Robustly Optimal Monetary Policy in a Microfounded New Keynesian Model: discussion

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Roadmap

- Relation to the literature
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- Main results hinge on local log-linear approximation
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- What are we missing by shutting down higher order moments?
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- What are we missing by shutting down higher order moments?
  → Asset Pricing implications of optimal monetary policy
Relation to the Literature

Clarida, Gali, Gertler (JEL, 1999)

1. The Central Bank minimizes

\[ \frac{1}{2} E_{t-1} \sum_{j=0}^{\infty} \beta^j \left[ \pi_{t+j}^2 + \lambda (x_{t+j} - x^*)^2 \right] \]

2. Private Sector’s Aggregate supply equation is

\[ \pi_t = \kappa x_t + \beta E_t \pi_{t+1} + \sigma_u w_t \]
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This Paper

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Where is the aggregate supply equation coming from?

What does it look like in a New-Keynesian model?
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- Are we missing any interesting economics?
Woodford (2010) revisited

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2. Private Sector’s Aggregate supply equation is

\[ \pi_t = \kappa x_t + \beta E_t m_{t+1} g(\pi_{t+1}) + \sigma_u w_t \]

where \( g(\pi_{t+1}) \) is a possibly non-linear function of \( \pi_{t+1} \).
My question in a nutshell

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What if $g(\pi_{t+1})$ is a quadratic function of $\pi_{t+1}$?
The key: figure out Distorted Probabilities
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- $\hat{f}(w) = m \cdot f(w)$ is the **distorted** pdf of AS shock
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Results:
- In baseline log-linear model:
  $\hat{f}(w) = \mathcal{N}(\mu_t, 1)$
  - Conditional Mean is time-varying
  - Conditional Variance is not time-varying
- In quadratic model:
  $\hat{f}(w) = \mathcal{N}(\mu_t + \nu_t, \sigma_t)$
  - Conditional Mean is time-varying
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I’ll spare you the details...

- Focus on linear policies: \( \pi_{t+1} = p_{0,t} + p_{1,t}w_{t+1} \).
- Distortion is of the form: \( \log m_{t+1} = c_t + \phi_{2,t} (\pi_{t+1} + \alpha \pi_{t+1}^2) \).
- Imposing the constraint \( 0 = \log E_t m_{t+1} \), implies that \( c_t = -\phi_{2,t} (1 + \alpha p_{0,t}) p_{0,t} + \frac{1}{2} \log \left( 1 - 2\phi_{2,t} \alpha p_{1,t}^2 \right) - \frac{1}{2} \phi_{2,t}^2 (1 + 2\alpha p_{0,t})^2 p_{1,t}^2 \left( 1 - 2\phi_{2,t} \alpha p_{1,t}^2 \right) \).
- Distorted beliefs are:
  \[
  \log \hat{f}(w) = \log m(w) + \log f(w) \\
  = -\frac{1}{2} \log 2\pi + \frac{1}{2} \log \left( 1 - 2\phi_{2,t} \alpha p_{1,t}^2 \right) \\
  - \frac{1}{2} \left( 1 - 2\phi_{2,t} \alpha p_{1,t}^2 \right) \left\{ w^2 - 2\frac{\phi_{2,t} p_{1,t} (1 + 2\alpha p_{0,t})}{\left( 1 - 2\phi_{2,t} \alpha p_{1,t}^2 \right)} + \phi_{2,t}^2 p_{1,t}^2 \left( 1 + 2\alpha p_{0,t} \right)^2 \right\} \\
  \]
- Re-scaling: \( \hat{f}(w) = N \left( \frac{\phi_{2,t} p_{1,t} (1 + 2\alpha p_{0,t})}{\left( 1 - 2\phi_{2,t} \alpha p_{1,t}^2 \right)}, \sqrt{1 - 2\phi_{2,t} \alpha p_{1,t}^2} \right) \), where
  \[
  \mu_t + \nu_t = \frac{\phi_{2,t} p_{1,t} (1 + 2\alpha p_{0,t})}{\left( 1 - 2\phi_{2,t} \alpha p_{1,t}^2 \right)} \text{ and } \sigma_t = \sqrt{1 - 2\phi_{2,t} \alpha p_{1,t}^2}.
  \]
Conditional variance is distorted: who cares?

- Think about where these distorted probabilities show up.
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- Think about where these distorted probabilities show up.
- Euler equations

\[
E_t \left[ m_{t+1} \frac{u'(c_{t+1})}{u'(c_t)} \frac{R_{t+1}}{SDF_{t+1}} \right] = 1
\]
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E_t \begin{bmatrix}
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  SDF_{t+1}
\end{bmatrix} = 1
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- The Stochastic Discount Factor is of the form:

\[
SDF_{t+1} = A + B \cdot \mu_t + C \cdot \sigma_t \cdot \epsilon_{t+1}
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- Optimal monetary policy introduces time-variation in the conditional variance of the stochastic discount factor.
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- The CB contributes to creating time-varying equity risk premia!
Great paper, great effort!

I think that there is more than you bargained for, if you take seriously non-linearities!

The link between optimal monetary policy and time-varying expected returns is important and very actual!

This channel is already built into the model: is it quantitatively important?