‘Learning, Long-Run Risks and Asset Price Jumps’ by R. Bansal and I. Shaliastovich

Discussion by Riccardo Colacito

UNC-Chapel Hill, visiting NYU-Stern
Many of the statistical challenges that plague econometricians presumably also plague market participants. Naive application of rational expectations equilibrium concepts may endow investors with too much knowledge about future growth prospects.

Hansen, Heaton, and Li (2005)
In formulas

\[ \Delta c_t = \mu_c + x_{t-1} + \epsilon_t^c \]
\[ x_t = \rho x_{t-1} + \epsilon_t^x \]

It may be hard to filter out a small persistent component from an almost i.i.d. process (Hansen (2007))
Bansal and Yaron (2004)

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- It may be hard to filter out a small persistent component from an almost i.i.d. process (Hansen (2007))
- Nevertheless agents are endowed with its full knowledge
Investors use consumption and dividend data to extract information about $x_t$

- Hansen and Sargent (2007)
Investors use consumption and dividend data to extract information about $x_t$

- Hansen and Sargent (2007)

This paper:
- CLL/HS are available for free
- BY can be purchased for a fee
Setup of the economy

- Representative agent chooses
  - consumption $C_t$
  - signal acquisition $s_t = \{0, 1\}$
- Investor has risk-sensitive preferences
  \[
  V_t = \log U_t = (1 - \beta) \log C_t + \beta \theta \log E_t^{s_t} \exp \left\{ \frac{V_{t+1}}{\theta} \right\}, \quad \theta = \frac{1}{1-\gamma}
  \]
- Budget constraint
  \[
  C_t = Y_t (1 - \chi s_t)
  \]
- Income process contains long-run risk
  \[
  \Delta y_{t+1} = \mu + x_t + \sigma \eta_{t+1} \\
  x_{t+1} = \rho x_t + \varphi_e \sigma \varepsilon_{t+1}
  \]
Economy starts off with $\hat{x}_t = x_t$
Economy starts off with $\hat{x}_t = x_t$

Innovations representation is available for free

\[ \Delta y_{t+1} = \mu + \hat{x}_t + u_{t+1} \]
\[ \hat{x}_{t+1} = \rho \hat{x}_t + K_t u_{t+1} \]
\[ K_t = \frac{\rho \omega_t^2}{\omega_t^2 + \sigma^2} \]
\[ \omega_{t+1}^2 = \sigma^2 \left( \varphi_e^2 + \rho^2 \frac{\omega_t^2}{\omega_t^2 + \sigma^2} \right) \]
Information dynamics

- Economy starts off with $\hat{x}_t = x_t$
- Innovations representation is available for free

\[
\begin{align*}
\Delta y_{t+1} & = \mu + \hat{x}_t + u_{t+1} \\
\hat{x}_{t+1} & = \rho \hat{x}_t + K_t u_{t+1} \\
K_t & = \frac{\rho \omega_t^2}{\omega_t^2 + \sigma^2} \\
\omega_{t+1}^2 & = \sigma^2 \left( \varphi_e^2 + \rho^2 \frac{\omega_t^2}{\omega_t^2 + \sigma^2} \right)
\end{align*}
\]

- The economy is periodically reset to BY: $\hat{x}_t = x_t$
Without signal purchasing

\[
m_{t+1} = E_t^{s^t} m_{t+1} - u_{t+1} + (\gamma - 1) BK_t u_{t+1} - (\gamma - 1) (f_{t+1} - E_t^{s^t} f_{t+1})
\]

where \( B \approx 260 \).
The model

**Stochastic Discount Factor**

**With** signal purchasing

\[
m_{t+1} = E_t^s m_{t+1} - u_{t+1} + (\gamma - 1) BK_t u_{t+1} - (\gamma - 1) (f_{t+1} - E_t^s f_{t+1})
\]

short-run risk  
long-run risk  
variance of filtering error

\[- (\gamma - 1) B (x_{t+1} - \hat{x}_{t+1})\]

revision of \(x\)

where \(B \approx 260\).
## Results

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Kurtosis</th>
<th>Freq Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market Return</td>
<td>7.98</td>
<td>16.61</td>
<td>21.19</td>
<td>3.38</td>
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<tr>
<td><strong>Simulation:</strong></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td><strong>Constant Volatility:</strong></td>
<td></td>
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<tr>
<td>Market Return</td>
<td>6.53</td>
<td>10.34</td>
<td>16.27</td>
<td>4.41</td>
</tr>
<tr>
<td>Market Return, no signal</td>
<td>6.75</td>
<td>10.43</td>
<td>3.01</td>
<td>&gt; 47.75</td>
</tr>
<tr>
<td><strong>Time-Varying Volatility:</strong></td>
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<tr>
<td>Market Return</td>
<td>6.64</td>
<td>10.31</td>
<td>34.20</td>
<td>3.08</td>
</tr>
<tr>
<td>Market Return, no signal</td>
<td>6.87</td>
<td>9.40</td>
<td>3.26</td>
<td>&gt; 48.64</td>
</tr>
</tbody>
</table>
Troublesome assumptions:

- Is there really a market for \( x_t \)?
- How do we think about the price of acquiring \( x_t \)?

Possible re-interpretation

- Periodically acquiring a more precise signal about \( x_t \)
- Would it still be possible to engineer large price jumps?
Where does $x_t$ show up?

- Dividends are exposed to long-run risks
- Price-dividend ratios, yield curve, consumption-output ratio contain information about $x_t$
  - Bansal, Kiku, and Yaron (2007)
  - Colacito and Croce (2007)
- Exposure of international quantities to long-run risks may explain exchange rate anomalies
  - Colacito and Croce (2007, 2008)
  - Bansal and Shaliastovich (2007)
International consumption data

\[
\begin{bmatrix}
\Delta c^1_t & \Delta c^2_t & \cdots & \Delta c^N_t
\end{bmatrix}' = \mu_c + x_t + \Sigma_c \varepsilon^c_t
\]

\[
x_t = \rho x_t + \sigma_x \varepsilon^x_t
\]

- \(\Sigma_c\) is diagonal
- \(x_t\) is common low frequency component
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Learning, Long-Run Risks, and Jumps
More series
More series

Re-interpreting

Signal extraction

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More series

Re-interpreting Signal extraction

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Re-interpreting Optimal filtering periods

Optimal number of filtering periods

Jump statistics and probability of jumps

Figure 3: Annual jump statistics (solid line with values on the left Y-axis) and the predicted probability of large price moves (dashed line with values on the right Y-axis) based on the macroeconomic, aggregate uncertainty. Stars indicate years with at least one large price move.

Figure 4: Optimal number of filtering periods, in years, in the constant volatility case as a function of the information cost parameter $\chi$. The risk aversion parameter $\gamma$ is set at 15 (lower dotted line), 10 (middle solid line) and 5 (upper dashed line).
Do stocks and consumption jump together?
Do stocks and consumption jump together?

Non-simultaneous jumps  Stock returns vs consumption growth

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Do stock returns and risk-free rates jump together?

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Understanding investors’ learning behavior in long-run risks model is important.

Results probably survive without assuming that $x_t$ is periodically observable.

How do we calibrate the cost?

How do we explain non-simultaneous jumps in returns?
Concluding remarks

- Understanding investors’ learning behavior in long-run risks model is important
- Results probably survive without assuming that $x_t$ is periodically observable
- How do we calibrate the cost?
- How do we explain non-simultaneous jumps in returns?
- How do we cope with parameter uncertainty?