

# Housing as a Measure for Long-Run Risk in Asset Pricing

by José Fillat

Discussion by  
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# The long-run risks recipe



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- 2 Small, but highly persistent predictive component of consumption growth.
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- equity premium puzzle (Bansal and Yaron (2004))
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# The long-run risks recipe



- 1 Epstein-Zin preferences: large elasticity of intertemporal substitution.
- 2 Small, but highly persistent predictive component of consumption growth. ✓
- 3 Euler equations hold at **quarterly** frequency. ✓

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# Battle plan



- The model
- Pricing implications
- Measuring long-run risks
- Suggestions

# Setup of the economy



- Preferences *à la* Hansen and Sargent

$$U_t = (1 - \beta) \log \left( C_t^{\frac{\varepsilon-1}{\varepsilon}} + w_t S_t^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \beta \theta \log E_t \exp \left\{ \frac{U_{t+1}}{\theta} \right\}$$

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- Budget constraint

$$p_t^C C_t + p_t^S S_t + q_t^C \theta_t^C + q_t^S \theta_t^S = (q_t^C + p_t^C C_t) \theta_{t-1}^C + (q_t^S + p_t^S S_t) \theta_{t-1}^S$$



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- Expenditure share

$$-\log \alpha_{t+1} = -\log p_{t+1}^C C_{t+1} / (p_{t+1}^C C_{t+1} + p_{t+1}^S S_{t+1})$$



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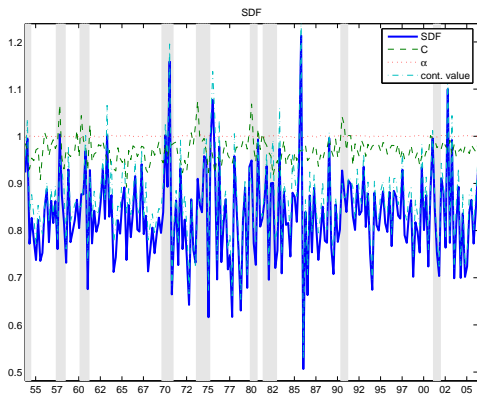
- Expenditure share

$$-\log \alpha_{t+1} = \mu^\alpha + \phi^\alpha x_{t+1} + x'_{t+1} \Psi x_{t+1}$$

## Stochastic Discount Factor



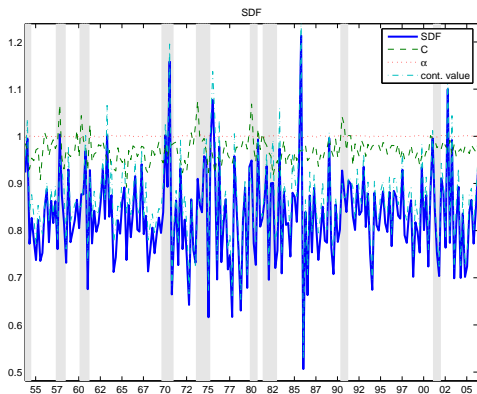
$$m_{t+1} = \log \frac{\partial U_t / \partial C_{t+1}}{\partial U_t / \partial C_t}$$



## Stochastic Discount Factor



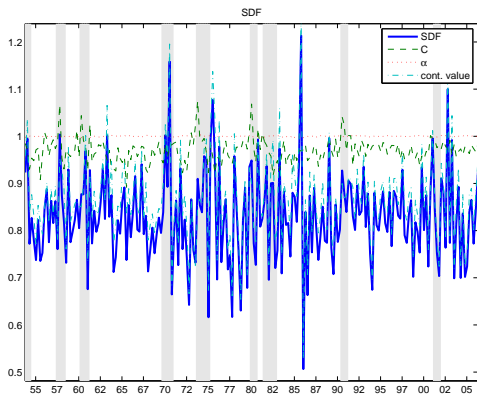
$$m_{t+1} = \log \beta - \Delta c_{t+1} + \log \frac{\exp \{U_{t+1}/\theta\}}{E_t \exp \{U_{t+1}/\theta\}} + \Delta \alpha_{t+1}$$



## Stochastic Discount Factor



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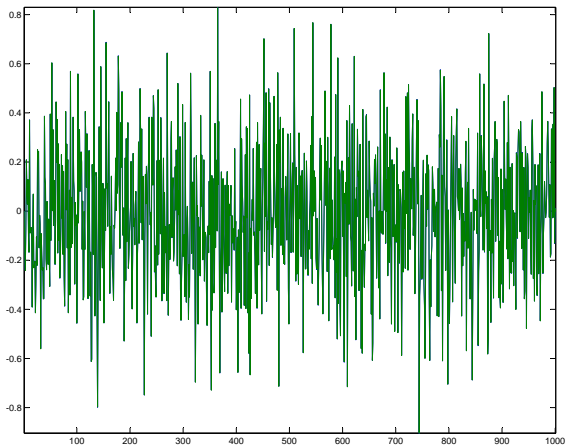
# Comparison



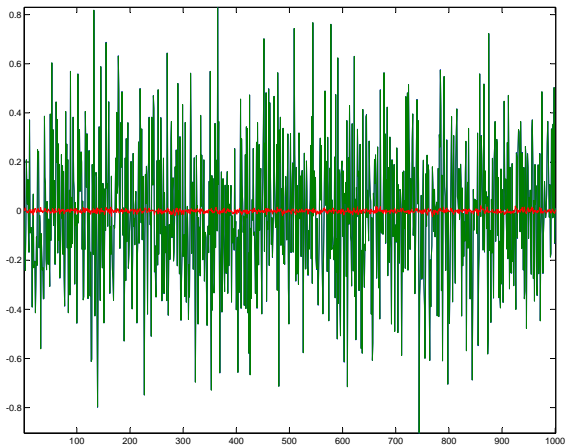
Compare two economies:

- 1 Preference for housing services is shut down.
- 2 Preference for housing services is turned on.

# Comparison of SDF's



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# Pricing results

- Volatility of SDF's

	No Durables	With Durables
$\sigma(m_{t+1})$	53.7802	53.7853



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- Equity premia

	1 Period			Limiting		
	$R^l$	$R^s$	$R^h$	$R^l$	$R^s$	$R^h$
RU + housing	5.2780	7.1620	-5.8615	5.6000	7.4894	-6.4424
Rec. Ut.	5.2143	7.1017	-5.7907	5.5893	7.4854	-6.4614
P. Ut	-0.0000	0.9801	0.1941	0.4011	0.7897	0.8834

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- Estimate long-run risks from

$$-\log \alpha_t = \mu^\alpha + \phi^\alpha x_t$$

$$x_t = \rho^x x_{t-1} + \varepsilon_t^x$$

# Estimating long-run risks



- Estimate long-run risks from

$$-\log \alpha_t = \begin{array}{c} 0.010 \\ (0.004) \end{array} + \phi^\alpha x_t$$

$$x_t = \begin{array}{c} 0.926 \\ (0.027) \end{array} x_{t-1} + \varepsilon_t^x$$

# Estimating long-run risks



- Estimate long-run risks from

$$-\log \alpha_t = \underset{(0.004)}{0.010} + \phi^\alpha x_t$$

$$x_t = \underset{(0.027)}{0.926} x_{t-1} + \varepsilon_t^x$$

- How does it compare to Bansal and Yaron's calibration?

$$0.926^{\frac{1}{3}} = 0.975 \approx 0.979$$

# Can $\alpha$ predict consumption?



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- Predictive regression

$$\Delta c_t = \phi^c(-\log \alpha_{t-1}) + \varepsilon_t^c$$

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$$\Delta c_t = \underset{(0.038)}{0.094} (-\log \alpha_{t-1}) + \varepsilon_t^c$$

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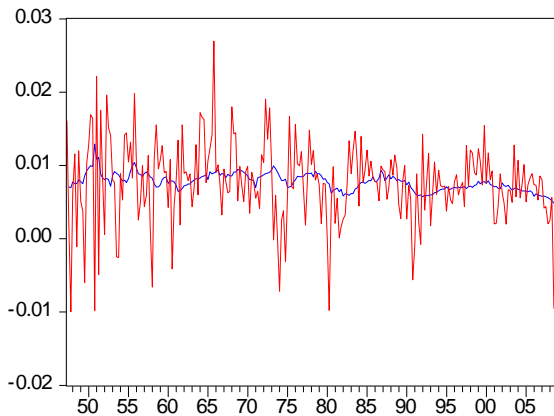
- Predictive regression

$$\Delta c_t = \underset{(0.038)}{0.094} (-\log \alpha_{t-1}) + \varepsilon_t^c$$

- How much are we explaining?

$$R^2 = 4.7\%$$

# Filtered long-run risks





- Less theoretical implications of preference for housing consumption
- More empirical pricing tests using this estimated measure of long-run risks