Risks for the Long Run and the Real Exchange Rate

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Set the stage

- Study the link between international stochastic discount factors and the depreciation of the US dollar:

$$\frac{M_{t+1}^{uk}}{M_{t+1}^{us}} = \frac{e_{t+1}}{e_t}$$

where

$$E_t\left[M_{t+1}^{i}R_{t+1}^{i}\right] = 1, \qquad \forall i \in \{us, uk\}$$

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     - e.g. with CRRA preferences $M_{t+1}^i = \delta \left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\gamma}$

Puzzle

- By no arbitrage

$$E_t\left[M^f_{t+1}R^f_{t+1}\right] \quad = 1 = \quad E_t\left[M^h_{t+1}R^h_{t+1}\right]$$

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$$\underbrace{\sigma^2_{m^f}}_{20\%} + \underbrace{\sigma^2_{m^h}}_{20\%} - 2 \underbrace{\rho_{m^f,m^h}}_{0.96} \underbrace{\sigma_{m^f}\sigma_{m^h}}_{20\%} \quad = \quad \underbrace{\sigma^2_\pi}_{1.5\%}$$

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$$\log M^i_{t+1} = -\gamma \Delta c^i_{t+1}$$

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The puzzle (cont'd)

- Brandt, Cochrane and Santa-Clara (2005):

	ρ_{m^f, m^h}	σ_π^2
Prices	high	low
Quantities	low	high

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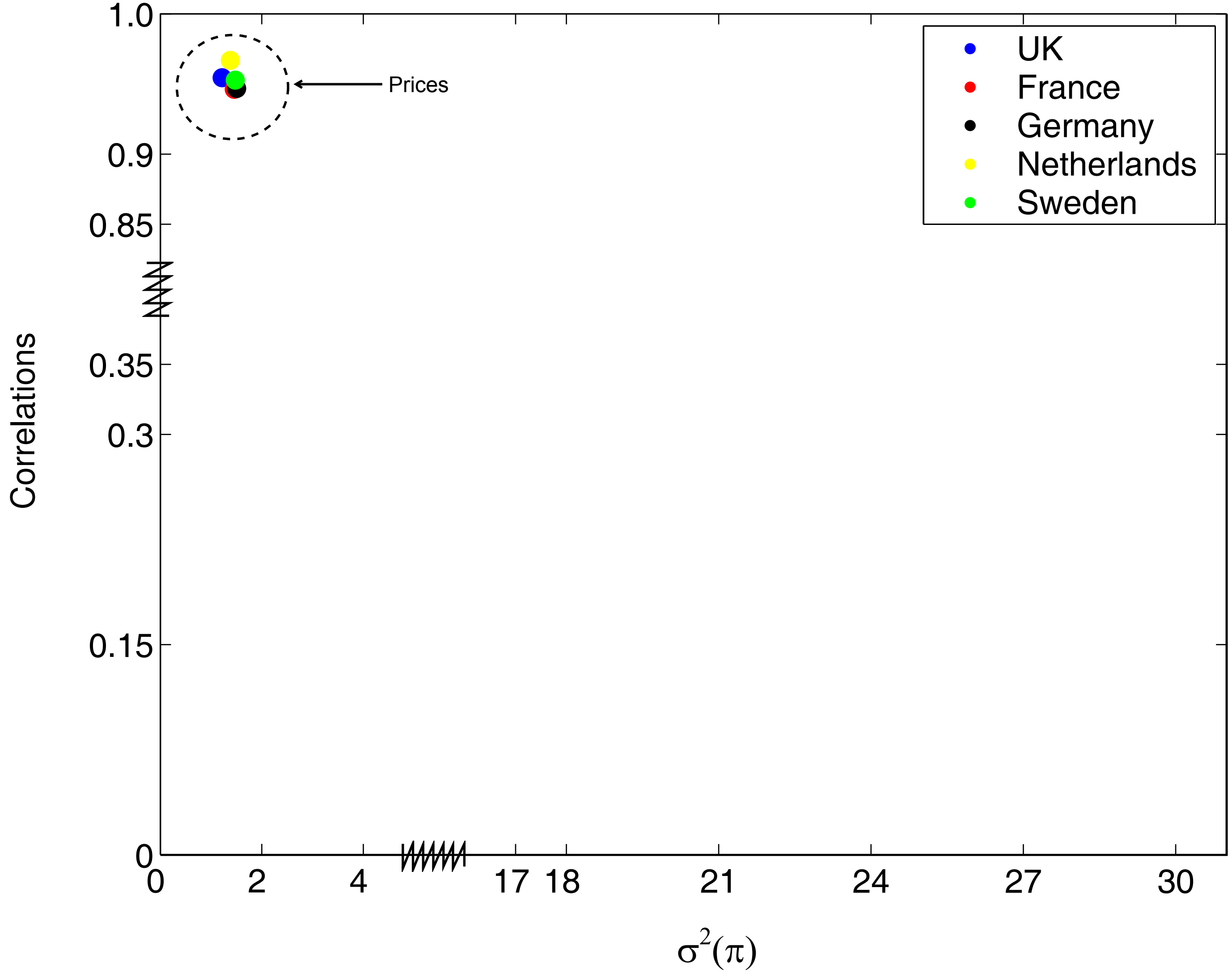
- How does this puzzle look in a cross section of countries?

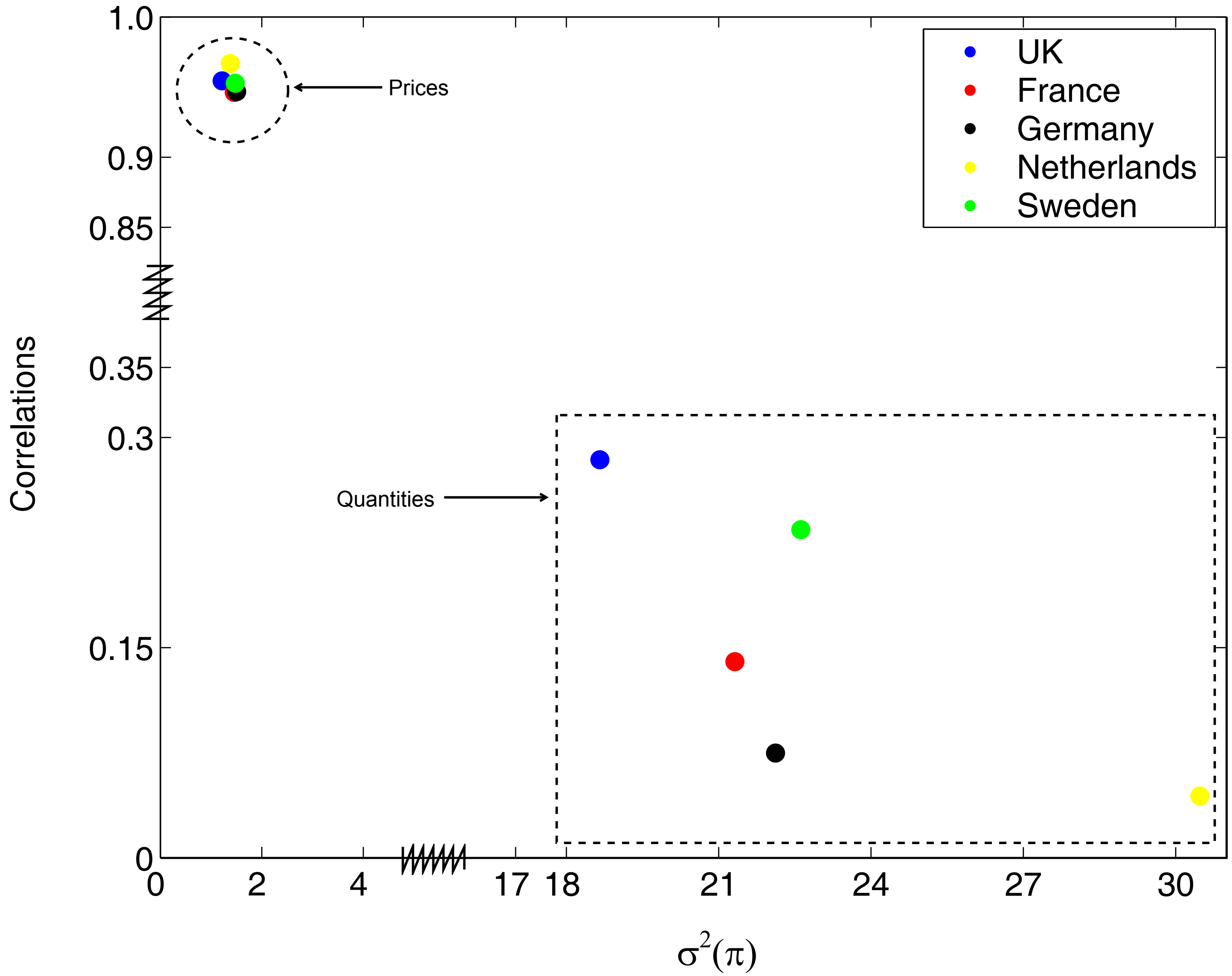
The puzzle in a cross section of countries

- HJ bound and depreciation rate: high correlation of SDF and low variance of depreciation rate.



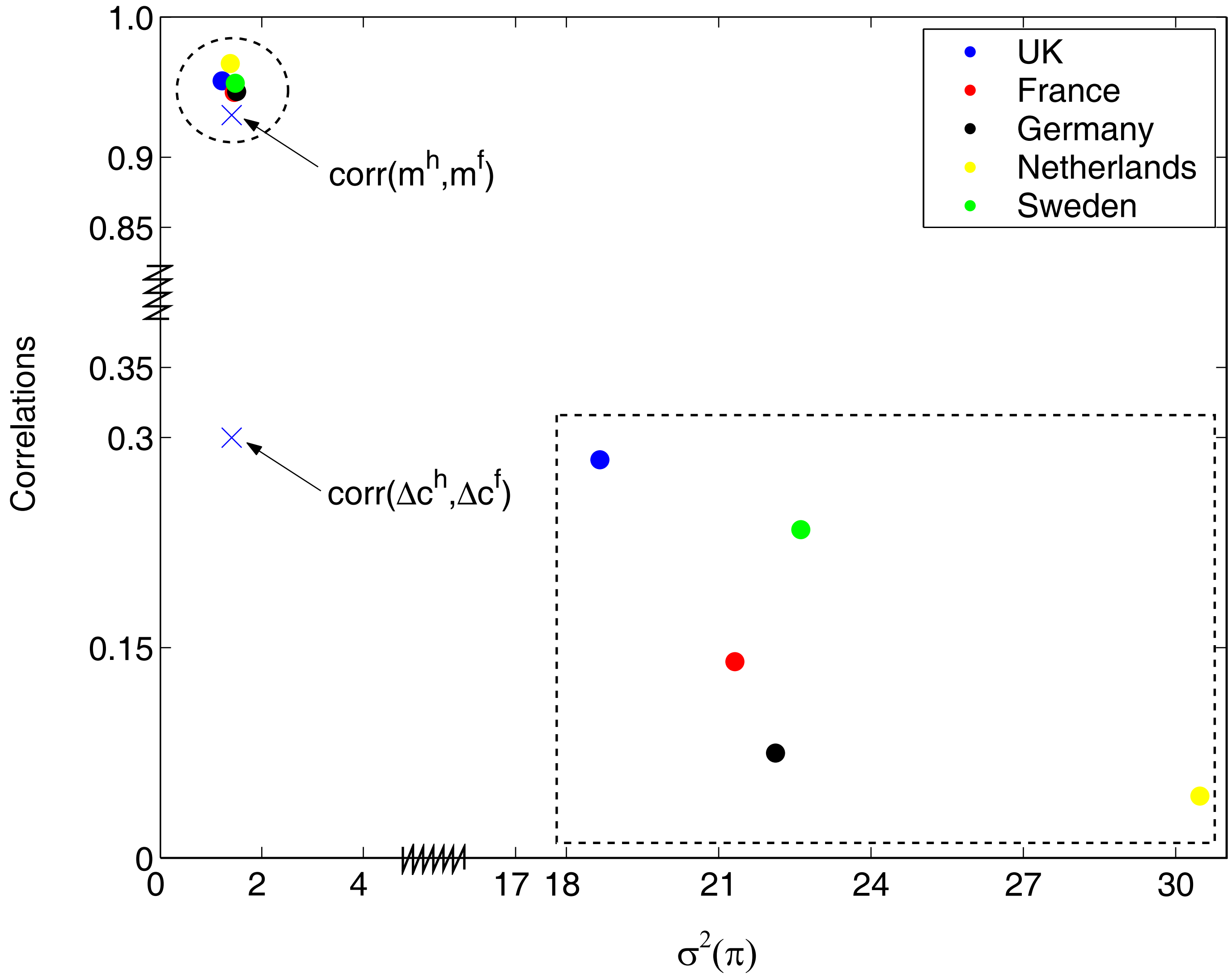
The puzzle in a cross section of countries

- Consumption data and CRRA preferences: low correlation of SDF and high variance of depreciation rate.



The puzzle in a cross section of countries

- This paper: high correlation of SDF, low correlation of Δc and low variance of depreciation rate.



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4. Extensions and future research...

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$$U_t^i = \left\{(1-\delta)(C_t^i)^{\frac{1-\gamma}{\theta}} + \delta\left[E_t(U_{t+1}^i)^{1-\gamma}\right]^{\frac{1}{\theta}}\right\}^{\frac{\theta}{1-\gamma}}, \forall i \in \{h, f\}$$

where

$$\theta = \frac{1-\gamma}{1-1/\psi}$$

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- What do stochastic discount factors look like?

Stochastic discount factors

- Assume $\psi = 1$:

$$U_t^i = (1-\delta)\log C_t^i + \frac{\delta}{1-\gamma}\log E_t\left[\exp\left(1-\gamma\right)U_{t+1}^i\right]$$

- The stochastic discount factors are

$$\begin{aligned}\log M_{t+1}^i &= \log \frac{\partial U^i / \partial C_{t+1}^i}{\partial U^i / \partial C_t^i} \\ &= \log\delta + \log\frac{C_t^i}{C_{t+1}^i} + \log\frac{\exp\{(1-\gamma)U_{t+1}^i\}}{E_t\left[\exp\{(1-\gamma)U_{t+1}^i\}\right]}\end{aligned}$$

- Brandt, Cochrane and Santa-Clara use:

$$\log M_{t+1}^i = \log\delta + \log\frac{C_t^i}{C_{t+1}^i}$$

Remainder of the economy

- Home country

$$\begin{aligned}\Delta c_t^h &= \mu_c + x_{t-1}^h + \sigma \varepsilon_{c,t}^h \\ x_t^h &= \rho x_{t-1}^h + \sigma \varphi_e \varepsilon_{x,t}^h\end{aligned}$$

- Foreign country

$$\begin{aligned}\Delta c_t^f &= \mu_c + x_{t-1}^f + \sigma \varepsilon_{c,t}^f \\ x_t^f &= \rho x_{t-1}^f + \sigma \varphi_e \varepsilon_{x,t}^f\end{aligned}$$

- Shocks are $i.i.d.$ within each country
- Shocks are correlated across countries
  - $\rho_c = corr(\varepsilon_{c,t}^h, \varepsilon_{c,t}^f)$
  - $\rho_x = corr(\varepsilon_{x,t}^h, \varepsilon_{x,t}^f)$

Calibration

δ	γ	ψ	θ	μ_c	σ	ρ	φ_e	ρ_x	ρ_c
.998	4.25	2	−6.5	.0015	.0068	.987	.048	1	.3

$$m_{t+1}^i = \theta \log \delta - \frac{\theta}{\psi} \Delta c_t^i + (\theta - 1) \log R_{c,t+1}^i$$

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$$x_t^i = \rho x_{t-1}^i + \sigma \varphi_e \varepsilon_{x,t}^i$$

Preferences:

- Low risk aversion (γ)
- IES from Bansal, Gallant and Tauchen (2004)
- Monthly model: high discounting

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x^i_t &= \rho x^i_{t-1} + \sigma \varphi_e \varepsilon^i_{x,t}
\end{aligned}
$$

Consumption process:

- Average consumption growth $\approx 2\%$
- Standard deviation of consumption growth $\approx 2.5\%$
- Variance explained by long run risk $\approx 7 - 8\%$

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Cross correlations of shocks:

- Correlation of consumption growths ≈ 0.3

Three ingredients

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  1. Use Epstein and Zin preferences:

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  - This paper: $m_{t+1}^{i} = E\left[\widetilde{g}(\Delta c_{t+1}^{i}, \Delta c_{t+2}^{i}, \Delta c_{t+3}^{i}, ...)|I_{t+1}\right]$

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  Alter the conditional distribution of $\left(\Delta c^h, \Delta c^f\right)$:

$$\begin{aligned} \Delta c^i_{t+1} &= \mu_c + x^i_t + \sigma \varepsilon^i_{c,t+1} \\ x^i_{t+1} &= \rho^i x^i_t + \sigma \varphi_e \varepsilon^i_{x,t+1} \end{aligned}$$

  by assuming

  2. High persistence ρ^i
  3. High cross country correlation $corr\left(\varepsilon^h_{x,t+1}, \varepsilon^f_{x,t+1}\right)$

Stochastic discount factors

$$m^i_{t+1} = \theta \log \delta - \frac{1}{\psi} x^i_t - \gamma \sigma \varepsilon^i_{c,t+1} + \frac{\delta(1-\gamma\psi)}{\psi(1-\rho\delta)} \sigma \varphi_e \varepsilon^i_{x,t+1}$$

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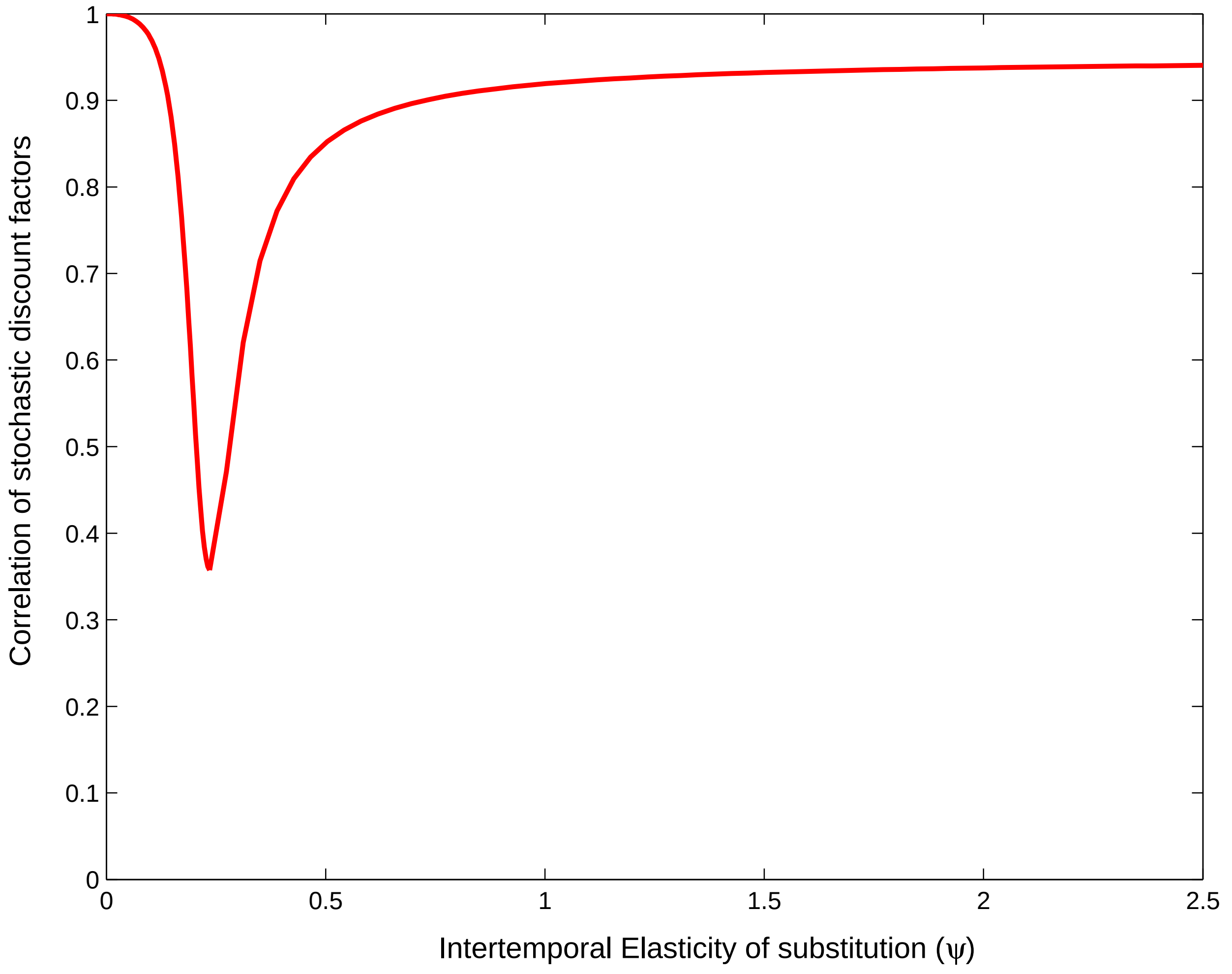

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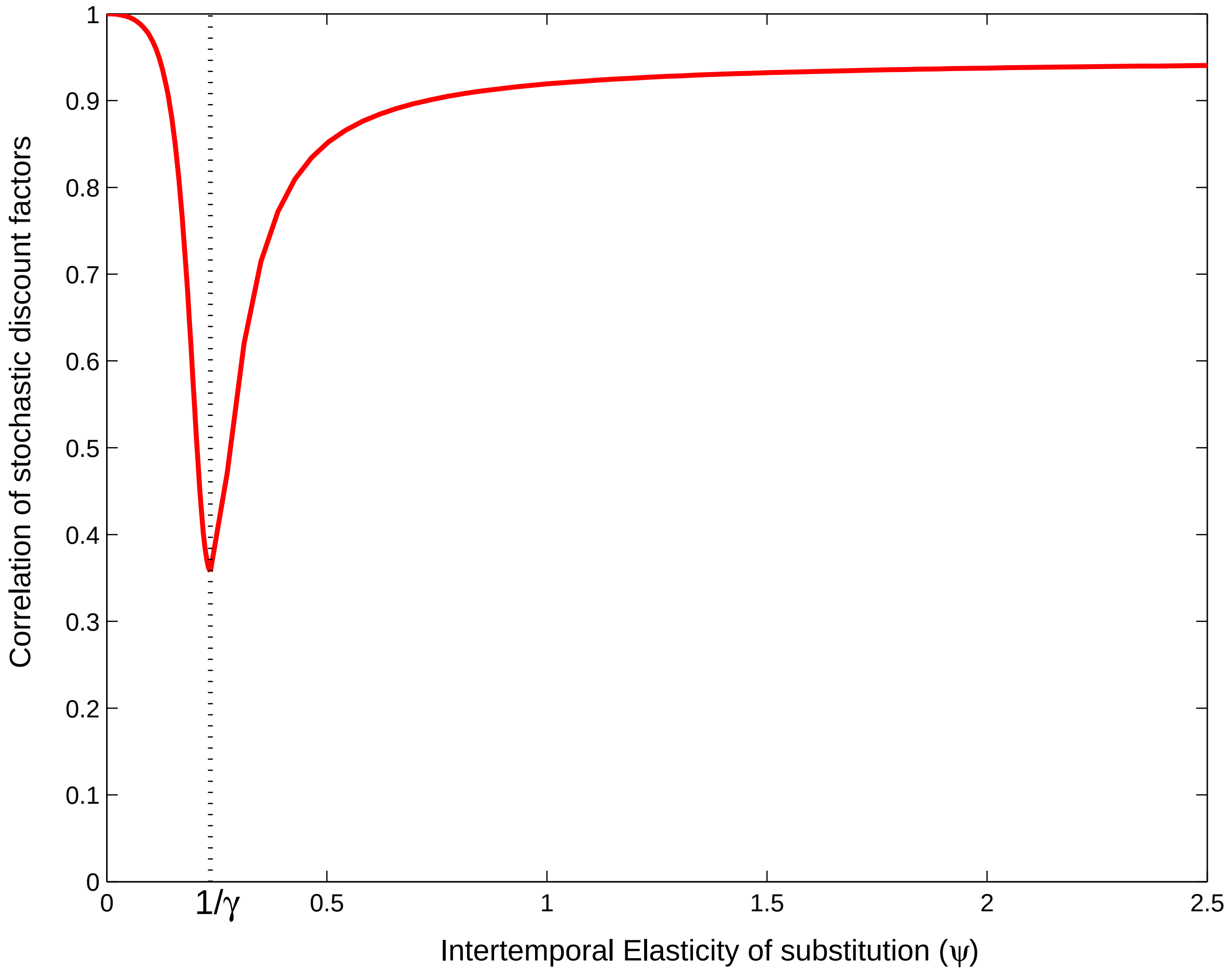

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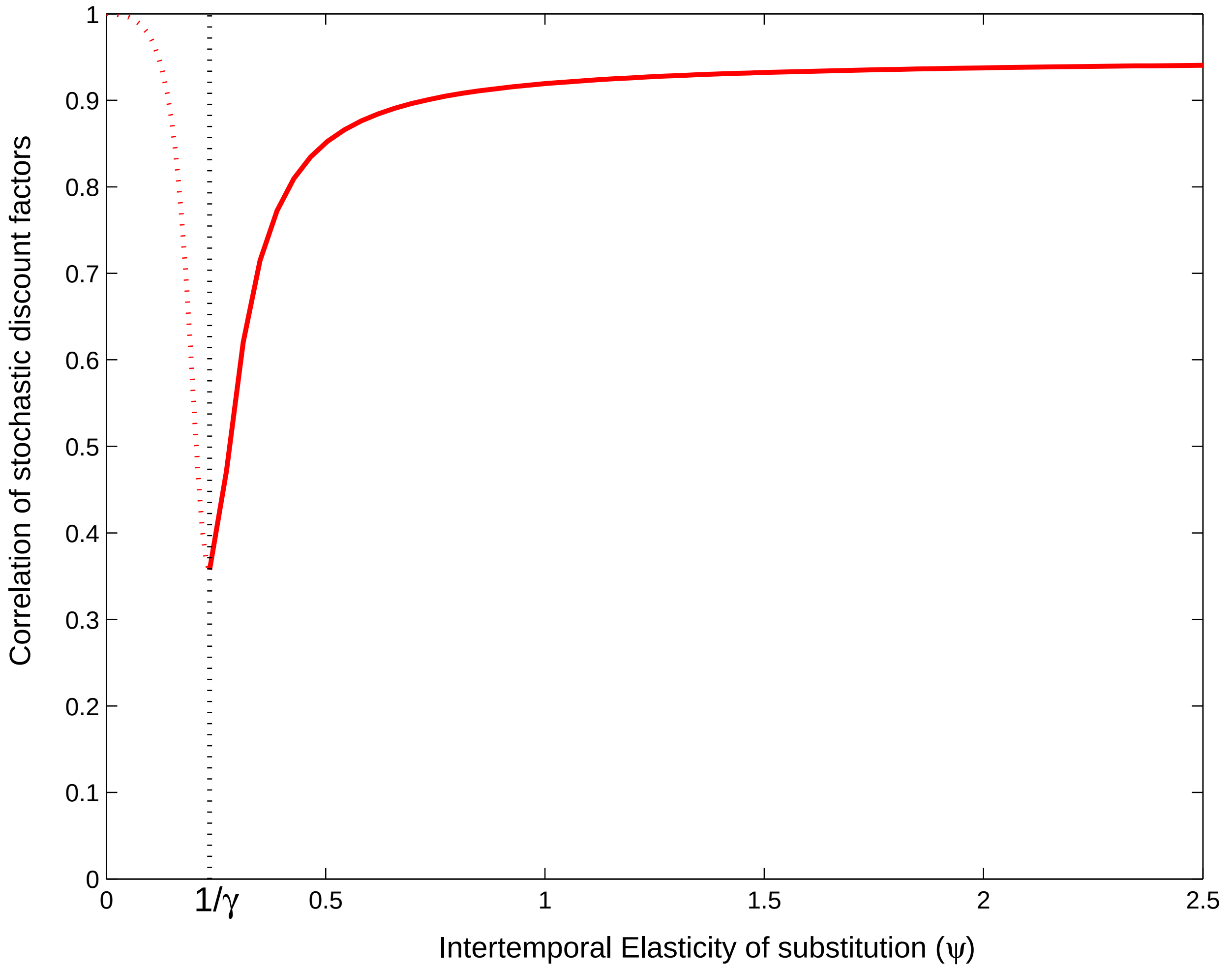

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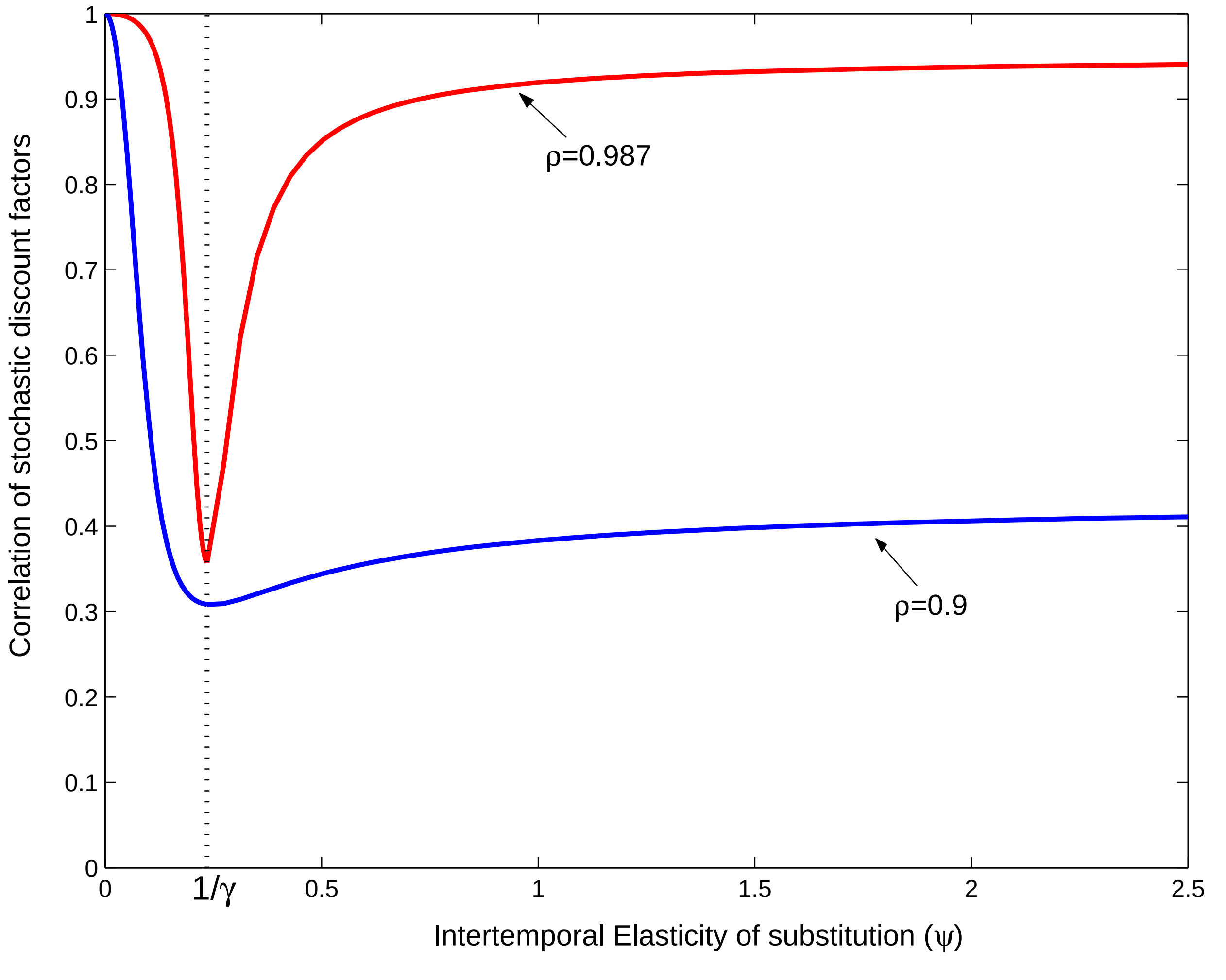

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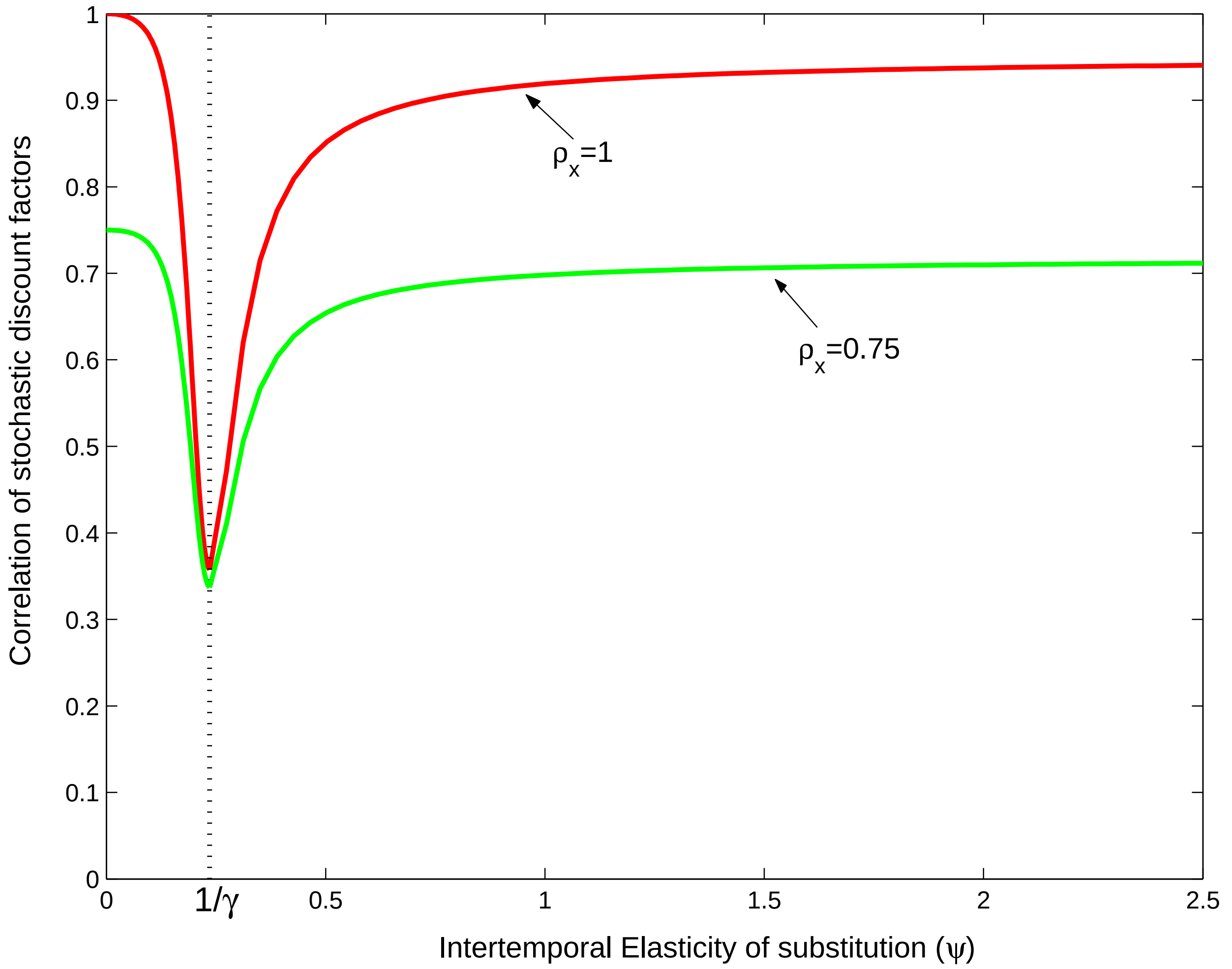

Exchange rate depreciation

$$Var\left(\frac{e_{t+1}}{e_t}\right) = \frac{2(1-\rho_x)}{\psi^2}\left\{\frac{1}{1-\rho^2} + \left[\frac{\delta(1-\gamma\psi)}{(1-\rho\delta)}\right]^2\right\}\varphi_e^2\sigma^2 + 2\gamma^2(1-\rho_c)\sigma^2$$

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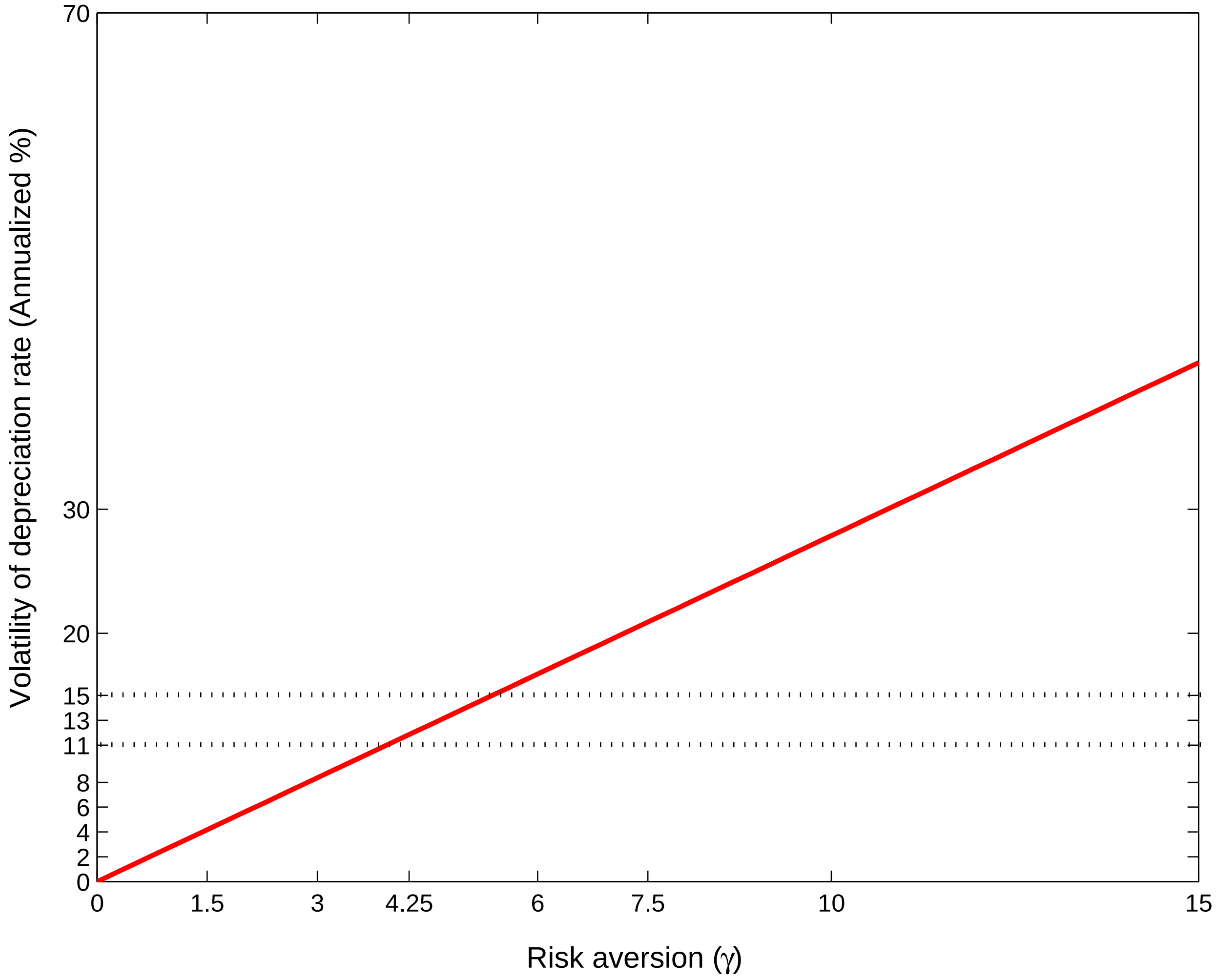

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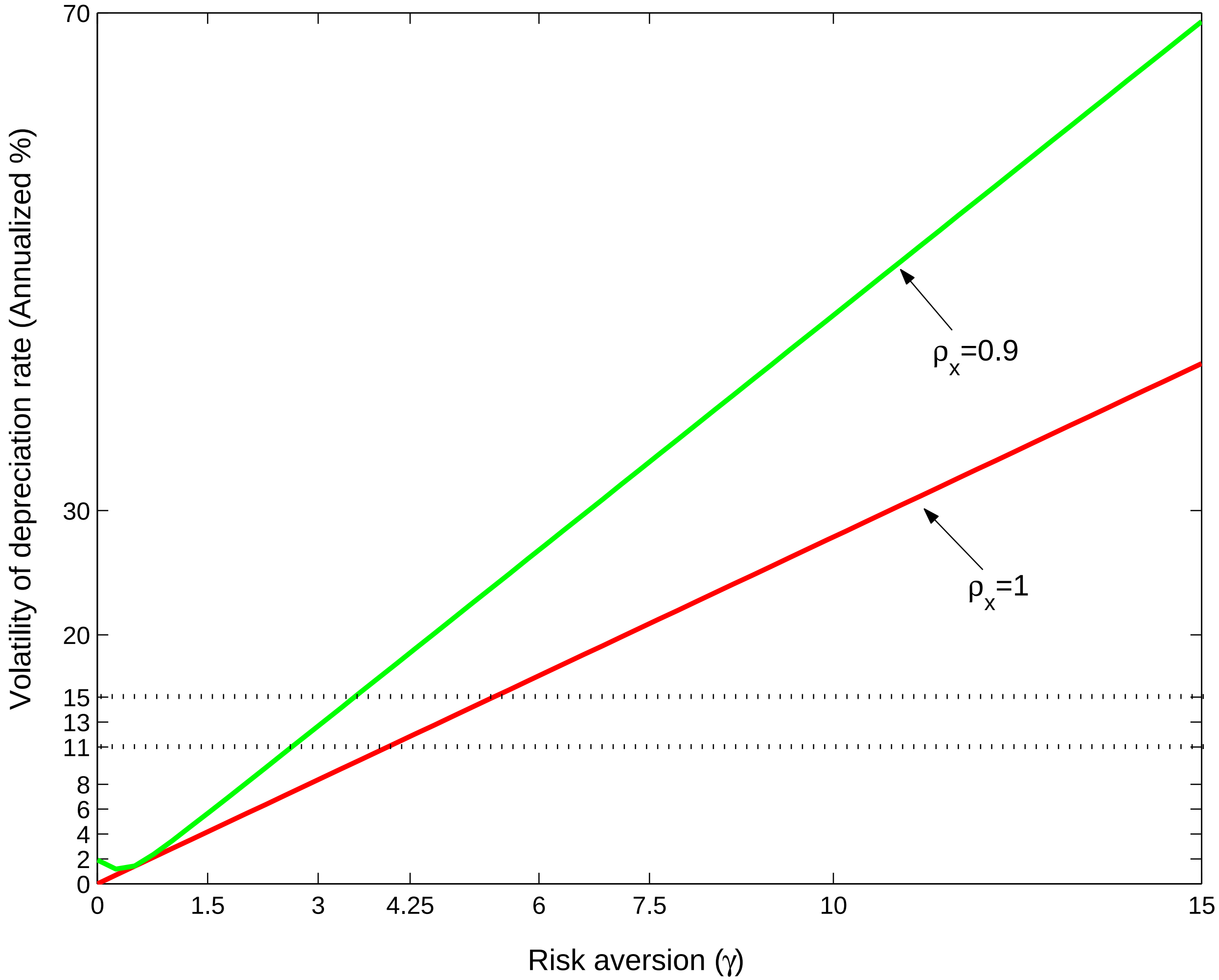

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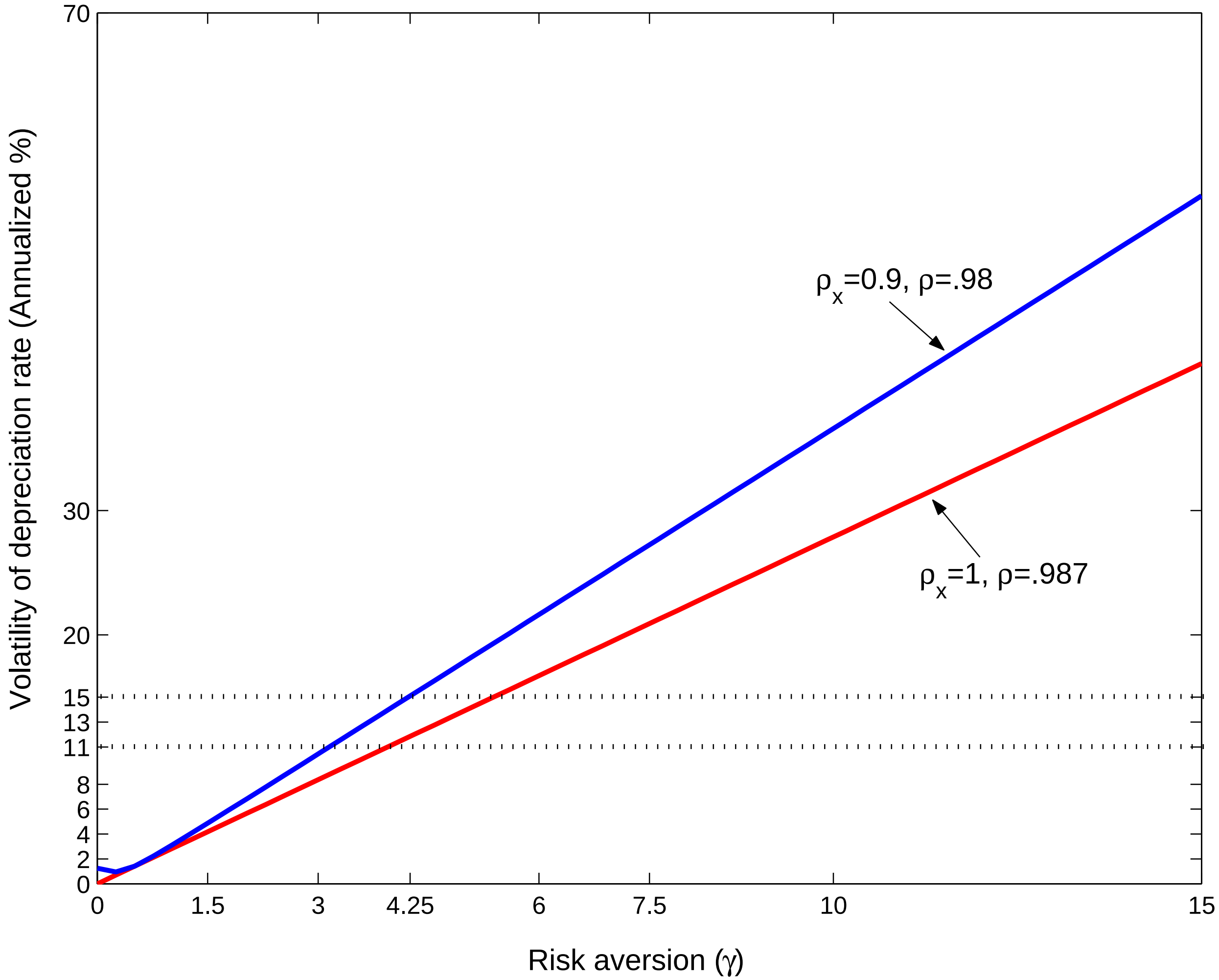

Every assumption counts

- Ingredients needed to solve the puzzle:
  1. Disentangle elasticity of substitution from risk aversion
  2. Highly persistent predictable component
  3. Highly correlated predictable components

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- Can we match key moments of international financial markets?

Introducing dividends

- The system becomes

$$\begin{aligned}\Delta c_t^i &= \mu_c + x_{t-1}^i + \sigma \varepsilon_{c,t}^i \\ \Delta d_t^i &= \mu_d + \lambda x_{t-1}^i + \sigma \varphi_d \varepsilon_{d,t}^i \\ x_t^i &= \rho x_{t-1}^i + \sigma \varphi_e \varepsilon_{x,t}^i\end{aligned}$$

$$\forall i \in \{h, f\}$$

- Shocks are $i.i.d.$ within each country
- Shocks are correlated across countries

Introducing dividends

- The system becomes

$$\begin{aligned}\Delta c_t^i &= \mu_c + x_{t-1}^i + \sigma \varepsilon_{c,t}^i \\ \Delta d_t^i &= .0007 + 3 \cdot x_{t-1}^i + \sigma \cdot 5 \cdot \varepsilon_{d,t}^i \\ x_t^i &= \rho x_{t-1}^i + \sigma \varphi_e \varepsilon_{x,t}^i\end{aligned}$$

$$\forall i \in \{h, f\}$$

- Shocks are $i.i.d.$ within each country
- Shocks are correlated across countries
- Calibrate coefficients of dividend growth to match:
  - Average dividend growth $\approx 1\%$
  - Standard deviation of dividend growth $\approx 12\%$
  - Leverage is 5
  - Small correlation of dividend growths: $corr\left(\varepsilon_{d,t}^h, \varepsilon_{d,t}^f\right) \approx 0$

Introducing dividends: results

	US	UK	Model
$\rho\left(m^h, m^f\right)$	-	-	0.93
$\sigma\left(\frac{e_{t+1}}{e_t}\right)$	11.21		11.83
$E\left(r_d - r_f\right)$	7.02	9.17	7.01
$\sigma\left(r_d - r_f\right)$	17.13	22.83	19.60
$\rho\left(r_d^h - r_f^h, r_d^f - r_f^f\right)$	0.60		0.58
$E\left(r_f\right)$	1.47	1.62	1.33
$\sigma\left(r_f\right)$	1.53	2.92	1.19
$\rho\left(r_f^h, r_f^f\right)$	0.65		1.00

Introducing stochastic volatility

- The system becomes $\forall i \in \{h, f\}$

$$
\begin{aligned}
\Delta c_t^i &= \mu_c + x_{t-1}^i + \sigma_t \varepsilon_{c,t}^i \\
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\left(\sigma_t^2\right)^i &= \overline{\sigma}^2 + \nu_1 \left[\left(\sigma_{t-1}^2\right)^i - \overline{\sigma}^2 \right] + \sigma_w \varepsilon_{\sigma,t}^i
\end{aligned}
$$

- Shocks are $i.i.d.$ within each country
- Shocks are correlated across countries
- Guidelines to calibrate stochastic volatility given by

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Var_t\left(r_{t+1}^d\right) = (1-\nu_1)k_0 + \nu_1 Var_{t-1}\left(r_t^d\right) + k_1 \sigma_w \varepsilon_{\sigma,t}
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Introducing stochastic volatility

- The system becomes $\forall i \in \{h, f\}$

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\left(\sigma_t^2\right)^i &= .0068^2 + .96\left[\left(\sigma_{t-1}^2\right)^i - .0068^2\right] + .23e^{-5} \varepsilon_{\sigma,t}^i
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- Cross correlation of $\varepsilon_{\sigma,t}$ has small impact on results.

Results

	US	UK	No stoch vol	W/Stoch vol
$\rho\left(m^h, m^f\right)$	-	-	0.93	0.92
$\sigma\left(\frac{e_{t+1}}{e_t}\right)$	11.21		11.83	12.67
$E\left(r_d - r_f\right)$	7.02	9.17	7.01	7.03
$\sigma\left(r_d - r_f\right)$	17.13	22.83	19.60	19.41
$\rho\left(r_d^h - r_f^h, r_d^f - r_f^f\right)$	0.60		0.58	0.57
$E\left(r_f\right)$	1.47	1.62	1.33	1.33
$\sigma\left(r_f\right)$	1.53	2.92	1.19	1.22
$\rho\left(r_f^h, r_f^f\right)$	0.65		1.00	0.98
$\sigma\left(r_c\right)$	-	-	4.74	4.75
$\rho\left(r_c^h, r_c^f\right)$	-	-	0.85	0.85

Estimating long run risks

- Can we justify high persistence and high correlation:

$$\begin{bmatrix} \Delta c_t^h \\ \Delta c_t^f \end{bmatrix} = \begin{bmatrix} \mu_c^h \\ \mu_c^f \end{bmatrix} + \begin{bmatrix} x_{t-1}^h \\ x_{t-1}^f \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ \rho_c & \sqrt{1-\rho_c^2} \end{bmatrix} \begin{bmatrix} \sigma^h \varepsilon_{c,t}^h \\ \sigma^f \varepsilon_{c,t}^f \end{bmatrix}$$

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- Roadmap:
  1. Use consumption data only
     - Use Kalman filter to get a recursive representation of the likelihood function
     - Multi-country provide inconclusive evidence
  2. Use consumption and price data
     - Focus on a set of moments of interest
     - Sharply identify departure from *i.i.d.*

Consumption data only: results

- Home = US and Foreign = UK
- Quarterly data from 1970 to 1998.

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Real Data	0.909	0.940	0.897	0.208
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(T=10000)	[0.983,0.990]	[0.983,0.991]	[0.953,1.0]	[0.285,0.318]

Consumption data only: results (cont'd)

- ... other parameters

	φ_e^h	φ_e^f	σ^h	σ^f
Calibrated	0.048	0.048	68	68
Real Data	0.351	0.184	38.1	77.9
(T=120)	[0.000,0.424]	[0.000,1.092]	[28.6,38.1]	[72.3,78.0]
Simulations	0.142	0.119	66.1	66.1
(T=120)	[0.000,0.475]	[0.000,1.165]	[51.2,71.6]	[37.4,68.2]
Simulations	0.049	0.049	67.9	68.0
(T=10000)	[0.039,0.059]	[0.041,0.057]	[67.1,69.1]	[67.3,68.7]

Does adding other countries help?

	US, UK and Germany	US, UK and Japan	US, UK, Germany and Japan
ρ^{US}	0.911	0.906	0.919
	[.530, .990]	[.552, .984]	[.796, .996]
ρ^{UK}	0.928	0.947	0.934
	[.049, .986]	[.214, .978]	[.746, .994]
ρ^{Ger}	0.932	-	0.993
	[.274, .985]	-	[.614, .996]
ρ^{Jpn}	-	0.989	0.99
	-	[.215, .989]	[.504, .998]
$\rho_x^{US,UK}$	0.920	0.973	0.913
	[.890, .999]	[.861, .999]	[.700, .999]
$\rho_x^{US,Ger}$	0.899	-	0.891
	[.874, 1.000]	-	[.700, .997]
$\rho_x^{US,Jpn}$	-	0.991	0.905
	-	[.877, 1.000]	[.701, 1.000]

Introducing prices

- Consumption information is not enough: we need price information.
- Select 22 moments to match:
  - Consumption: $Var\left(\Delta c_t^h\right)$, $cov\left(\Delta c_t^h, \Delta c_{t-1}^h\right)$, $cov\left(\Delta c_t^h, \Delta c_{t-2}^h\right)$, $Var\left(\Delta c_t^f\right)$, $cov\left(\Delta c_t^f, \Delta c_{t-1}^f\right)$, $cov\left(\Delta c_t^f, \Delta c_{t-2}^f\right)$, $cov\left(\Delta c_t^h, \Delta c_t^f\right)$, $cov\left(\Delta c_t^h, \Delta c_{t-1}^f\right)$, $cov\left(\Delta c_{t-1}^h, \Delta c_t^f\right)$
  - Dividend: $Var\left(\Delta d_t^h\right)$, $Var\left(\Delta d_t^f\right)$, $cov\left(\Delta d_t^h, \Delta d_t^f\right)$
  - Excess returns: $Var\left(r_t^{d,h} - r_t^{f,h}\right)$, $cov\left(r_t^{d,h} - r_t^{f,h}, r_{t-1}^{d,h} - r_{t-1}^{f,h}\right)$, $Var\left(r_t^{d,f} - r_t^{f,f}\right)$, $cov\left(r_t^{d,f} - r_t^{f,f}, r_{t-1}^{d,f} - r_{t-1}^{f,f}\right)$, $cov\left(r_t^{d,h} - r_t^{f,h}, r_t^{d,f} - r_t^{f,f}\right)$
  - Risk free rates: $Var\left(r_t^{f,h}\right)$, $Var\left(r_t^{f,f}\right)$
  - Depreciation rate: $Var\left(\frac{e_{t+1}}{e_t}\right)$
- Use Simulated Method of Moments

Prices and consumption: results

	Consumption only		Whole Model	
	Point Estimate	95% CI	Point Estimate	95% CI
ρ^h	0.736	[0.349,0.996]	0.997	[0.927,1.000]
ρ^f	0.904	[0.015,0.997]	0.996	[0.912,0.999]
φ_e^h	1.422	[0.190,17.318]	0.024	[0.005,0.213]
φ_e^f	0.182	[0.000,3.502]	0.041	[0.003,0.157]
σ^h	27.629	[4.527,34.799]	32.792	[28.036,36.676]
σ^f	79.916	[44.407,87.421]	79.569	[69.767,88.879]
ρ_x^{hf}	0.999	[0.353,1.000]	0.998	[0.853,1.000]
ρ_c^{hf}	0.222	[-0.988,0.999]	0.349	[0.215,0.492]
	Unconditional moments		Unconditional moments	
$\sigma\left(\frac{e_{t+1}}{e_t}\right)$	-		11.692	
$\rho\left(m^h, m^f\right)$	-		0.922	

Future Research

What's next?

- Extend the list of moments that can be matched
- Relax assumption of complete home bias in consumption
- Where does x_t come from?

Extending the basic model

- Set $\Psi = 1$ (risk sensitive preferences):

$$U_t^i = \log c_t^i + \frac{\delta}{(1-\gamma^i)(1-\delta)} \log E_t \left[\exp \left\{ (1-\gamma^i)(1-\delta) U_{t+1}^i \right\} \right]$$

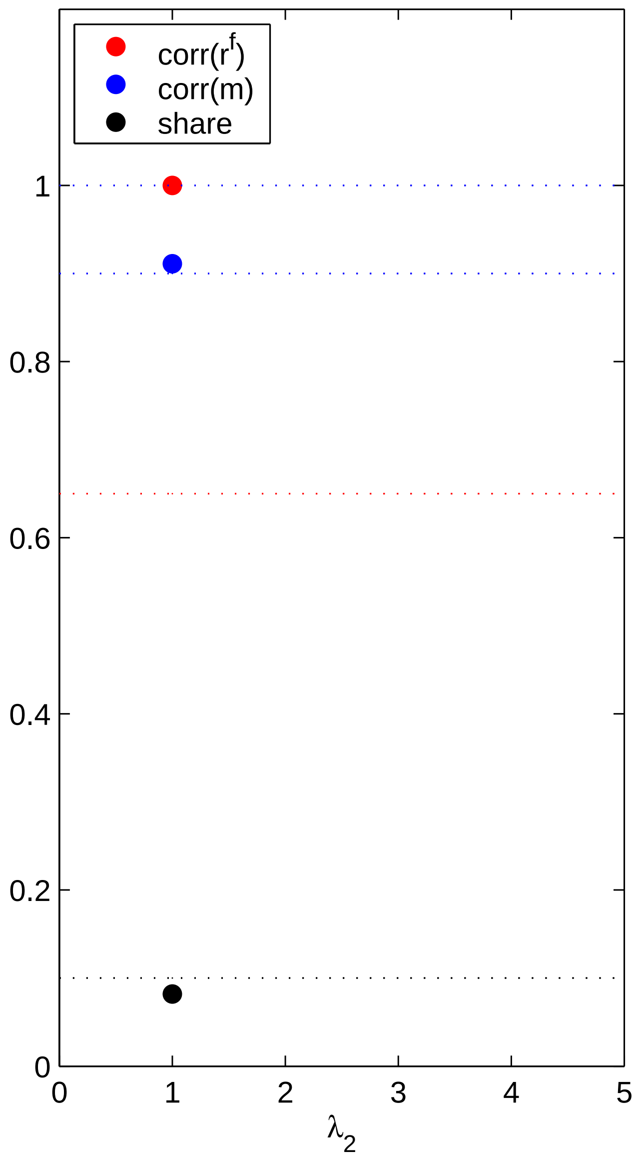
- Two common factors

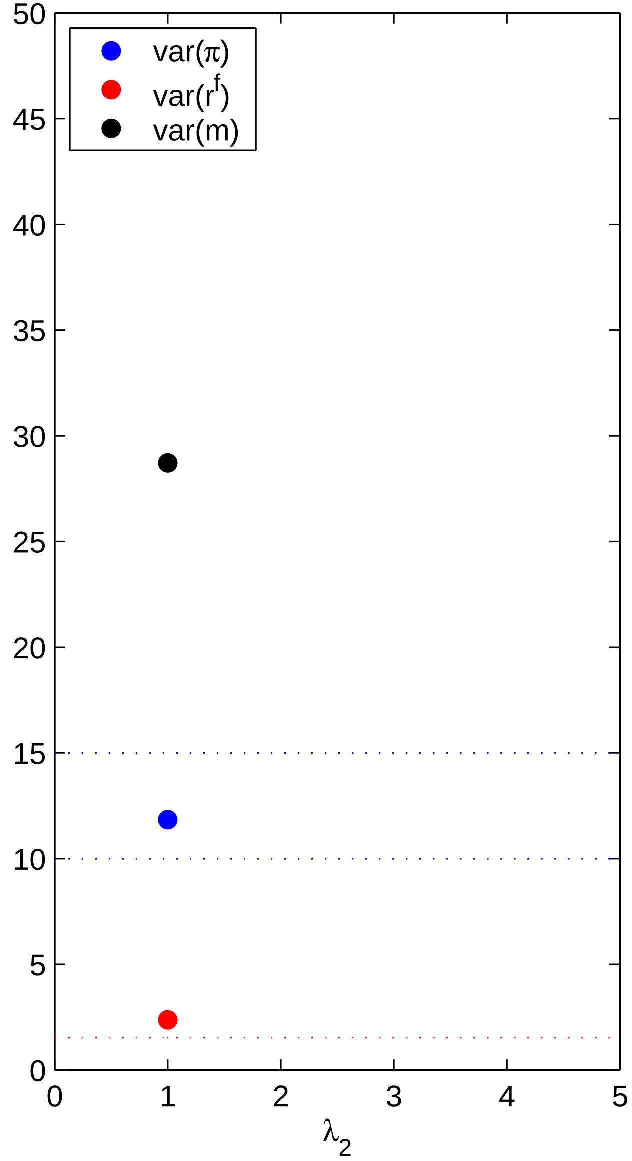
$$\begin{aligned}
\Delta c_t^h &= \mu^{c,h} + \lambda_1 z_{1,t-1} + \lambda_2 z_{2,t-1} + \varepsilon_t^{c,h} \\
\Delta c_t^f &= \mu^{c,f} + \lambda_2 z_{1,t-1} + \lambda_1 z_{2,t-1} + \varepsilon_t^{c,f} \\
z_{1,t} &= \rho_1 z_{1,t-1} + \varepsilon_t^{z_1} \\
z_{2,t} &= \rho_2 z_{2,t-1} + \varepsilon_t^{z_2}
\end{aligned}$$

- If $\lambda_1 = 1$, $\lambda_2 = 0$, $\rho\left(\varepsilon_t^{c,h}, \varepsilon_t^{c,f}\right) = 0.3$, $\rho\left(\varepsilon_t^{z_1}, \varepsilon_t^{z_2}\right) = 1$, $\rho\left(\varepsilon_t^{c,h}, \varepsilon_t^{z_1}\right) = \rho\left(\varepsilon_t^{c,f}, \varepsilon_t^{z_2}\right) = 0$, then we have the basic model.

Two factors

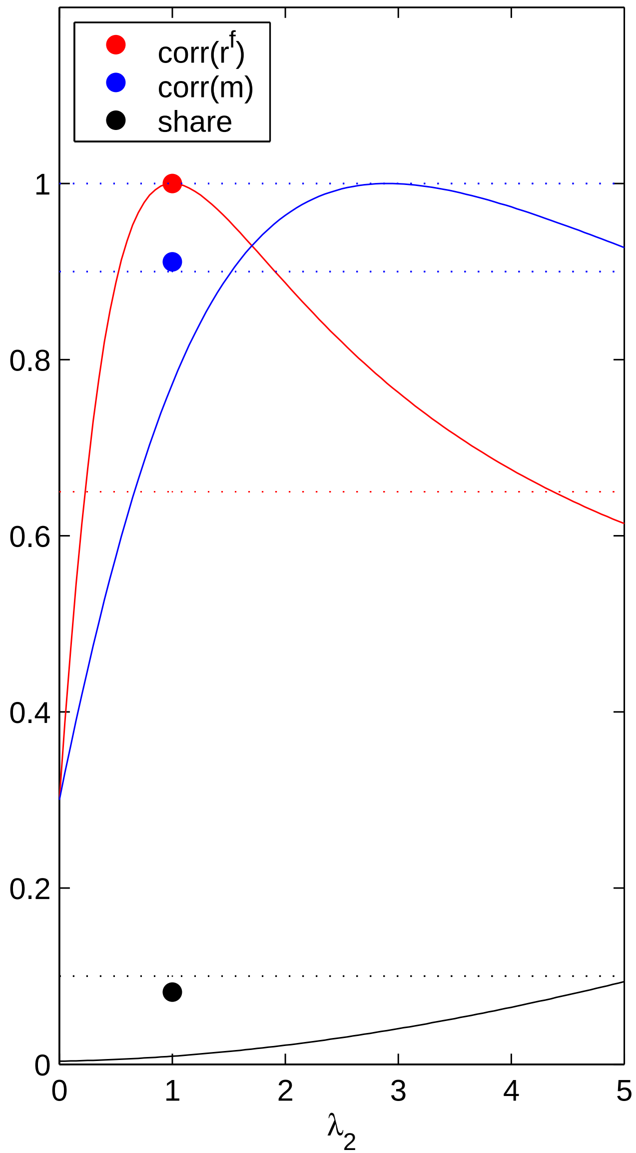
- One factor: yields are perfectly correlated.

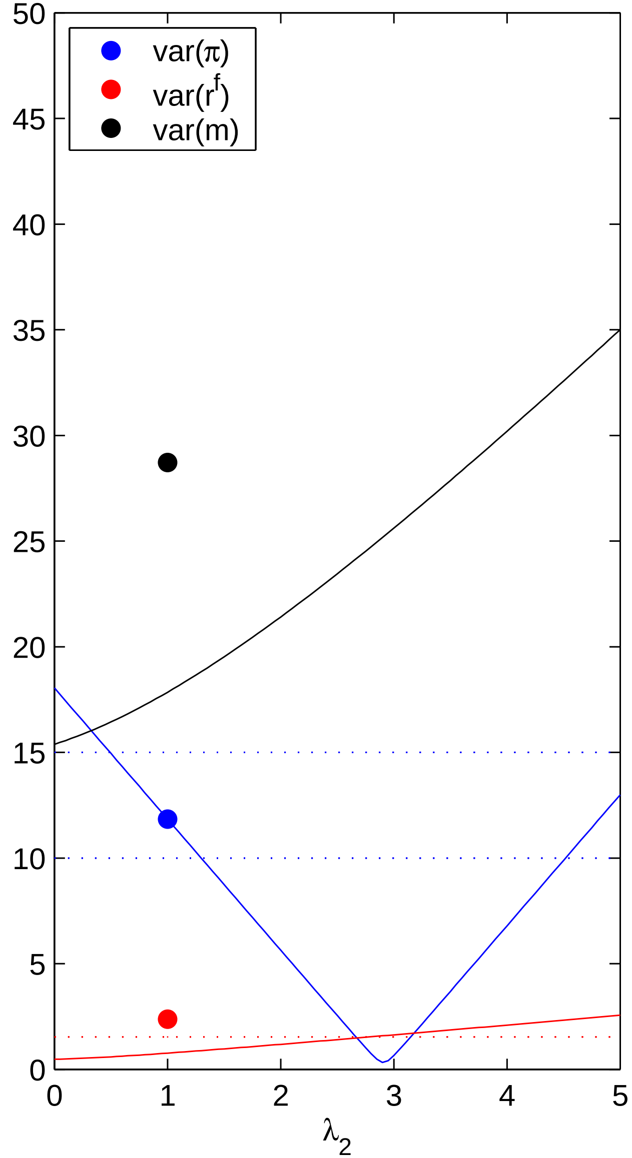




Two factors

- Two factors: low $corr(r^{f^h}, r^{f^f})$ and high $corr(m^h, m^f)$.





Yields and long run risks

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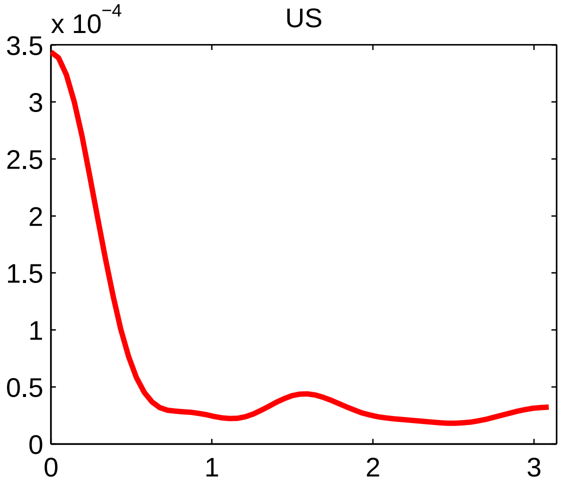

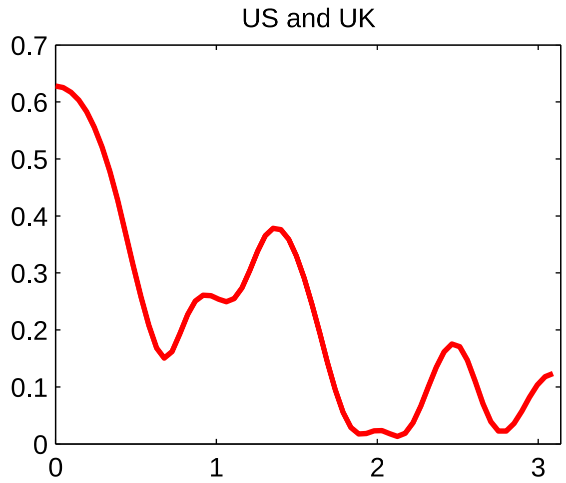

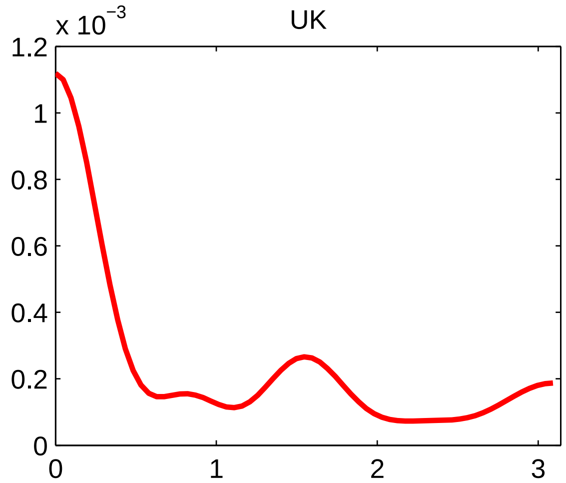

Concluding remarks

- Key ingredients
  - Separate elasticity of substitution from risk aversion
  - Highly persistent predictable component
  - Highly correlated predictable components
- It is possible to explain
  - low volatility of the depreciation of the US dollar
  - high equity premium
  - high persistence of the risk free rate
  - high correlation of int'l financial markets
  - correlation of bonds
  - low correlation of consumption growths
  - low persistence of consumption growths