Benefits from U.S. Monetary Policy Experimentation in the Days of Samuelson and Solow and Lucas

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Motivation

- When a policy maker has multiple submodels, Bayes’ law and a Bellman equation tell him to experiment. Nevertheless, Blinder, Lucas, and others have told policy makers not to experiment (i.e., to ignore the Bellman equation).

- We study the benefits from listening to Bellman (and not Lucas and Blinder).
Central Bank chooses \( v_t \) to

\[
\min E_0 \sum_{t=0}^{\infty} \beta^t (U_t^2 + \lambda v_t^2), \text{ s.t.}
\]

- **Model 1:**
  \[
  U_t = \bar{U}_1 + A_1 U_{t-1} + B_1 \pi_t + C_{U,1} \eta_{1,t} \\
  \pi_t = v_{t-1} + C_{\pi} \eta_{3,t}
  \]

- **Model 2:**
  \[
  U_t = \bar{U}_2 + A_2 U_{t-1} + B_2 (\pi_t - v_{t-1}) + C_{U,2} \eta_{2,t} \\
  \pi_t = v_{t-1} + C_{\pi} \eta_{3,t}
  \]
The economy

- Central Bank chooses \( v_t \) to

\[
\min E_0 \sum_{t=0}^{\infty} \beta^t (U_t^2 + \lambda v_t^2), \text{ s.t.}
\]

- Model 1 \((\alpha_t)\):

\[
U_t = \bar{U}_1 + A_1 U_{t-1} + B_1 \pi_t + C_{U,1} \eta_{1,t} \\
\pi_t = v_{t-1} + C_{\pi} \eta_{3,t}
\]

- Model 2 \((1 - \alpha_t)\):

\[
U_t = \bar{U}_2 + A_2 U_{t-1} + B_2 (\pi_t - v_{t-1}) + C_{U,2} \eta_{2,t} \\
\pi_t = v_{t-1} + C_{\pi} \eta_{3,t}
\]

- Bayesian updating:

\[
\alpha_t = B(\alpha_{t-1}, U_t)
\]
Central Bank chooses $v_t$ to

$$\min E_0 \sum_{t=0}^{\infty} .995^t (U_t^2 + 0.1v_t^2), \text{ s.t.}$$

- **Model 1** ($\alpha_t$):
  $$U_t = 0.0023 + 0.7971U_{t-1} - 0.2761\pi_t + 0.0054\eta_{1,t}$$
  $$\pi_t = v_{t-1} + 0.0055\eta_{3,t}$$

- **Model 2** ($1 - \alpha_t$):
  $$U_t = 0.0007 + 0.8468U_{t-1} - 0.2489(\pi_t - v_{t-1}) + 0.0055\eta_{2,t}$$
  $$\pi_t = v_{t-1} + 0.0055\eta_{3,t}$$

- **Bayesian updating**:
  $$\alpha_t = B(\alpha_{t-1}, U_t)$$
Plan of the talk

- Use Bayes’ law to get a transition equation for $\alpha_t$
- Bellman equations
  1. Bayesian problem
  2. Anticipated utility
- Policy and value functions
- Experiments
Using Bayes’ law:

\[ \log \frac{\alpha_t}{1 - \alpha_t} = \log \frac{\alpha_{t-1}}{1 - \alpha_{t-1}} + \log \frac{p_1(U_t | U_{t-1}, v_{t-1})}{p_2(U_t | U_{t-1}, v_{t-1})} \]

Timing protocol

\[
\ldots \quad v_{t-1} \quad \pi_t, U_t \quad \alpha_t \quad v_t
\]
Evolution of $\alpha_t$

Using Bayes’ law:

$$\log \frac{\alpha_t}{1 - \alpha_t} = \log \frac{\alpha_{t-1}}{1 - \alpha_{t-1}} + \log \frac{p_1(U_t|U_{t-1}, v_{t-1})}{p_2(U_t|U_{t-1}, v_{t-1})}$$

Timing protocol

$$\ldots \quad v_{t-1} \quad \pi_t, U_t \quad \alpha_t \quad v_t$$

$\alpha_t$ is a martingale from the point of view of the Bayesian agent.

If the prior attaches nonzero probability to the model that actually generates the economy, then $\alpha_t$ converges almost surely under that measure.
Bayesian Problem

\[ V \left( U_t, \alpha_t \right) = \max_{v_t} \left\{ -\left( U_t^2 + \lambda v_t^2 \right) \right. \]
\[ + \beta \alpha_t \int V \left( U_{1,t+1}, B(\alpha_t, U_{1,t+1}) \right) dF(\varepsilon_{1,t+1}) \]
\[ + \beta (1 - \alpha_t) \int V \left( U_{2,t+1}, B(\alpha_t, U_{2,t+1}) \right) dF(\varepsilon_{2,t+1}) \}\]

subject to:

\[ U_{1,t+1} = \overline{U}_1 + A_1 U_t + B_1 v_t + C_1 \varepsilon_{1,t+1} \]
\[ U_{2,t+1} = \overline{U}_2 + A_2 U_t + C_2 \varepsilon_{2,t+1} \]
Bellman equation:

\[ W (U_t, \alpha) = \max_{v_t} \left\{ - (U_t^2 + \lambda v_t^2) \right. \]

\[ + \beta \alpha \int W (U_{1,t+1}, \alpha) dF(\varepsilon_{1,t+1}) \]

\[ + \beta (1 - \alpha) \int W (U_{2,t+1}, \alpha) dF(\varepsilon_{2,t+1}) \right\} \]

subject to:

\[ U_{1,t+1} = \overline{U}_1 + A_1 U_t + B_1 v_t + C_1 \varepsilon_{1,t+1} \]

\[ U_{2,t+1} = \overline{U}_2 + A_2 U_t + C_2 \varepsilon_{2,t+1} \]
Anticipated Utility

- Bellman equation:

\[
W (U_t, \alpha) = \max_{v_t} \left\{ - (U_t^2 + \lambda v_t^2) \right. \\
+ \beta \alpha \int W (U_{1,t+1}, \alpha) dF(\varepsilon_{1,t+1}) \\
+ \beta (1 - \alpha) \int W (U_{2,t+1}, \alpha) dF(\varepsilon_{2,t+1}) \left. \right\}
\]

- Policy function

\[
v_t = w(U_t, \alpha)
\]
Bellman equation:

\[ W(U_t, \alpha) = \max_{v_t} \left\{ -(U_t^2 + \lambda v_t^2) \right. \]
\[ + \beta \alpha \int W(U_{1,t+1}, \alpha) \, dF(\varepsilon_{1,t+1}) \]
\[ + \beta (1 - \alpha) \int W(U_{2,t+1}, \alpha) \, dF(\varepsilon_{2,t+1}) \right\} \]

Policy function

\[ v_t = w(U_t, \alpha_t) \]
\[ \alpha_t = B(\alpha_{t-1}, U_t) \]
Anticipated Utility cont’d

- Value of the economy with AU:

\[
\tilde{W}(U_t, \alpha_t) = -U_t^2 - \lambda w(U_t, \alpha_t)^2
\]

\[
+ \beta \alpha_t \int \tilde{W}(U_{1,t+1}, B(\alpha_t, U_{1,t+1})) \, dF(\varepsilon_{1,t+1})
\]

\[
+ \beta (1 - \alpha_t) \int \tilde{W}(U_{2,t+1}, B(\alpha_t, U_{2,t+1})) \, dF(\varepsilon_{2,t+1})
\]

subject to:

\[
U_{1,t+1} = \bar{U}_1 + A_1 U_t + B_1 v_t + C_1 \varepsilon_{1,t+1}
\]

\[
U_{2,t+1} = \bar{U}_2 + A_2 U_t + C_2 \varepsilon_{2,t+1}
\]
Policy functions

(a) Bayesian Problem

(b) Anticipated Utility
Policy functions (Slices)

\(\alpha \approx 0\)  
\(\alpha = 0.2\)  
\(\alpha = 0.40\)  
\(\alpha = 0.80\)  
\(\alpha \approx 1\)

Value functions

Prior on Samuelson and Solow ($\alpha$)
Value of experimentation
Suppose that one model generates the data:

- How long does it take to learn it?
- How much faster can we learn it with experimentation?
- How different are inflation and unemployment in the learning process?
Forgetting Lucas

Prior on Samuelson and Solow ($\alpha$)

Optimal inflation ($v$)

Unemployment gap ($U$)

Forgetting Lucas

Prior on Samuelson and Solow ($\alpha$)

Optimal inflation ($v$)

Unemployment gap ($U$)
Prior on Samuelson and Solow ($\alpha$)

Optimal inflation ($v$)

Unemployment gap ($U$)

Conclusions

- Benefits of experimentation, but not too big.
- Samuelson-Solow is less sensitive to small doubts.
- Limitations:
  1. Only two models are on the table.
  2. Models’ parameters are assumed to be known.
- Results vary with $\lambda$.
- Cogley, Colacito, Hansen, and Sargent (2005) compute decisions that are robust to:
  1. misspecifications of the dynamics within each submodel
  2. misspecifications of the prior distribution over the two submodels