Risk Sharing for the Long-Run

A General Equilibrium Approach to International Finance with Recursive Preferences and Long-Run Risks

Riccardo Colacito and Mariano Max Croce

THE UNIVERSITY
of NORTH CAROLINA
at CHAPEL HILL

Finance Cavalcade, University of Michigan, 5/2011

Motivation

- We would like to explain:
  1. The forward premium anomaly: the tendency of high interest rate currencies to appreciate
  2. The Backus and Smith anomaly: the lack of correlation between consumption differentials and FX movements

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- A general equilibrium model: quantities (consumption, NX,...) and prices (assets' returns, FX,...) are outcome of utility maximization problem

- The model should be consistent with:
  - low int'l correlation of consumption and output
  - smoothness of exchange rates
  - large int'l equity risk premia
  - large int'l correlation of returns
  - volatility of Net Exports
  - ...

Roadmap of the talk

1. Setup of the model
   - Preferences
   - Endowments
2. Market structures
   - Complete Markets
   - Portfolio Autarky
3. Calibration
4. Results

Preferences

- Two countries: home (h) and foreign (f)
- Agents have risk-sensitive preferences

$$U_{i,t} \quad = \quad (1-\delta)\log C_{i,t} + \delta\theta \log E_t \exp\left\{\frac{U_{i,t+1}}{\theta}\right\}, \qquad \forall i \in \{h,f\}$$

where $\theta = 1/(1-\gamma)$.

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where $\theta = 1/(1-\gamma)$. If $\theta \to -\infty$: time additive case.

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$$U_{i,t} \approx (1-\delta)\log C_{i,t} + \delta E_t[U_{i,t+1}] + \frac{\delta}{2\theta} V_t[U_{i,t+1}], \quad \forall i \in \{h, f\}$$

where $\theta = 1/(1-\gamma)$. Conditional Variance matters.

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- Preferences are defined over the consumption aggregate

$$C_{h,t} = (x_{h,t})^{\alpha}(y_{h,t})^{1-\alpha} \quad \text{and} \quad C_{f,t} = (x_{f,t})^{1-\alpha}(y_{f,t})^{\alpha}$$

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- Consumption bias: $\alpha > 1/2$.

Endowments

- Endowments' growth is *almost* i.i.d.

$$\begin{aligned}\Delta \log X_t &= \mu_x + z_{1,t-1} + \varepsilon_{x,t} \\ \Delta \log Y_t &= \mu_y + z_{2,t-1} + \varepsilon_{y,t}\end{aligned}$$

where $z_{1,t}$ and $z_{2,t}$ are small, predictable components

$$\begin{aligned}z_{1,t} &= \rho_1 z_{1,t-1} + \varepsilon_{1,t} \\ z_{2,t} &= \rho_2 z_{2,t-1} + \varepsilon_{2,t}\end{aligned}$$

- Shocks are homoskedastic.

Markets

- *Home* is endowed with good X;
- *Foreign* is endowed with good Y;
- Complete set of one period ahead state contingent securities;
- Budget constraints:

$$x_{h,t} + p_t y_{h,t} + \sum_{s_{t+1}} Q_{t+1}(s_{t+1}) A_{t+1}(s_{t+1}) \leq X_t + A_t \qquad [Home]$$

$$x_{f,t} + p_t y_{f,t} - \sum_{s_{t+1}} Q_{t+1}(s_{t+1}) A_{t+1}(s_{t+1}) \leq p_t Y_t - p_t A_t \qquad [Foreign]$$

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- We shall solve the Pareto problem first...

Planner's problem

Efficient allocations are the solution to the planner's problem

$$
\begin{aligned}
\text{choose} \quad & \{x_{h,t}, x_{f,t}, y_{h,t}, y_{f,t}\}_{t=0}^{+\infty} \\
\text{to max} \quad & Q = \mu_h U_{h,0} + \mu_f U_{f,0} \\
\text{s.t.} \quad & x_{h,t} + x_{f,t} = X_t \\
& y_{h,t} + y_{f,t} = Y_t, \quad \forall t \geq 0
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- μ_h and μ_f correspond to an initial distribution of assets.
- Notation: $S = \mu_h / \mu_f$.

Allocations

Time Additive Preferences

Let $k = \frac{\alpha}{1-\alpha}$:

$$x_t^h = \frac{kS}{1+kS} X_t, \qquad x_t^f = \frac{1}{1+kS} X_t$$

$$y_t^h = \frac{S}{k+S} Y_t, \qquad y_t^f = \frac{k}{k+S} Y_t$$

where

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Allocations

Risk Sensitive Preferences

Let $k = \frac{\alpha}{1-\alpha}$:

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where

$$S_t = S_{t-1} \cdot \frac{\delta \exp\{U_{h,t}/\theta\}}{E_{t-1}\exp\{U_{h,t}/\theta\}} \Big/ \frac{\delta \exp\{U_{f,t}/\theta\}}{E_{t-1}\exp\{U_{f,t}/\theta\}}$$

Economic Interpretation

1. What is S_t?

2. How does S_t move?

3. Why does S_t move?

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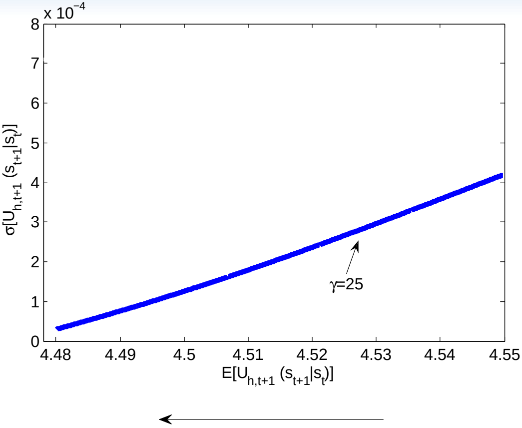
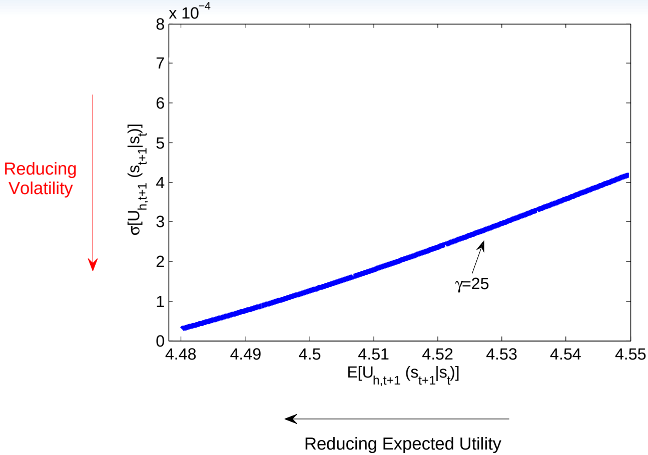


Reducing Expected Utility

Why does S_t move?



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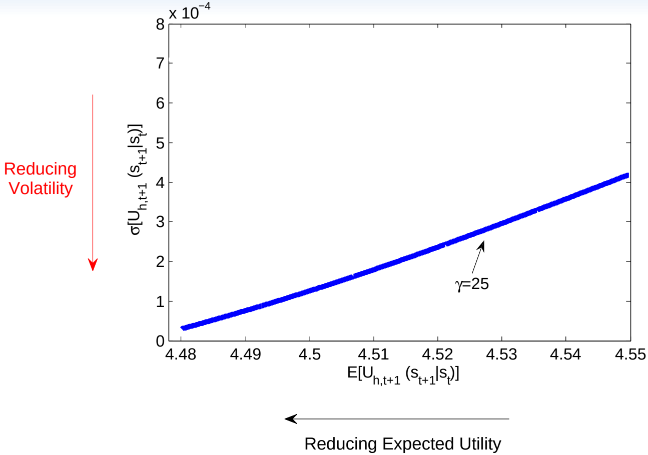
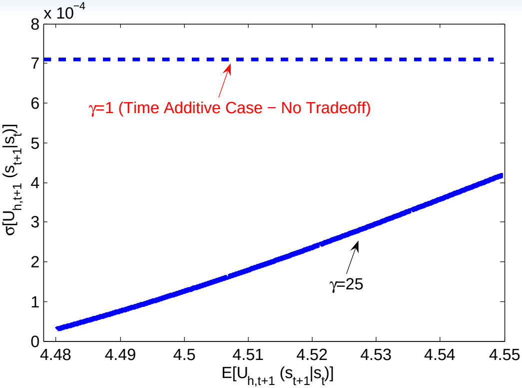


Trade-off between Expected Utility and Utility Variance

Why does S_t move?



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   - Agents are willing to trade-off lower consumption today for smoother future utility profiles.
   - Volatilities are high in bad times and low in good time.

Consumption growth

Time Additive Preferences

$$
\begin{aligned}
\Delta c_t^h &= \alpha \Delta x_t + (1-\alpha)\Delta y_t \\
\Delta c_t^f &= (1-\alpha)\Delta x_t + \alpha \Delta y_t
\end{aligned}
$$

Consumption growth

Risk Sensitive Preferences

$$\Delta c_t^h = \Delta c_t^{h,TA} + \lambda_c^h s_{t-1} + \sigma_t^h \varepsilon_t$$
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$$\Delta c_t^h = \Delta c_t^{h,TA} + \lambda_c^h s_{t-1} + \sigma_1^h g(\varepsilon_{1,t}, \varepsilon_{2,t}, \varepsilon_{x,t}, \varepsilon_{y,t})$$
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1. Additional endogenous predictive component

Consumption growth

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1. Additional endogenous predictive component
2. Contemporaneous response to long-run shocks

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3. Endogenous time-varying volatility

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- Both current

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- Agents are extremely sensitive to long-run news
- Long-run risks should be very correlated to replicate FX volatility

The Backus and Smith Anomaly

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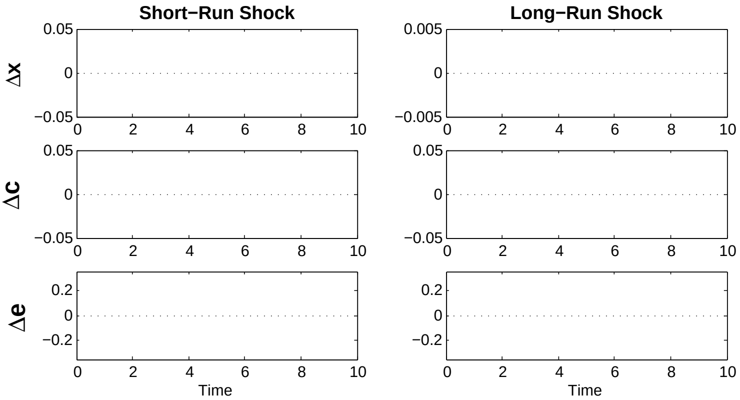

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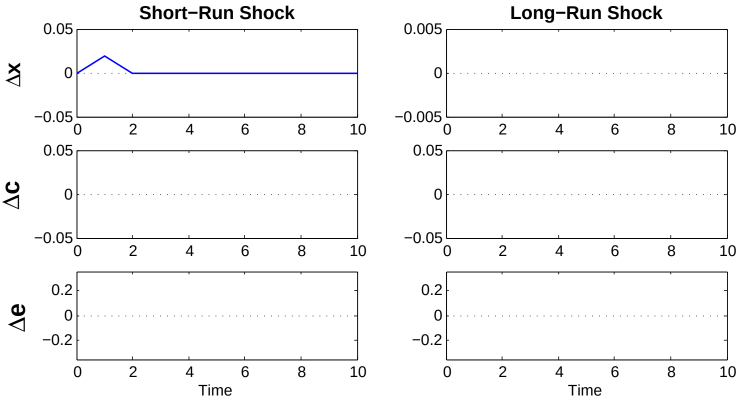

→ Short-run shock to X: home country is happy!

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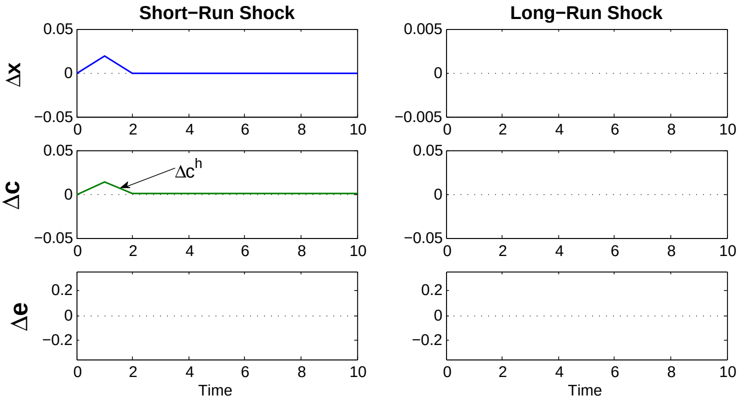

$\rightarrow$ Home increases consumption more than foreign.

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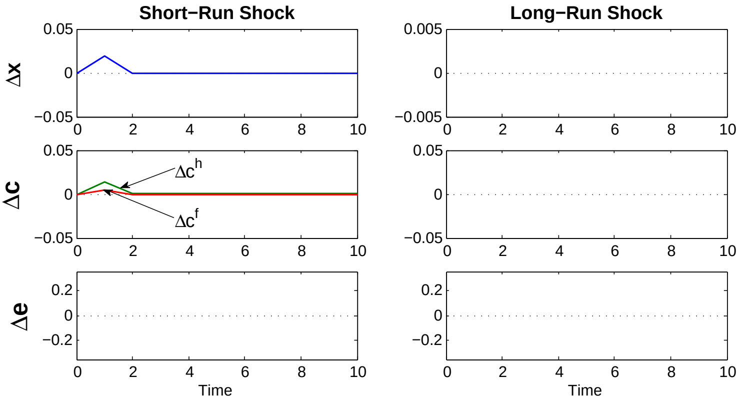

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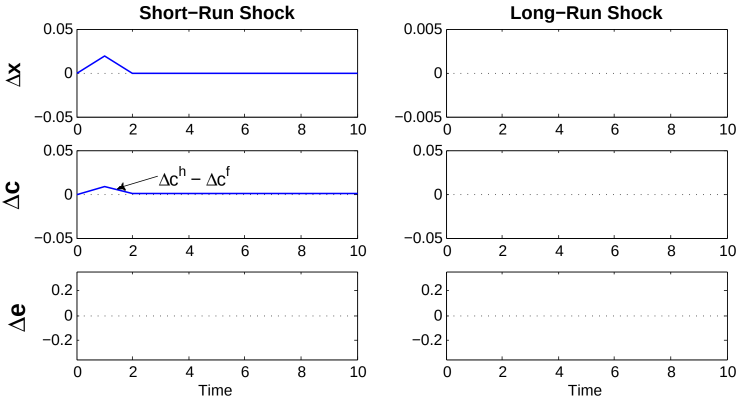

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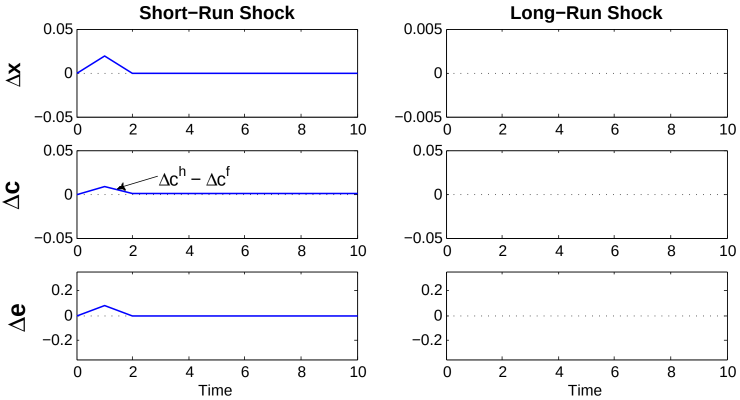

→ Home currency depreciates

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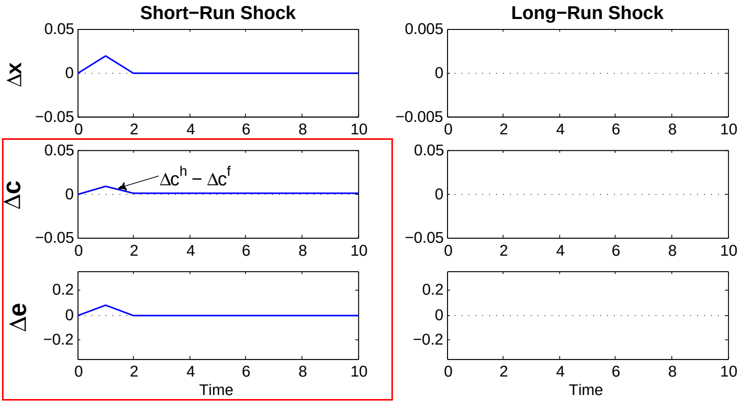

→ Home currency depreciates: $corr(\Delta c^h - \Delta c^f, \Delta e)$ is positive

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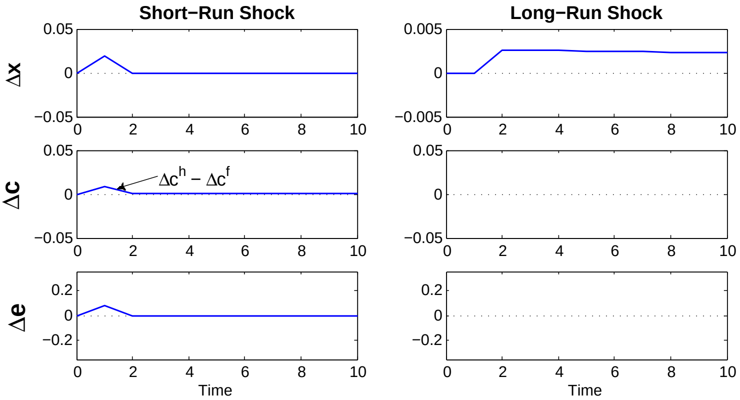

→ **Long**-run shock to X: home country is **very** happy!

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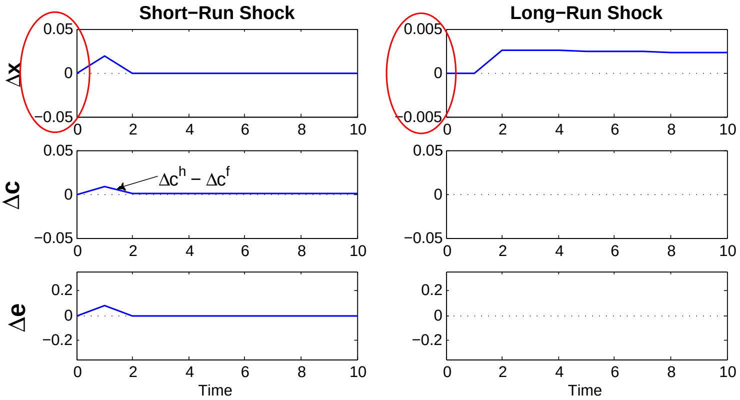

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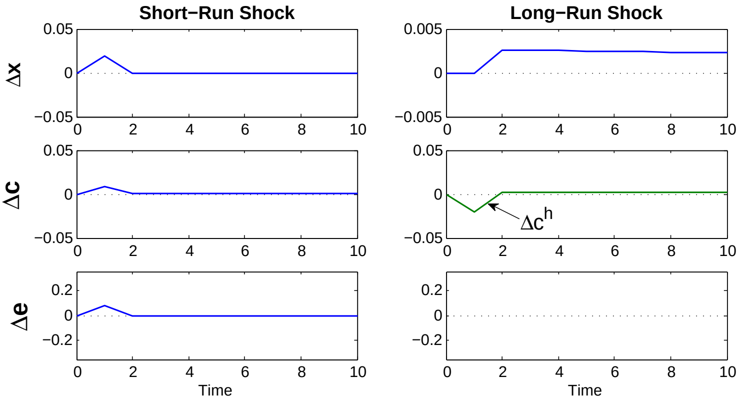

→ Home consumption falls to restore equilibrium.

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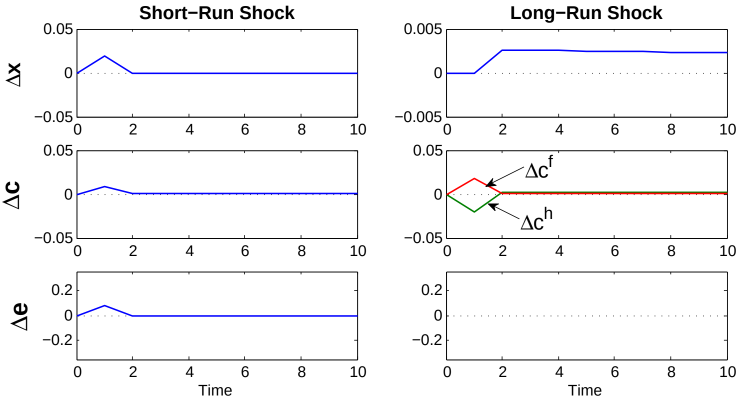

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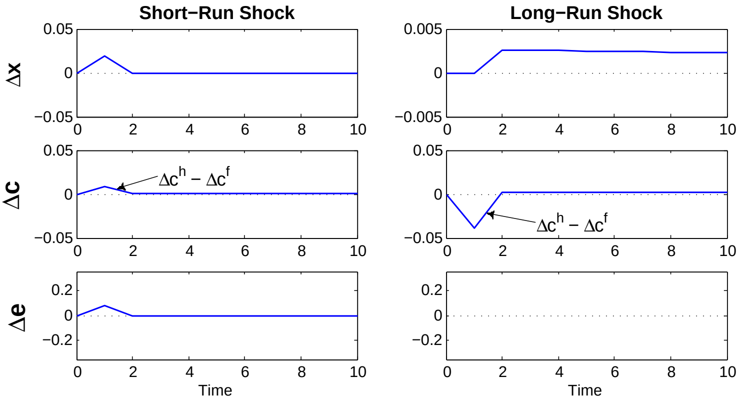

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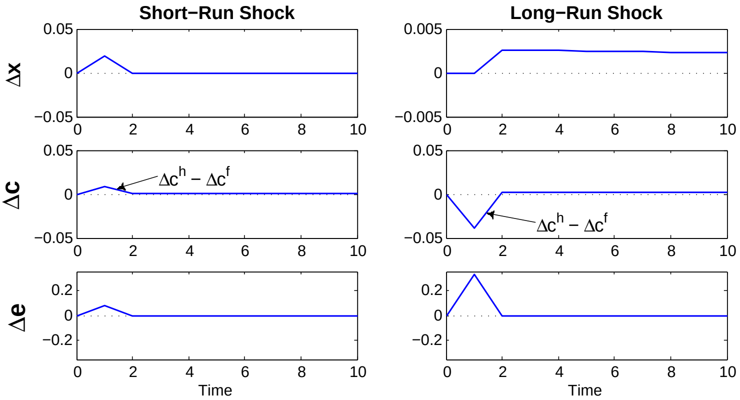

→ Home currency depreciates

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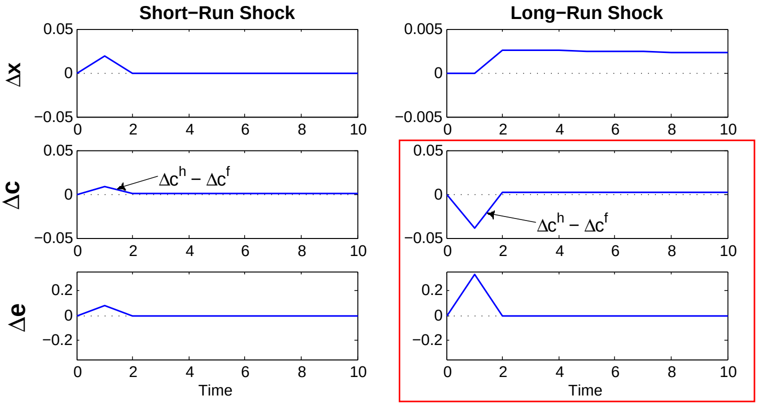

$\rightarrow$ Home currency depreciates: $corr(\Delta c^h - \Delta c^f, \Delta e)$ is negative

The Backus and Smith Anomaly

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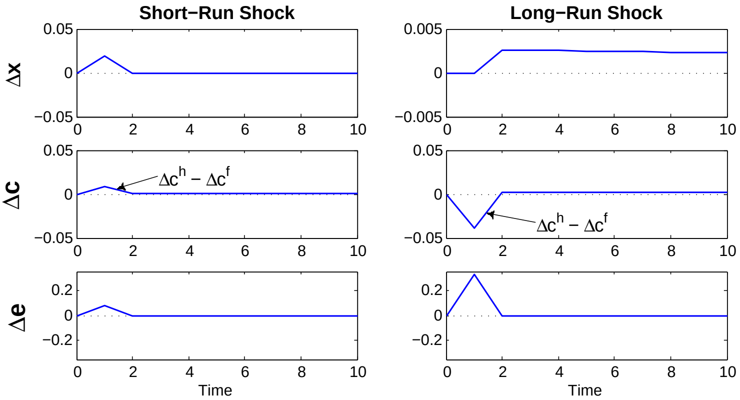

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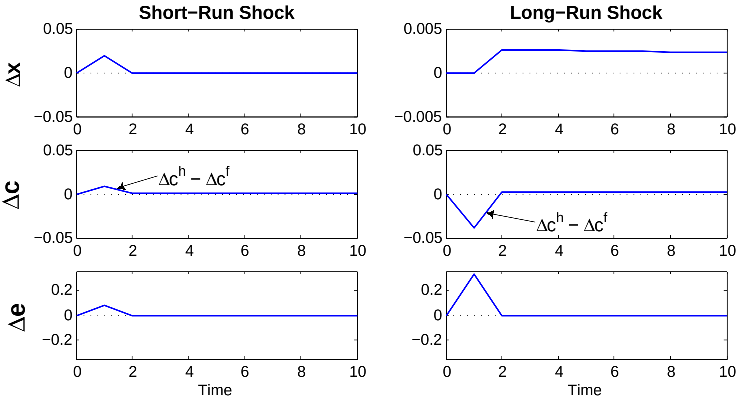

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Interest rate differential

$$\uparrow r_{t-1}^{h} - r_{t-1}^{f} \approx \uparrow E_{t-1}\left[\Delta c_t^h - \Delta c_t^f\right] + \frac{1}{2}\left(V_{t-1}\left[\Delta c_t^f\right] - V_{t-1}\left[\Delta c_t^h\right]\right)\uparrow$$

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Expected FX growth

$$\uparrow\downarrow E_{t-1}\left[\Delta e_t\right] = \uparrow E_{t-1}\left[\Delta c_t^h - \Delta c_t^f\right] + \frac{1}{2\theta^2}\left(Var_{t-1}\left[U_t^h\right] - Var_{t-1}\left[U_t^f\right]\right)\downarrow$$

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$$\uparrow r^h_{t-1} - r^f_{t-1} \approx \uparrow E_{t-1}\left[\Delta c^h_t - \Delta c^f_t\right] + \frac{1}{2}\left(V_{t-1}\left[\Delta c^f_t\right] - V_{t-1}\left[\Delta c^h_t\right]\right)\uparrow$$

Expected FX growth

$$\uparrow\downarrow E_{t-1}\left[\Delta e_t\right] = \uparrow E_{t-1}\left[\Delta c^h_t - \Delta c^f_t\right] + \frac{1}{2\theta^2}\left(Var_{t-1}\left[U^h_t\right] - Var_{t-1}\left[U^f_t\right]\right)\downarrow$$

$\rightarrow$ Assume that Home has good long-run news ($\varepsilon_{1,t}\uparrow$)

Forward Premium Anomaly

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Calibration

	Pre-1970	Post-1970

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Financial Regime	Portfolio Autarky	Complete Markets

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		Pre-1970	Post-1970
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μ	Mean Output growth	0.165%	
σ	Std. dev. of idiosyncratic output growth	0.540%	
σ_x	Std. dev. of long-run output growth	$4\% \cdot \sigma$	
α	Degree of consumption home bias	0.980	
δ	Subjective discount factor	0.998	
γ	Risk Aversion	7	
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ρ	Persistence of long-run risks	0.988	
ρ_{12}	Correlation of long-run risks	0.5	.9
ρ_{xy}	Correlation of short-run risks	-0.5	0.05

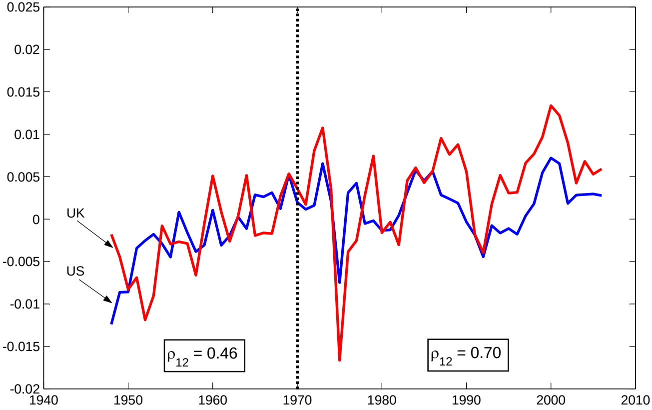
Correlation of Long-Run Risks: methodology

Methodology:

- Define GDP as sum of consumption and Net Export
- For each country: regress ΔGDP on lagged Δc, pd, cy
- Use projection as measure of long-run risks
- Apply to US and UK

Long-Run Risks

→ Using all NX



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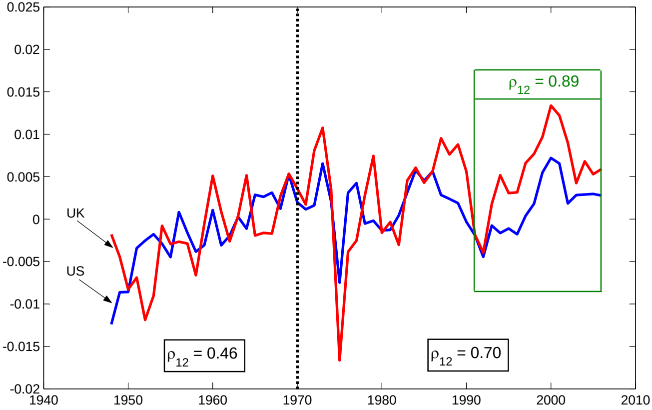
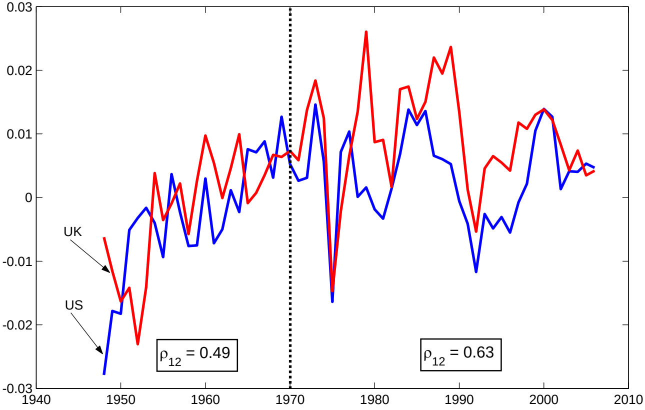


Long-Run Risks

→ Using only bilateral NX



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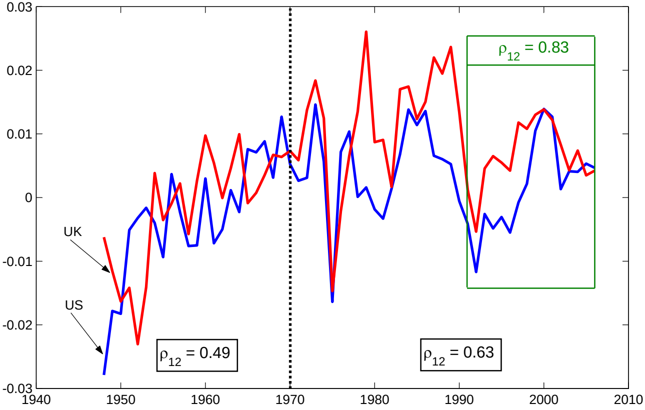

Results: quantities

	Pre-1970		Post-1970	
	Model (Portfolio Autarky)	Data (US,UK)	Model (Complete Markets)	Data (US,UK)
$Std[\Delta y^h]$				
$ACF_1[\Delta y^h]$				
$Std[\Delta c^h]$				
$ACF_1[\Delta c^h]$				
$corr[\Delta y_t^h, \Delta y_t^f]$				
$corr[\Delta c_t^h, \Delta c_t^f]$				
$Std[NX^h/Y^h]$				
$ACF_1[NX^h/Y^h]$				

Results: quantities

	Pre-1970		Post-1970	
	Model (Portfolio Autarky)	Data (US,UK)	Model (Complete Markets)	Data (US,UK)
$Std[\Delta y^h]$	1.920	2.483	1.920	1.865
$ACF_1[\Delta y^h]$	0.430	0.064	0.431	0.023
$Std[\Delta c^h]$	1.880	1.405	1.780	1.705
$ACF_1[\Delta c^h]$	0.430	0.283	0.463	0.396
$corr[\Delta y_t^h, \Delta y_t^f]$				
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$corr[\Delta c_t^h, \Delta c_t^f]$	-0.070	0.021	0.639	0.584
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$Std[NX^h/Y^h]$	0.000	1.426	0.810	3.020
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β_{UIP}				

Results: prices

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	Model (Portfolio Autarky)	Data (US,UK)	Model (Complete Markets)	Data (US,UK)
$Std[m^h]$	38.100	-	36.200	-
$Std[\Delta e]$	3.280	4.391	14.500	11.900
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$E[r_f^h]$	2.710	0.775	2.700	1.295
$Std[r_f^h]$	1.210	0.884	1.210	1.200
$corr[r_{f,t}^h, r_{f,t}^f]$	0.500	0.512	0.870	0.672
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$corr[\Delta c_t^h - \Delta c_t^f, \Delta e_t]$	-	-	-0.190	-0.020
β_{UIP}				

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Epstein and Zin Preferences

- Risk Sensitive preferences are a special case of Epstein and Zin preferences.
- What happens when the elasticity of intertemporal substitution is different from 1?
- Main results are confirmed, larger equity risk premium...

Epstein and Zin Preferences: Results

ψ	**1.5**	**1**	**0.66**
γ	10	10	10
$Std\,(\Delta e)$	18.02	18.14	27.57
$\mathrm{E}[r_c - r^f]$	3.24	0.28	-1.27
$\mathrm{E}[r_f]$	1.48	3.6	5.27
$\mathrm{Std}(r_f)$	0.85	1.47	2.06
$\mathrm{corr}(r_f^h, r_f^f)$	0.86	0.93	0.95
$\mathrm{corr}(\Delta c^h - \Delta c^f, \Delta e)$	-0.13	0	0.63
β_{UIP}	-2.56	-2.65	-3.29

Concluding Remarks

A two-countries model with:

- complete markets
- two goods
- long-run risks in the endowments
- recursive preferences

1. generates
   - dynamic risk-sharing scheme
   - endogenously time varying second moments
2. replicates a number of international finance facts
3. introduce frictions and investments