Market Prices of Risk with Diverse Beliefs, Learning, and Catastrophes: discussion

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Introduction

Three states for aggregate endowment growth:
→ **High** growth, **Mild** recession, **Deep** recession
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  → **High** growth, **Mild** recession, **Deep** recession

- Two agents:
  → Agree on probability of positive growth vs. any kind of recession
  → One agent is less well informed
    - More pessimistic about a deep recession
    - Learns over time using Bayes’ rule
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Two agents:
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Financial Markets:
1. Complete Markets
   → Agent with wrong beliefs loses wealth over time
   → Market Price of Risk is low
2. One non-state contingent bond
   → Agent with correct beliefs loses wealth over time
   → Market Price of Risk is high
Three states for aggregate endowment growth:
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Financial Markets:
1. **Complete Markets**
   → Agent with **wrong** beliefs loses wealth over time
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2. One non-state contingent bond
   → Agent with **correct** beliefs loses wealth over time
   → Market Price of Risk is high
3. **Two bonds**
   → A catastrophe bond + a non-catastrophe bond
Model(s)

- Two periods model:

\[ U_i = \log(C_i) + \beta \left[ \pi_{i,H} \cdot \log(C'_{i,H}) + \pi_{i,M} \cdot \log(C'_{i,M}) + \pi_{i,D} \cdot \log(C'_{i,D}) \right] \]
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- Endowments are half of aggregate \( Y \)
Model(s)

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- Endowments are half of aggregate \( Y \)

- Final period’s budget constraints: consume endowments \( \pm \) savings
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- Date zero budget constraints depends on markets’ structure
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  - One bond: \( C_i + b_i \cdot p_b = 1/2 \cdot Y \)
Model(s)

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- Endowments are half of aggregate \( Y \)

- Final period’s budget constraints: consume endowments \( \pm \) savings

- Date zero budget constraints depends on markets’ structure

1. One bond: \( C_i + b_i \cdot p_b = \frac{1}{2} \cdot Y \)

3. Three bonds: \( C_i + b_{H,H} \cdot p_{b_H} + b_{M,M} \cdot p_{b_M} + b_{D,D} \cdot p_{b_D} = \frac{1}{2} \cdot Y \)
Two periods model:

\[ U_i = \log(C_i) + \beta \left[ \pi_{i,H} \log(C'_{i,H}) + \pi_{i,M} \log(C'_{i,M}) + \pi_{i,D} \log(C'_{i,D}) \right] \]

Endowments are half of aggregate \( Y \)

Final period’s budget constraints: consume endowments \( \pm \) savings

Date zero budget constraints depends on markets’ structure

1. One bond: \( C_i + b_i \cdot p_b = 1/2 \cdot Y \)

2. Two bonds: \( C_i + b_{\text{no disaster}} \cdot p_{b_{\text{no disaster}}} + b_{\text{disaster}} \cdot p_{b_{\text{disaster}}} = 1/2 \cdot Y \)

3. Three bonds: \( C_i + b_H \cdot p_{b_H} + b_M \cdot p_{b_M} + b_D \cdot p_{b_D} = 1/2 \cdot Y \)
Two periods model:

$$U_i = \log(C_i) + \beta \left[ \pi_{i,H} \cdot \log(C'_{i,H}) + \pi_{i,M} \cdot \log(C'_{i,M}) + \pi_{i,D} \cdot \log(C'_{i,D}) \right]$$

- Endowments are half of aggregate $Y$
- Final period’s budget constraints: consume endowments $\pm$ savings
- Date zero budget constraints depends on markets’ structure
  1. One bond: $C_i + b_i \cdot p_b = 1/2 \cdot Y$
  2. Two bonds: $C_i + b_{\text{no disaster}} \cdot p_{b_{\text{no disaster}}} + b_{\text{disaster}} \cdot p_{b_{\text{disaster}}} = 1/2 \cdot Y$
  3. Three bonds: $C_i + b_{H} \cdot p_{b_H} + b_{M} \cdot p_{b_M} + b_{D} \cdot p_{b_D} = 1/2 \cdot Y$

- Bonds are in zero net supply
Calibration

- Aggregate endowments and probabilities for date 1

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<thead>
<tr>
<th></th>
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<tr>
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<td>1.03</td>
<td>0.99</td>
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<td>True probabilities</td>
<td>0.50</td>
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- Date 0 aggregate endowment is equal to 1
- Discount factor $\beta$ is 0.95
What is going to happen?
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- Agent 1 is the pessimist in the disaster market
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- Agent 1 is the pessimist in the disaster market \( (b_{\text{dis}} = 0.1501 > 0) \)
- Agent 1 is the optimist in the no-disaster market \( (b_{\text{no\,dis}} = -.001 < 0) \)
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- Agent 1 is the pessimist in the disaster market \( (b_{\text{dis}} = 0.1501 > 0) \)
- Agent 1 is the optimist in the no-disaster market \( (b_{\text{no dis}} = -0.001 < 0) \)

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<td>( E[C'_1] - E[C'_2] )</td>
<td>0.0001</td>
<td>-0.001</td>
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<tr>
<td>Agent 1 MPR</td>
<td>0.089</td>
<td><strong>0.051</strong></td>
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<td>Agent 2 MPR</td>
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- Make Agent 1 more pessimistic
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- Agent 1 is now the pessimist in both markets
Agent 1: assets holdings

- **Non catastrophe bond(s)**
  - Baseline case:
    - Probability of state H:
      - 0
      - 0.25
      - 0.5
      - -0.002
      - -0.001
      - 0
      - 0.001
      - 0.002
      - 0.003
  - One bond:
    - Baseline case:
      - Probability of state H:
        - 0
        - 0.25
        - 0.5
        - -0.002
        - -0.001
        - 0
        - 0.001
        - 0.002
        - 0.003

- **Catastrophe bond**
  - Baseline case:
    - Probability of state H:
      - 0
      - 0.25
      - 0.5
      - 0.148
      - 0.149
      - 0.15
      - 0.151
      - 0.148
      - 0.149
      - 0.15

Graphs showing the probability of state H for non-catastrophe and catastrophe bonds with one and two bonds.
Average difference of terminal consumptions (agent 1 minus agent 2)
Market Prices of Risk

**Introduction**

**Model(s)**

- Eqm with 2 bonds
- Changing beliefs

**Conclusion**

**Diagram:**
- **RE MPR (Agent 1, pessimist)**
- **RE MPR (Agent 2)**
  - One bond
  - Two bonds
  - Three bonds
Concluding Remarks

- We may not have to shut down the market for catastrophes.
- The key is to make the less well informed agent pessimistic in both markets.
- In this example: results with two bonds are similar to results with one bond.