symmetric specification across countries:

\[
\Delta a_t = \mu + \tau(a_{t-1} - a_{t-1}^*) + \sigma \varepsilon_{a,t} + \varepsilon_{x,t} - 1
\]

\[
z_{x,t} = \rho z_{x,t-1} + \sigma_x \varepsilon_x, t
\]

\[
\varepsilon_{a,t}, \varepsilon_{x,t} \sim iid N(0,1)
\]

Production

\[ X_t^{Tot} = K_t^\alpha (A_t N_t)^{1-\alpha} \]

\[ = X_t + I_{x,t} + X^*_t + I_{y,t} \]

Domestic Use \hspace{1cm} Foreign Use
Production

\[ X_t^{Tot} = K_t^\alpha (A_t N_t)^{1-\alpha} \]

\[ = X_t + I_{x,t} + X^*_t + I_{y,t} \]

\[ = \underbrace{X_t + Y_t P_t}_{\text{Consumption}} + \underbrace{I_{x,t} + I^*_x P_t}_{\text{Investment}} + \underbrace{X^*_t + I_{y,t}}_{\text{Exports}} - \underbrace{P_t (Y_t + I^*_x,t)}_{\text{Imports}} \]

\( P_t \) is the terms of trade
Capital & Investment

\[ K_t = (1 - \delta)K_{t-1} + e^{\omega t} G_{t-1} \]

\[ G_t = \begin{bmatrix}
  v^{1 - \frac{1}{\xi}} I_{x,t} \\
  (1 - v)^{1 - \frac{1}{\xi}} I_{x,t} \\
\end{bmatrix}
\]

\begin{align*}
\begin{bmatrix}
  v^{1 - \frac{1}{\xi}} I_{x,t} \\
  (1 - v)^{1 - \frac{1}{\xi}} I_{x,t} \\
\end{bmatrix}
\end{align*}

Domestic Investment

Foreign Investment

\[ E_t \begin{bmatrix}
  M_{X,t} + 1 \\
  MPK_t + 1 + (1 - \delta) Q^*_K, t \\
\end{bmatrix}
\]

Terms of trade (EGG)
Capital & Investment

\[ K_t = (1 - \delta)K_{t-1} + e^{\omega_t}G_{t-1} \]

\[ G_t = \begin{bmatrix} \nu l_{x,t}^{1-\frac{1}{\xi}} + (1 - \nu) l_{x,t}^{*1-\frac{1}{\xi}} \\ \text{Domestic Investment} & \text{Foreign Investment} \end{bmatrix} \]

\[ \omega_{t+1} = -\left(\frac{1}{\alpha} - 1\right)(\Delta a_{t+1} - \mu) \]

(EGG)  

(ACL)
Capital & Investment

\[ K_t = (1 - \delta)K_{t-1} + e^{\omega_t} G_{t-1} \]

\[ G_t = \begin{bmatrix} \nu I_{x,t}^{\frac{1}{1-\frac{1}{\xi}}} + (1 - \nu) I_{x,t}^{\frac{1}{1-\frac{1}{\xi}}} \end{bmatrix} \]

Domestic Investment

Foreign Investment

(EGG)

\[ \omega_{t+1} = - \left( \frac{1}{\alpha} - 1 \right) (\Delta a_{t+1} - \mu) \]

(ACL)

\[ \frac{\partial G_t^{-1}}{\partial I_{x,t}} = E_t \left[ M_{t+1}^X (MPK_{t+1} + (1 - \delta) Q_{K,t+1}) e^{\omega_{t+1}} \right] \]

\[ \frac{\partial G_t^{*^{-1}}}{\partial I_{y,t}} = E_t \left[ M_{t+1}^X (MPK_{t+1}^{*} + (1 - \delta) Q_{K,t+1}^{*}) e^{\omega_{t+1}^{*}} P_{t+1} \right] \]
Capital & Investment

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(EGG)

\[ \omega_{t+1} = -\left(\frac{1}{\alpha} - 1\right)(\Delta a_{t+1} - \mu) \]  

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Terms of trade

\[ P_{t+1} \]
## Results

<table>
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<tr>
<th>Model</th>
<th>(4)</th>
<th>(5b)</th>
<th>(6)</th>
<th>Data</th>
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<th>43✓</th>
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<td>$E\left[\frac{I_y P}{I}\right]$</td>
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| $E[IyP/I]$              | 15  | 47✓  | 43✓  | 40   |
| $E[Y·P/C]$              | 15  | 3✓   | 3✓   | 5    |
| $\rho(\Delta \frac{NXQ}{GDP}, \Delta GDP)$ | -0.53 | -0.06✓ | -0.14✓ | -0.27 |
| $\sigma(\Delta e)$     | 0.54 | 9✓   | 10✓  | 11   |
| $\rho(C) - \rho(GDP)$  | 0.10 | -0.12✓ | -0.06✓ | -0.17 |
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| $\rho(C) - \rho(GDP)$ | 0.10 | -0.12✓ | -0.06✓ | -0.17 |
| $E[r^{ex}]$ | 0.08 | 0.22 | 3.46✓ | 5.01 |
| $\beta_{UIP}$ | 1.04 | 0.81 | 0.51✓ | -0.72 |
Key Differences

Short Run Shock

Long Run Shock

\[ \Delta a_t \]

\[ \Delta \ln I_t \]

\[ \frac{NX_t}{GDP_t} \]

Model (4): EZ–BKK

Model (6): Benchmark

Student Version of MATLAB
Empirical Analysis

- Focus on G-7 countries
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- Estimate short-run shocks, $\varepsilon_{a,t}$:

$$\Delta \ln A_t = c + \beta_1 \cdot pd_{t-1} + \beta_2 \cdot rf_{t-1} + \varepsilon_{a,t}$$
Empirical Analysis

- Focus on G-7 countries

- Estimate short-run shocks, $\varepsilon_{a,t}$:

\[
\Delta \ln A_t = c + \beta_1 \cdot pd_{t-1} + \beta_2 \cdot rf_{t-1} + \epsilon_{a,t}
\]

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\[
x_t = \rho x_{t-1} + \epsilon_{x,t}
\]
Empirical Analysis

- Focus on G-7 countries

- Estimate short-run shocks, $\varepsilon_{a,t}$:

  $$\Delta \ln A_t = c + \beta_1 \cdot pd_{t-1} + \beta_2 \cdot rf_{t-1} + \varepsilon_{a,t}$$

- Estimate long-run shocks, $\varepsilon_{x,t}$:

  $$x_t = \rho x_{t-1} + \varepsilon_{x,t}$$

- Estimate response of investments

  $$\Delta \ln I_{US}^t - \Delta \ln I_{World}^t = c + \beta_1 (\varepsilon_{US}^t - \varepsilon_{t,World}^t) + \beta_2 (\varepsilon_{x,t}^{US} - \varepsilon_{x,t}^{World})$$
Empirical Analysis

- Focus on G-7 countries
- Estimate short-run shocks, $\varepsilon_{a,t}$:
  \[
  \Delta \ln A_t = c + \beta_1 \cdot pd_{t-1} + \beta_2 \cdot rf_{t-1} + \varepsilon_{a,t}
  \]
- Estimate long-run shocks, $\varepsilon_{x,t}$:
  \[
  x_t = \rho x_{t-1} + \varepsilon_{x,t}
  \]
- Estimate response of investments
  \[
  \Delta \ln I_{t}^{US} - \Delta \ln I_{t}^{World} = c + \beta_1 (\varepsilon_{t}^{US} - \varepsilon_{t}^{World}) + \beta_2 (\varepsilon_{x,t}^{US} - \varepsilon_{x,t}^{World})
  \]
- Estimate response of NX of investments
  \[
  \frac{NX_{I,t}}{GDP_t} = c + \beta_1 (\varepsilon_{t}^{US} - \varepsilon_{t}^{World}) + \beta_2 (\varepsilon_{x,t}^{US} - \varepsilon_{x,t}^{World})
  \]
### Empirical Evidence

#### Panel A: Response of Investments

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#### Panel B: Response of Net Exports of Investments

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Summary

- **Empirical Contribution:** investment outflows upon good long-run news
Summary

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- **Model Contribution:** international production economy with recursive risk-sharing and long-run risk
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Summary

Empirical Contribution: investment outflows upon good long-run news

Model Contribution: international production economy with recursive risk-sharing and long-run risk

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Summary

- **Empirical Contribution**: investment outflows upon good long-run news.

- **Model Contribution**: international production economy with recursive risk-sharing and long-run risk.
  - good news for the long (short) run $\rightarrow$ capital outflows (inflows)
  - quantity anomaly turned into a regularity.


- **Broader point**: conveying the need of introducing long-run risk considerations in international investment theory.
Relation to literature

Prior work by the authors:

  - International production and investment flows

- Ai Croce Li (RFS 2012): Closed production economy with vintage capital to explain EP and VP & investment vintages
  - International perspective

Previous papers:

- Raffo (JIE 2008): NX empirically driven by quantities (GHH preferences)
- Erceg Guerrieri Gust (JEDC 2008): Home bias is strong in consumption, mild in investment
  - LRR-based AP perspective
- Tretvoll (2012): Robust-BKK with short-run risk only
Wages

\[ W_t = MRS_{t}^{C,L} \]
\[ W_t = \frac{\partial X_t^{Tot}}{\partial N_t} \]
\[ MRS_{t}^{C,L} = \frac{\partial \tilde{C}_t / \partial L_t}{\partial \tilde{C}_t / \partial C_t} \]
\[ (1 - \alpha) \frac{X_t^{Tot}}{N_t} = \phi \left( 1 + \frac{1}{f} \right) N_t^{\frac{1}{T}} A_{t-1} e^{\phi \Delta a_t} \]
Time varying Pareto weights

\[ S_t = S_{t-1} \frac{M_t}{M_t^*} \frac{C_t / C_{t-1}}{C_t^* / C_{t-1}^*} \]

\[ = \frac{1 - \lambda}{\lambda} \left( \frac{C_t}{C_t^*} \right)^{1 - \frac{1}{3}} \left( \frac{X_t}{X_t^*} \right)^{\frac{1}{3}} \]

\[ = \frac{\lambda}{1 - \lambda} \left( \frac{C_t}{C_t^*} \right)^{1 - \frac{1}{3}} \left( \frac{Y_t}{Y_t^*} \right)^{\frac{1}{3}} \]
The terms of trade is the price of imports over the price of exports:

\[ P_t = \frac{1 - \lambda}{\lambda} \left( \frac{X_t}{Y_t} \right)^{\frac{1}{2}} \]
Domestic SDF

\[ M_{t+1} = \frac{\partial U_{t+1}/\partial C_{t+1}}{\partial U_t/\partial C_t} \]

\[ = \beta \left( \tilde{C}_{t+1} \right)^{-\frac{1}{\psi}} \left( \frac{U_{t+1}}{E_t \left[ U_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi} - \gamma} \]

\[ M_{t+1}^{X} = \frac{\partial U_{t+1}/\partial X_{t+1}}{\partial U_t/\partial X_t} \]

\[ = \left( \frac{C_{t+1}}{C_t} \frac{X_t}{X_{t+1}} \right)^{\frac{1}{3}} M_{t+1} \]
Ordinarily equivalent transformation: \( V_t = \frac{U_t^{1 - \frac{1}{\psi}}}{1^{1 - \frac{1}{\psi}}} \)

\[
V_t = \frac{1 - \beta}{1 - \frac{1}{\psi}} \tilde{C}_t^{1 - \frac{1}{\psi}} + \beta E_t \left[ V_t^{1 - \gamma} \right]^{1 - \frac{1}{\psi}} \frac{1 - \frac{1}{\psi}}{1 - \gamma}
\]

\[
U_t = \left[ (1 - \beta) \tilde{C}_t^{1 - \frac{1}{\psi}} + \beta E_t \left[ U_{t+1}^{1 - \gamma} \right]^{1 - \frac{1}{\psi}} \right]^{1 - \frac{1}{\psi}} \frac{1}{1 - \gamma}
\]
## Calibration

### TABLE 2: Calibrated Parameter Values

<table>
<thead>
<tr>
<th>Model:</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(5b)</th>
<th>(6)</th>
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<tr>
<td>Subjective discount factor</td>
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<td>Consumption-bundle elasticity</td>
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<td>Capital Income Share</td>
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</table>

Notes - This table reports the parameter values used for our calibrations. All models are calibrated at an annual frequency. Model (1) refers to the original BKK economy. Model (6) is our benchmark.