Market Prices of Risk with Diverse Beliefs, Learning, and Catastrophes: discussion

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  1. Complete Markets
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     $\rightarrow$ Market Price of Risk is low
  2. One non-state contingent bond
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  3. Two bonds
     $\rightarrow$ A catastrophe bond + a non-catastrophe bond

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- Two periods model:

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- Bonds are in zero net supply

Calibration

- Aggregate endowments and probabilities for date 1

	H	M	D
Growth rate of Y	1.03	0.99	0.90
True probabilities	0.50	0.495	0.005
Agent 1 beliefs	0.50	0.490	0.010
Agent 2 beliefs	0.50	0.495	0.005

- Date 0 aggregate endowment is equal to 1
- Discount factor β is 0.95

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	One bond	**Two bonds**	Three bonds
$E[C_1'] - E[C_2']$	0.0001	**-0.001**	-0.001
Agent 1 MPR	0.089	**0.051**	0.051
Agent 2 MPR	0.022	**0.052**	0.051

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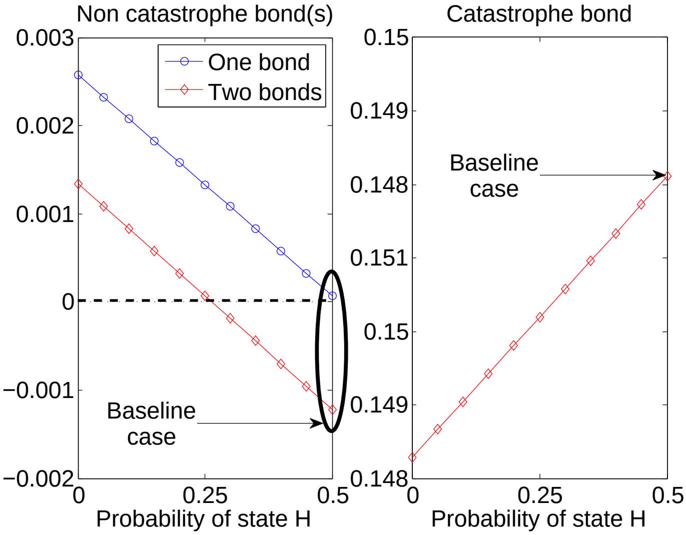
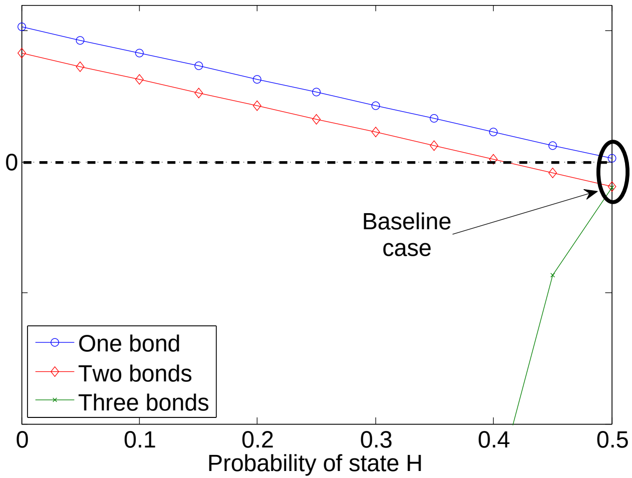
- Agent 1 is now the pessimist in both markets

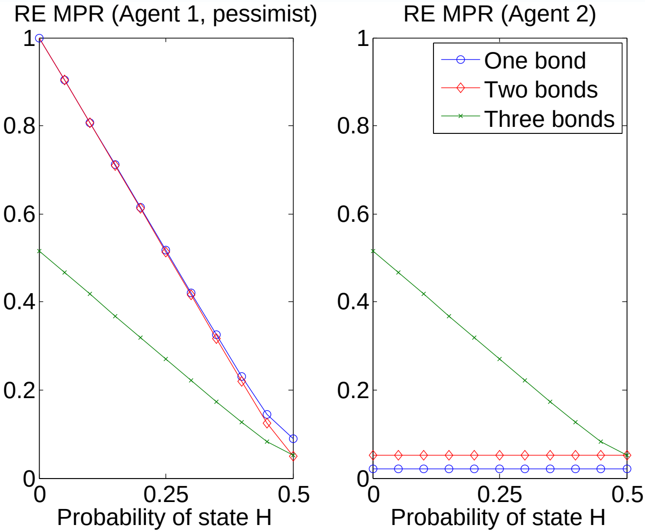
Agent 1: assets holdings



Average difference of terminal consumptions (agent 1 minus agent 2)



Market Prices of Risk



Concluding Remarks

- We may not have to shut down the market for catastrophes
- The key is to make the less well informed agent pessimistic in both markets
- In this example: results with two bonds are similar to results with one bond