Housing as a Measure for Long-Run Risk in Asset Pricing

by José Fillat

Discussion by

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UNC, Chapel Hill
The long-run risks recipe

1. Epstein-Zin preferences: large elasticity of intertemporal substitution.

2. Small, but highly persistent predictive component of consumption growth.

3. Euler equations hold at monthly frequency.
The long-run risks recipe

1. Epstein-Zin preferences: large elasticity of intertemporal substitution.

2. Small, but highly persistent predictive component of consumption growth.

3. Euler equations hold at monthly frequency.

This combination can explain:

- equity premium puzzle (Bansal and Yaron (2004))
- value premium (Kiku (2006))
- exchange rates behavior (Colacito and Croce (2008))
- ...
The long-run risks recipe

1. Epstein-Zin preferences: large elasticity of intertemporal substitution.

2. Small, but highly persistent predictive component of consumption growth. ✓

3. Euler equations hold at quarterly frequency. ✓

This combination can explain:
- equity premium puzzle (Bansal and Yaron (2004))
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- ...
Battle plan

- The model
- Pricing implications
- Measuring long-run risks
- Suggestions
Setup of the economy

- Preferences à la Hansen and Sargent

\[
U_t = (1 - \beta) \log \left( C_t^\frac{\varepsilon-1}{\varepsilon} + w_t S_t^\frac{\varepsilon-1}{\varepsilon} \right)^\frac{\varepsilon}{\varepsilon-1} + \beta \theta \log \mathbb{E}_t \exp \left\{ \frac{U_{t+1}}{\theta} \right\}
\]
Setup of the economy

- Preferences *à la* Hansen and Sargent

\[ U_t = (1 - \beta) \log \left( C_t^{\varepsilon-1} + w_t S_t^{\varepsilon-1} \right) \frac{\varepsilon}{\varepsilon-1} + \beta \theta \log E_t \exp \left\{ \frac{U_{t+1}}{\theta} \right\} \]

- Budget constraint

\[ p_t^C C_t + p_t^S S_t + q_t^C \theta_t^C + q_t^S \theta_t^S = \left( q_t^C + p_t^C C_t \right) \theta_{t-1}^C + \left( q_t^S + p_t^S S_t \right) \theta_{t-1}^S \]
Preferences à la Hansen and Sargent

\[ U_t = (1 - \beta) \log \left( \frac{C_t^{\varepsilon-1}}{\varepsilon} + w_t S_t^{\varepsilon-1} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \beta \theta \log E_t \exp \left\{ \frac{U_{t+1}}{\theta} \right\} \]

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Consumption growth

\[ \Delta c_{t+1} = \mu_c + \phi^c x_t + \sigma^c \varepsilon_t^{c} \]

\[ x_t = \rho x_{t-1} + \sigma^x \varepsilon_t^{x} \]
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\Delta c_{t+1} = \mu_c + \phi^c x_t + \sigma^c \epsilon_{t+1}^c
\]

\[
x_t = \rho x_{t-1} + \sigma^x \epsilon_{t+1}^x
\]

- Expenditure share

\[
- \log \alpha_{t+1} = - \log p_{t+1}^C C_{t+1} / (p_{t+1}^C C_{t+1} + p_{t+1}^S S_{t+1})
\]
Setup of the economy

- **Preferences á la Hansen and Sargent**
  \[ U_t = (1 - \beta) \log \left( C_t^{\frac{\varepsilon - 1}{\varepsilon}} + w_t S_t^{\frac{\varepsilon - 1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon - 1}} + \beta \theta \log E_t \exp \left\{ \frac{U_{t+1}}{\theta} \right\} \]

- **Budget constraint**
  \[ p_t^C C_t + p_t^S S_t + q_t^C \theta_t^C + q_t^S \theta_t^S = \left( q_t^C + p_t^C C_t \right) \theta_{t-1}^C + \left( q_t^S + p_t^S S_t \right) \theta_{t-1}^S \]

- **Consumption growth**
  \[ \Delta c_{t+1} = \mu_c + \phi_c x_t + \sigma_c \varepsilon_{t+1} \]
  \[ x_t = \rho x_{t-1} + \sigma x \varepsilon_{t+1} \]

- **Expenditure share**
  \[ -\log \alpha_{t+1} = \mu^\alpha + \phi^\alpha x_{t+1} + x'_{t+1} \Psi x_{t+1} \]
The model

Pricing kernel

Stochastic Discount Factor

\[ m_{t+1} = \log \frac{\partial U_t / \partial C_{t+1}}{\partial U_t / \partial C_t} \]

Figure 7: Stochastic Discount Factor.

I present the results for three models. First, the model RU+housing corresponds to the model developed in Section 2, where agents have recursive preferences over a non-separable bundle of housing and non-housing consumption modeled separately. The second model, RU, corresponds to a model where agents have recursive preferences over non-housing consumption. It is a slight modification of Bansal and Yaron (2004) and Hansen, Heaton, and Li (2008), since they consider preferences over non-durables and services. The third model is the standard Lucas-Breeden power utility model. The risk prices associated with the shocks that drive the economy are summarized in Table 6. The first two columns show the local risk prices. Local risk prices are computed as the limit when the horizon shrinks to zero of the derivative of the risk premia with respect to the risk exposure; intuitively, is the price of 38

\( 0.5 / 12 \)
The model

Pricing kernel

\[ m_{t+1} = \log \beta - \Delta c_{t+1} + \log \frac{\exp \{U_{t+1}/\theta\}}{E_t \exp \{U_{t+1}/\theta\}} + \Delta \alpha_{t+1} \]
The model

Pricing kernel

Stochastic Discount Factor

\[ m_{t+1} = \log \beta - \Delta c_{t+1} + \log \frac{\exp \{ U_{t+1}/\theta \}}{E_t \exp \{ U_{t+1}/\theta \}} + \Delta \alpha_{t+1} \]
Compare two economies:

1. Preference for housing services is shut down.
2. Preference for housing services is turned on.
Comparison of SDF's
Comparison of SDF’s
Volatility of SDF’s

<table>
<thead>
<tr>
<th></th>
<th>No Durables</th>
<th>With Durables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(m_{t+1})$</td>
<td>53.7802</td>
<td>53.7853</td>
</tr>
</tbody>
</table>
### Volatility of SDF's

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</table>

### Equity premia

<table>
<thead>
<tr>
<th></th>
<th>1 Period</th>
<th>Limiting</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$R^1$</td>
<td>$R^o$</td>
</tr>
<tr>
<td>RU + housing</td>
<td>5.2780</td>
<td>7.1620</td>
</tr>
<tr>
<td>Rec. Ut.</td>
<td>5.2143</td>
<td>7.1017</td>
</tr>
<tr>
<td>P. Ut</td>
<td>-0.0000</td>
<td>0.9801</td>
</tr>
</tbody>
</table>
Estimating long-run risks

How does it compare to Bansal and Yaron’s calibration?

\[
\frac{9}{12} = 0.75 \approx 0.979
\]
Estimating long-run risks

- Estimate long-run risks from

\[- \log \alpha_t = \mu^\alpha + \phi^\alpha x_t\]

\[x_t = \rho^x x_{t-1} + \varepsilon^x_t\]
Estimating long-run risks

Estimate long-run risks from

\[- \log \alpha_t = 0.010 + \phi^\alpha x_t\]

\[(0.004)\]

\[x_t = 0.926 x_{t-1} + \varepsilon_t^x\]

\[(0.027)\]

How does it compare to Bansal and Yaron's calibration?

\[0.926 \approx 0.979\]
Estimating long-run risks

- Estimate long-run risks from

\[ -\log \alpha_t = 0.010 + \phi^\alpha x_t \]

(0.004)

\[ x_t = 0.926 x_{t-1} + \varepsilon_t^x \]

(0.027)

- How does it compare to Bansal and Yaron’s calibration?

\[ 0.926^{\frac{1}{3}} = 0.975 \approx 0.979 \]
Can $\alpha$ predict consumption?
Can $\alpha$ predict consumption?

Predictive regression

$$\Delta c_t = \phi^c(-\log \alpha_{t-1}) + \varepsilon^c_t$$
Can $\alpha$ predict consumption?

Predictive regression

$$\Delta c_t = 0.094 \left( - \log \alpha_{t-1} \right) + \varepsilon_t^c$$

$$(0.038)$$
Can $\alpha$ predict consumption?

**Predictive regression**

$$\Delta c_t = 0.094 \left( -\log \alpha_{t-1} \right) + \epsilon_t^c$$

(0.038)

**How much are we explaining?**

$$R^2 = 4.7\%$$
Filtered long-run risks
Suggestions

- Less theoretical implications of preference for housing consumption
- More empirical pricing tests using this estimated measure of long-run risks