Robustly Optimal Monetary Policy in a Microfounded New Keynesian Model: discussion

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THE UNIVERSITY *of* NORTH CAROLINA *at* CHAPEL HILL

ECB conference on "Information, Beliefs and Economic Policy", Frankfurt, 12/2011

Roadmap

- Relation to the literature

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- Main results hinge on local log-linear approximation

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  $\rightarrow$ Asset Pricing implications of optimal monetary policy

Relation to the Literature

Clarida, Gali, Gertler (JEL, 1999)

1. The Central Bank minimizes

$$\frac{1}{2} E_{t-1} \sum_{j=0}^{\infty} \beta^j \left[\pi_{t+j}^2 + \lambda (x_{t+j} - x^*)^2 \right]$$

2. Private Sector's Aggregate supply equation is

$$\pi_t \quad = \quad \kappa x_t + \beta E_t \pi_{t+1} + \sigma_u w_t$$

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- Are we missing any interesting economics?

My question in a nutshell

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$$\pi_t = \kappa x_t+\beta E_t m_{t+1} g\left(\pi_{t+1}\right)+\sigma_u w_t$$

where $g\left(\pi_{t+1}\right)$ is a possibly non-linear function of π_{t+1}.

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What if $g\left(\pi_{t+1}\right)$ is a quadratic function of π_{t+1}?

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I'll spare you the details...

- Focus on linear policies: $\pi_{t+1} = p_{0,t} + p_{1,t} w_{t+1}$.
- Distortion is of the form: $\log m_{t+1} = c_t + \phi_{2,t}\left(\pi_{t+1} + \alpha\pi_{t+1}^2\right)$.
- Imposing the constraint $0 = \log E_t m_{t+1}$, implies that $c_t = -\phi_{2,t}\left(1+\alpha p_{0,t}\right) p_{0,t} + \frac{1}{2}\log\left(1-2\phi_{2,t}\alpha p_{1,t}^2\right) - \frac{1}{2}\phi_{2,t}^2\left(1+2\alpha p_{0,t}\right)^2 p_{1,t}^2\left(1-2\phi_{2,t}\alpha p_{1,t}^2\right)$.
- Distorted beliefs are:

$$
\begin{aligned}
\log \widehat{f}(w) &= \log m(w) + \log f(w) \\
&= -\frac{1}{2}\log 2\pi + \frac{1}{2}\log\left(1-2\phi_{2,t}\alpha p_{1,t}^2\right) \\
&\quad -\frac{\left(1-2\phi_{2,t}\alpha p_{1,t}^2\right)}{2}\left\{w^2 - 2\frac{\phi_{2,t}p_{1,t}\left(1+2\alpha p_{0,t}\right)}{\left(1-2\phi_{2,t}\alpha p_{1,t}^2\right)} + \phi_{2,t}^2 p_{1,t}^2\left(1+2\alpha p_{0,t}\right)^2\right\}
\end{aligned}
$$

- Re-scaling: $\widehat{f}(w) = N\left(\frac{\phi_{2,t}p_{1,t}\left(1+2\alpha p_{0,t}\right)}{\left(1-2\phi_{2,t}\alpha p_{1,t}^2\right)}, \sqrt{1-2\phi_{2,t}\alpha p_{1,t}^2}\right)$, where $\mu_t + \nu_t = \frac{\phi_{2,t}p_{1,t}\left(1+2\alpha p_{0,t}\right)}{\left(1-2\phi_{2,t}\alpha p_{1,t}^2\right)}$ and $\sigma_t = \sqrt{1-2\phi_{2,t}\alpha p_{1,t}^2}$.

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$$SDF_{t+1}=A+B\cdot\mu_t+C\cdot{\color{red}\sigma_t}\cdot\varepsilon_{t+1}$$

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- The CB contributes to creating time-varying equity risk premia!

Concluding Remarks

- Great paper, great effort!
- I think that there is more than you bargained for, if you take seriously non-linearities!
- The link between optimal monetary policy and time-varying expected returns is important and very actual!
- This channel is already built into the model: is it quantitatively important?