Robustness and Monetary Policy Experimentation

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Motivation

- When a policy maker has multiple submodels, Bayes' law and a Bellman equation tell him to experiment. Nevertheless, Blinder, Lucas, and others have told policy makers not to experiment (i.e., to ignore the Bellman equation).
- In Cogley, Colacito, and Sargent (2007), we studied the benefits from listening to Bellman (and not Lucas and Blinder).
- We now study experimentation when the policy maker doubts both models and his prior over them.

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- When a policy maker has multiple submodels, Bayes' law and a Bellman equation tell him to experiment. Nevertheless, Blinder, Lucas, and others have told policy makers not to experiment (i.e., to ignore the Bellman equation).
- In Cogley, Colacito, and Sargent (2007), we studied the benefits from listening to Bellman (and not Lucas and Blinder).
- We now study experimentation when the policy maker doubts both models and his prior over them.
- **Policy maker wants robust decision rules.**

Plan of the talk

- Two Bellman equations

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  1. Bayesian problem

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Plan of the talk

- Two Bellman equations
  1. Bayesian problem
  2. Robust problem
- Policy and value functions
- Different θ_1's and θ_2's: risk sensitivity parameters.
- Different λ's: relative importance of unemployment and inflation.

The economy

- Central Bank chooses v_t to

$$\min \quad E_0 \sum_{t=0}^{\infty} .995^t (U_t^2 + \lambda v_t^2), s.t.$$

- **Model z=1** *a Samuelson-Solow model that specifies a permanently exploitable inflation-unemployment tradeoff*

- **Model z=2** *a Lucas-Phelps model with no exploitable trade-off*

The economy

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- **Model z=1**

$$\begin{aligned} U_{t+1} &= 0.0023 + 0.7971 U_t - 0.2761 \pi_{t+1} + 0.0054 \eta_{1,t+1} \\ \pi_{t+1} &= v_t + 0.0055 \eta_{3,t} \end{aligned}$$

- **Model z=2**

$$\begin{aligned} U_{t+1} &= 0.0007 + 0.8468 U_t - 0.2489 (\pi_{t+1} - v_t) + 0.0055 \eta_{2,t+1} \\ \pi_{t+1} &= v_t + 0.0055 \eta_{3,t+1} \end{aligned}$$

The economy

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- **Model z=1**

$$U_1^* \quad = \quad \overline{U}_1 + A_1 U + B_1 v + C_1 \varepsilon_1^*$$

- **Model z=2**

$$U_2^* \quad = \quad \overline{U}_2 + A_2 U + C_2 \varepsilon_2^*$$

The economy

- Central Bank chooses v_t to

$$\min \quad E_0 \sum_{t=0}^{\infty} \beta^t r(U_t + \lambda v_t), s.t.$$

- **Model z=1** (α)

$$U_1^* = \overline{U}_1 + A_1 U + B_1 v + C_1 \varepsilon_1^*$$

- **Model z=2** $(1-\alpha)$

$$U_2^* = \overline{U}_2 + A_2 U + C_2 \varepsilon_2^*$$

- Bayesian updating:

$$\alpha^* = \pi_\alpha(\alpha, U^*)$$

Evolution of α_t

- Using Bayes' law:

$$\log \frac{\alpha_t}{1-\alpha_t} = \log \frac{\alpha_{t-1}}{1-\alpha_{t-1}} + \log \frac{p_1(U_t|U_{t-1}, v_{t-1})}{p_2(U_t|U_{t-1}, v_{t-1})}$$

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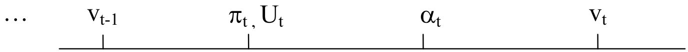
- Timing protocol



Bayesian Problem

$$V(U,\alpha) = \max_{v} \left\{ r(U,\alpha) + \beta \left[\alpha \int V(U_1^*,\alpha^*)\, dF(\varepsilon_1^*) + (1-\alpha) \int V(U_2^*,\alpha^*)\, dF(\varepsilon_2^*) \right] \right\}$$

subject to:

$$\begin{aligned}
U_1^* &= \overline{U}_1 + A_1 U + B_1 v + C_1 \varepsilon_1^* \\
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\alpha^* &= \pi_\alpha(\alpha, U_z^*), \quad z = \{1,2\}
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Bayesian Problem

$$V(s,\alpha) = \max_{v}\left\{r(U,v) + \mathbf{E}_z\left[\mathbf{E}_{U^*,\alpha^*}(\beta V(U^*,\alpha^*)|U,v,\alpha,z)|U,v,\alpha\right]\right\}$$

subject to:

$$\begin{aligned} U^* &= \pi_U(U,v,z,\varepsilon^*) \\ \alpha^* &= \pi_\alpha(\alpha,\pi_U(U,v,z,\varepsilon^*)) \\ z &= \{1,2\} \end{aligned}$$

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$\mathbf{T}^1$ operator: misspecification of a submodel

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$$\mathbf{T}^1(V(U^*,\alpha^*))(U,\alpha,v,z;\theta_1) = -\theta_1 \log \mathbf{E}_{U^*,\alpha^*}\left[\exp\left(\frac{-V(U^*,\alpha^*)}{\theta_1}\right)\Big|(U,\alpha,v,z)\right]$$

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This is the indirect utility function for a penalized utility minimization problem that yields a worst-case case distortion to the distribution over (U^*,α^*) conditional on z that is proportional to

$$\exp\left(\frac{-V(U^*,\alpha^*)}{\theta_1}\right)$$

$\mathbf{T}^2$ operator: prior misspecification

$$\mathbf{T}^2(V(U^*,\alpha^*))(U,\alpha,v;\theta_2) = -\theta_2 \log \mathbf{E}_z \left[\exp\left(\frac{-V(U^*,\alpha^*)}{\theta_2} \right) \Big| (U,\alpha,v) \right]$$

$\mathbf{T}^2$ operator: prior misspecification

$$\mathbf{T}^2(V(U^*,\alpha^*))(U,\alpha,v;\theta_2) = -\theta_2 \log \mathbf{E}_z\left[\exp\left(\frac{-V(U^*,\alpha^*)}{\theta_2}\right)\Big|(U,\alpha,v)\right]$$

The associated distortion to the worst-case prior over z is proportional to

$$\exp\left(\frac{-V(U,\alpha,v,z)}{\theta_2}\right)$$

Robust Bellman equation

$$V(U,\alpha) = \max_{v} \left\{ r(U,v) + \mathbf{T}^2 \left[\mathbf{T}^1 \left[(\beta V(U^*, \alpha^*)(U,v,\alpha,z;\theta_1)) \right] (U,v,\alpha;\theta_2) \right] \right\}$$

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- θ_1 measures concern about misspecification of a submodel.
- θ_2 measures concern about misspecification of the prior α.

Robust Bellman equation

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- θ_1 measures concern about misspecification of a submodel.
- θ_2 measures concern about misspecification of the prior α.
- Idea: replace $\mathbf{E}_{U^*,\alpha^*}$ with $\mathbf{T}^1$ and $\mathbf{E}_z$ with $\mathbf{T}^2$.

Quantitative findings

Quantitative findings

Roadmap

1. Risk sensitivity operator $\mathbf{T}^2$ only.
2. Risk sensitivity operator $\mathbf{T}^1$ only.
3. Both risk sensitivity operators are turned on.

$\mathbf{T}^2$ only: messages

Slants α toward worst case model

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Slants α toward worst case model

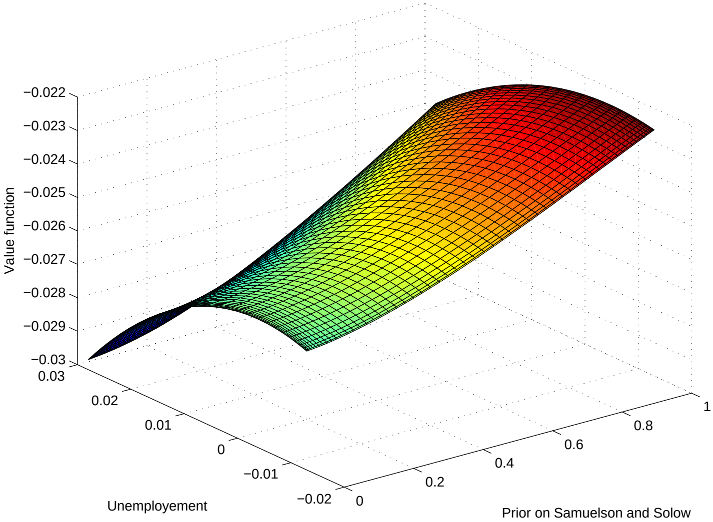
- When λ **is small**, Lucas model is worst case model.
- When λ **is big**, the SS is the worst-case model.

T^2 only: messages

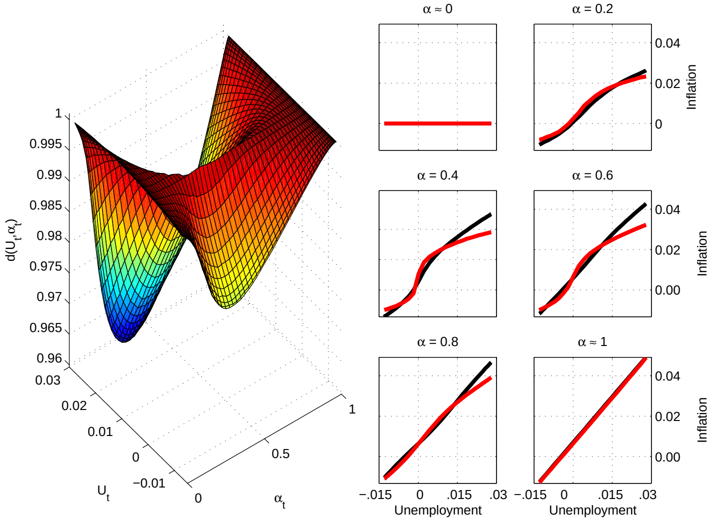
Slants α toward worst case model

- When λ **is small**, Lucas model is worst case model.
  $\rightarrow$ Therefore, robust policy is **less** countercyclical.
- When λ **is big**, the SS is the worst-case model.
  $\rightarrow$ Therefore, robust policy is **more** countercyclical.

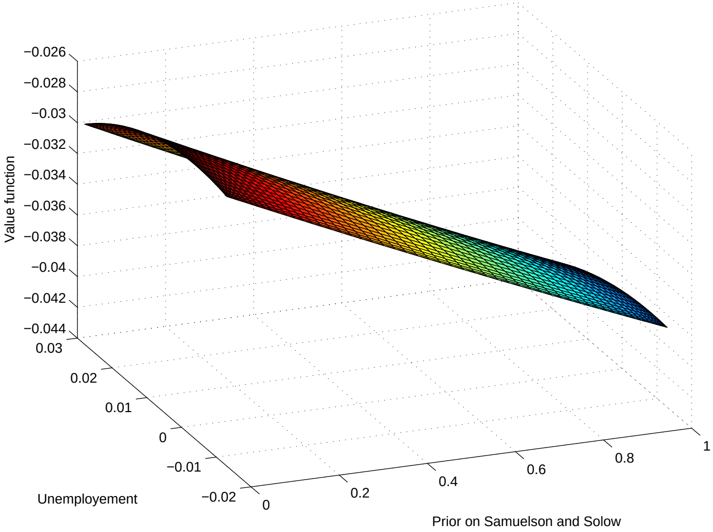
Non-Robust value function, $\lambda = 0.1$



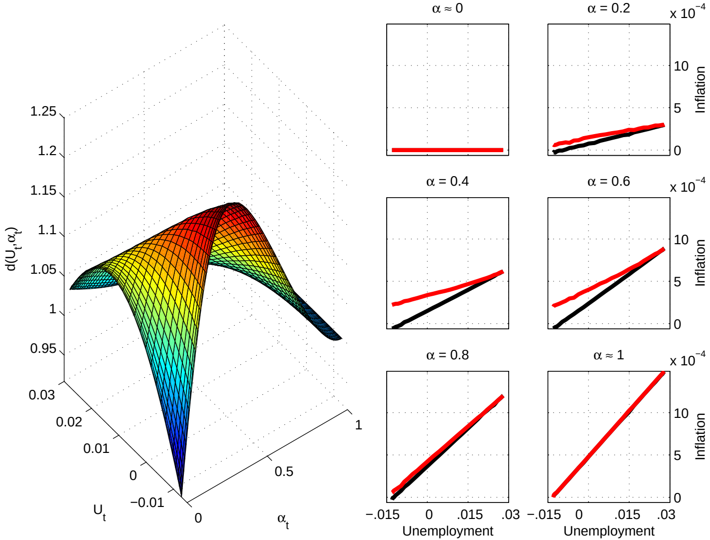
T^2 only, $\lambda = 0.1$ and $\theta_2 = .1$



Value function (no robustness) for $\lambda = 16$



T^2 only, $\lambda = 16$ and $\theta_2 = 0.001$



$\mathbf{T}^1$ only: messages

Worst case slants shock distribution toward higher probabilities of deviation-amplifying shock when U is large.

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- Affects both conditional means and variances.

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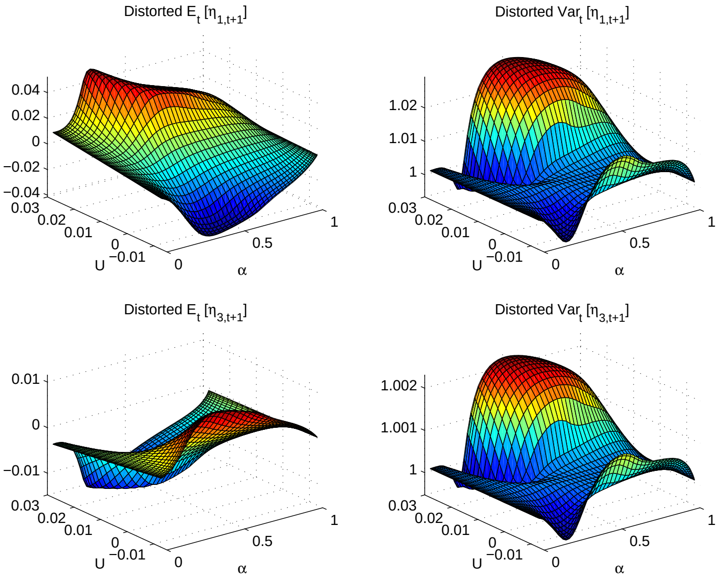
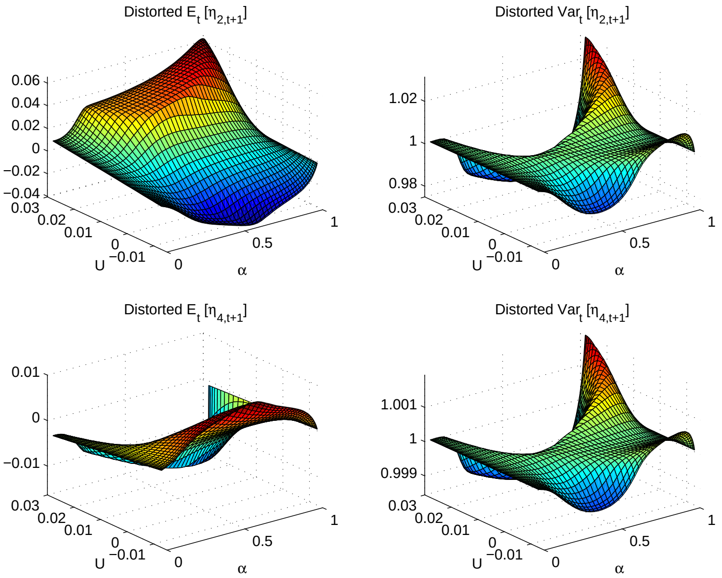
Worst case slants shock distribution toward higher probabilities of deviation-amplifying shock when U is large.

- Affects both conditional means and variances.
- The robust policy maker adopts a more aggressive countercyclical stance.

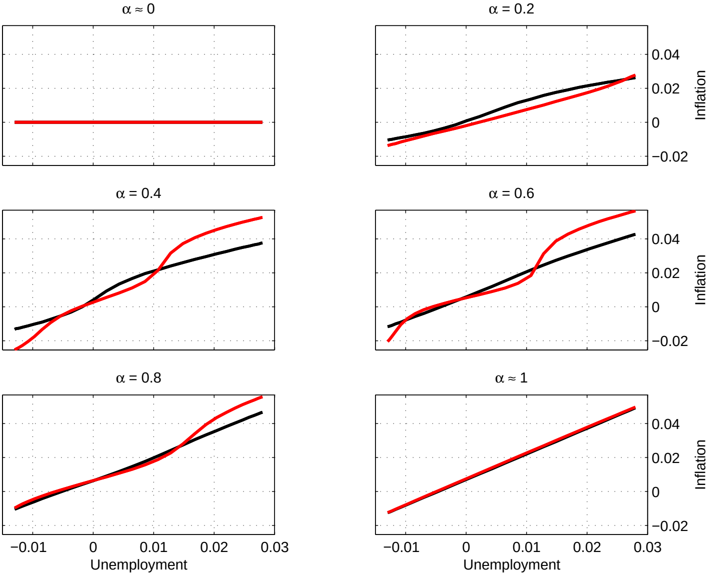
Distorted shocks to SS model ($\theta_1 = .1$)



Distorted shocks to Lucas model ($\theta_1 = .1$)



T^1 only robust policy: $\theta_1 = .1$



$\mathbf{T}^1$ and $\mathbf{T}^2$: prediction

- $\mathbf{T}^1$ only: policy is **more** countercyclical.
- $\mathbf{T}^2$ only: policy is **less** countercyclical.

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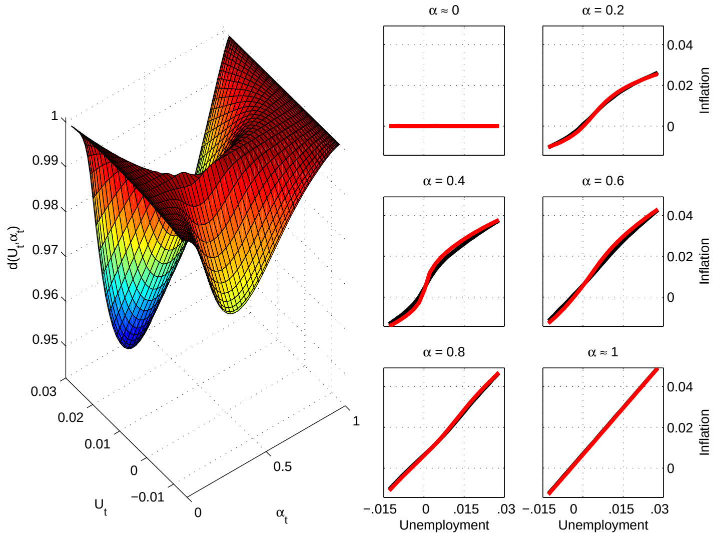
Optimal Bayesian decision rule with experimentation is robust to a mixture of concerns about the two types of misspecification.

Both T^j's, $\theta_1 = .1$ and $\theta_2 = .1$



Conclusions

- Robustness attained by calculating bounds on value functions.
- This automatically leads to a worst case analysis.
- The $\mathbf{T}^1$ operator checks robustness of a submodel.
  $\Rightarrow$ Calls for more countercyclical policy.
- The $\mathbf{T}^2$ operator checks robustness w.r.t. prior over submodels.
  $\Rightarrow$ Calls for less countercyclical policy.
- Bayesian policy with experimentation is robust to both fears of misspecification.