International Asset Pricing with Risk-Sensitive Agents

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THE UNIVERSITY of NORTH CAROLINA at CHAPEL HILL

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Motivation

Established empirical facts

- Volatility of exchange rates is time-varying
- Correlation of int'l stock market returns is time-varying
- Equity risk premia are time-varying
- Volatility of consumption growth is time-varying
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- time-additive preferences
- *i.i.d.* homoscedastic endowments

fails to replicate empirical findings...

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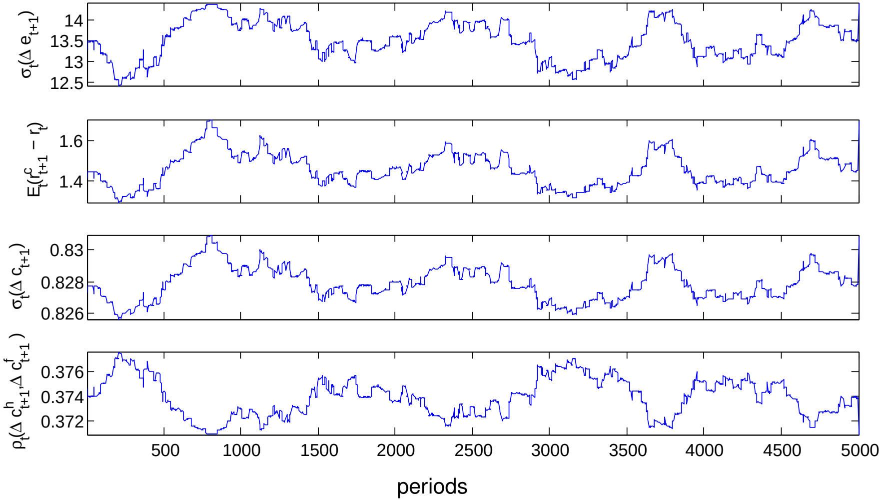
Established theoretical fact: a model with

- complete markets ✓
- ~~time-additive preferences~~ recursive preferences
- *i.i.d.* homoscedastic endowments ✓

can replicate empirical findings...

Endogenous heteroscedasticity and predictability



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  - Social planner's problem: **allocations** and time-varying Pareto weights

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  5. Small probability of large endowment drop
- Qualitative implications of the model

Setup of the Economy

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- Agents have risk-sensitive preferences

$$U_{i,t} = (1-\delta)\log C_{i,t} + \delta\theta \log E_t \exp\left\{\frac{U_{i,t+1}}{\theta}\right\}, \qquad \forall i \in \{h, f\}$$

where $\theta = 1/(1-\gamma)$.

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where $\theta = 1/(1-\gamma)$. If $\theta \to -\infty$: time additive case.

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$$U_{i,t} \quad \approx \quad (1-\delta)\log C_{i,t} + \delta E_t[U_{i,t+1}] + \frac{\delta}{2\theta} V_t[U_{i,t+1}], \quad \forall i \in \{h, f\}$$

where $\theta = 1/(1-\gamma)$. Conditional Variance matters.

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- Preferences are defined over the consumption aggregate

$$C_{h,t} = (x_{h,t})^{\alpha}(y_{h,t})^{1-\alpha} \quad \text{and} \quad C_{f,t} = (x_{f,t})^{1-\alpha}(y_{f,t})^{\alpha}$$

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- Consumption bias: $\alpha > 1/2$.
- Complete markets.
- Endowments are *i.i.d.* homoscedastic
  1. Two states: $HL = \{X = 103, Y = 100\}$ and $LH = \{X = 100, Y = 103\}$
  2. Introduce rare events

Risk Sharing Scheme:
Allocations

Planner's problem

Efficient allocations are the solution to the planner's problem

$$\begin{aligned}
\text{choose} \quad & \{x_{h,t}, x_{f,t}, y_{h,t}, y_{f,t}\}_{t=0}^{+\infty} \\
\text{to max} \quad & Q = \mu_h U_{h,0} + \mu_f U_{f,0} \\
\text{s.t.} \quad & x_{h,t} + x_{f,t} = X_t \\
& y_{h,t} + y_{f,t} = Y_t, \quad \forall t \geq 0
\end{aligned}$$

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\end{aligned}
$$

- μ_h and μ_f correspond to an initial distribution of assets.
- Notation: $S = \mu_h / \mu_f$.

What to look for...

$$\mu_h \cdot \frac{\alpha}{x^j_{h,t}} = \frac{1-\alpha}{x^j_{f,t}} \cdot \mu_f$$

What to look for...

$$\frac{\exp\left\{U^j_{h,t}/\theta\right\}}{E_{t-1}\left[\exp\left\{U_{h,t}/\theta\right\}\right]}\cdot\mu_h\cdot\frac{\alpha}{x^j_{h,t}} = \frac{1-\alpha}{x^j_{f,t}}\cdot\mu_f\cdot\frac{\exp\left\{U^j_{f,t}/\theta\right\}}{E_{t-1}\left[\exp\left\{U_{f,t}/\theta\right\}\right]}$$

What to look for...

$$\underbrace{\boxed{\frac{\exp\left\{U^j_{h,t}/\theta\right\}}{E_{t-1}\left[\exp\left\{U_{h,t}/\theta\right\}\right]}}}_{\mu^j_{h,t}} \cdot \mu_h \cdot \frac{\alpha}{x^j_{h,t}} = \frac{1-\alpha}{x^j_{f,t}} \cdot \mu_f \cdot \underbrace{\boxed{\frac{\exp\left\{U^j_{f,t}/\theta\right\}}{E_{t-1}\left[\exp\left\{U_{f,t}/\theta\right\}\right]}}}_{\mu^j_{f,t}}$$

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$\mu^j_{h,t}$ $\mu^j_{f,t}$

$$\boxed{\rightarrow\ U^j_{h,t} > U^j_{f,t}:\ \frac{\mu_{h,t}}{\mu_{f,t}}\downarrow}$$

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$$\uparrow \qquad\qquad\qquad\qquad \uparrow$$

$$\mu^j_{h,t} \qquad\qquad\qquad\qquad \mu^j_{f,t}$$

$$\boxed{\begin{array}{l}\rightarrow\ U^j_{h,t} > U^j_{f,t}\text{: } \frac{\mu_{h,t}}{\mu_{f,t}}\downarrow \\ \rightarrow\ U^j_{h,t} < U^j_{f,t}\text{: } \frac{\mu_{h,t}}{\mu_{f,t}}\uparrow\end{array}}$$

What to look for...

$$\frac{x^j_{h,t}}{x^k_{h,t}} = \frac{x^j_{f,t}}{x^k_{f,t}}$$

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Time Additive Allocations

$$\left[\left(\frac{X^j_t}{X^k_t}\right)^{\alpha}\left(\frac{Y^j_t}{Y^k_t}\right)^{(1-\alpha)}\right]^{(1-\delta)/\theta}$$

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$$\exp\left\{\frac{U^{j}_{h,t}-U^{k}_{h,t}}{\theta}\right\}\cdot\frac{x^{j}_{h,t}}{x^{k}_{h,t}} \neq \frac{x^{j}_{f,t}}{x^{k}_{f,t}}\cdot\exp\left\{\frac{U^{j}_{f,t}-U^{k}_{f,t}}{\theta}\right\}$$

Time Additive Allocations

$$\left[\left(\frac{X^{j}_{t}}{X^{k}_{t}}\right)^{\alpha}\left(\frac{Y^{j}_{t}}{Y^{k}_{t}}\right)^{(1-\alpha)}\right]^{(1-\delta)/\theta}$$

$$\boxed{\alpha > \frac{1}{2}}$$

Time Additive Allocations

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What to look for...

$$\exp\left\{\frac{U_{h,t}^{LH}-U_{h,t}^{HL}}{\theta}\right\}\cdot\frac{x_{h,t}^{LH}}{x_{h,t}^{HL}} \neq \frac{x_{f,t}^{LH}}{x_{f,t}^{HL}}\cdot\exp\left\{\frac{U_{f,t}^{LH}-U_{f,t}^{HL}}{\theta}\right\}$$

Time Additive Allocations

$$\left(\frac{103}{100}\right)^{(2\alpha-1)(\delta-1)/\theta}$$

$$\alpha > \frac{1}{2}$$

Time Additive Allocations

$$\left(\frac{100}{103}\right)^{(2\alpha-1)(\delta-1)/\theta}$$

$\rightarrow$ Eg 2 states: $HL=\{X=103, Y=100\}$, $LH=\{X=100, Y=103\}$

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$U_{h,t}^{LH} \uparrow$, $U_{h,t}^{HL} \downarrow$

$\alpha > \frac{1}{2}$

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$U_{h,t}^{LH}\uparrow,\ U_{h,t}^{HL}\downarrow$

$\alpha > \frac{1}{2}$

$U_{f,t}^{HL}\uparrow,\ U_{f,t}^{LH}\downarrow$

$\rightarrow$ Consumption home bias matters

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$\rightarrow$ Consumption home bias matters

$\rightarrow$ Utility smoothing matters

Allocations

$\boxed{\text{Time Additive Preferences}}$

Let $k = \frac{\alpha}{1-\alpha}$:

$$x_t^h = \frac{kS}{1+kS} X_t, \qquad x_t^f = \frac{1}{1+kS} X_t$$
$$y_t^h = \frac{S}{k+S} Y_t, \qquad y_t^f = \frac{k}{k+S} Y_t$$

where

$$S = \mu_h / \mu_f$$

Allocations

Risk Sensitive Preferences

Let $k = \frac{\alpha}{1-\alpha}$:

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$$y_t^h = \frac{S_t}{k+S_t} Y_t, \qquad y_t^f = \frac{k}{k+S_t} Y_t$$

where

$$S_t = S_{t-1} \cdot \frac{\delta \exp\{U_{h,t}/\theta\}}{E_{t-1} \exp\{U_{h,t}/\theta\}} \bigg/ \frac{\delta \exp\{U_{f,t}/\theta\}}{E_{t-1} \exp\{U_{f,t}/\theta\}}$$

Questions

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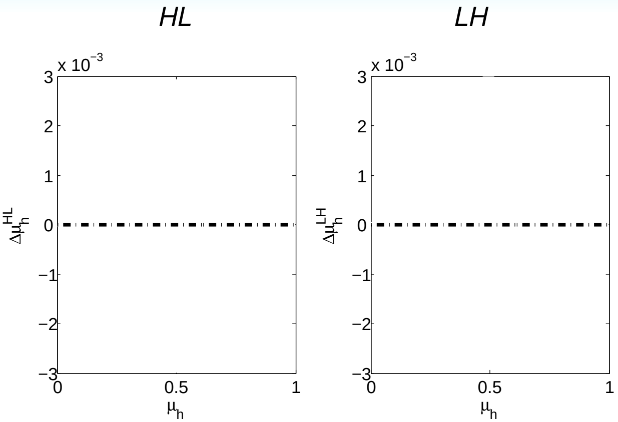
2. **Why** does S_t move?
3. **What** is S_t?

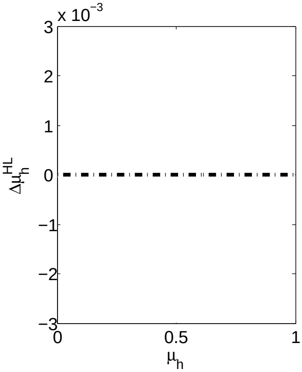
   $\rightarrow$ Decentralized Economy.

How does S_t move?



How does S_t move?

HL



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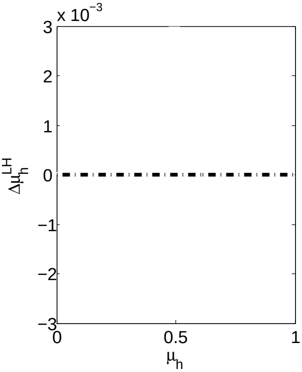

$\longrightarrow$ **Abundant** X, scarce Y:

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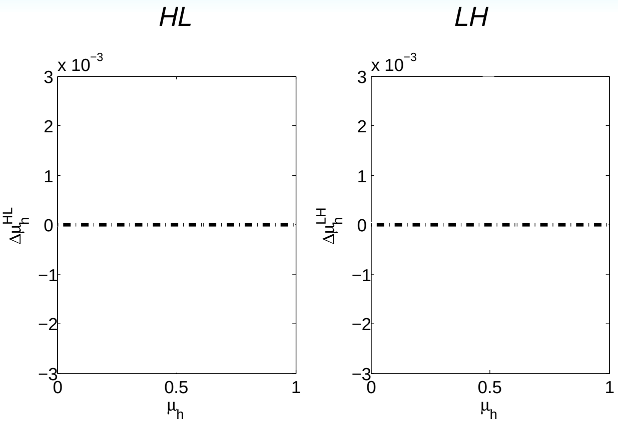
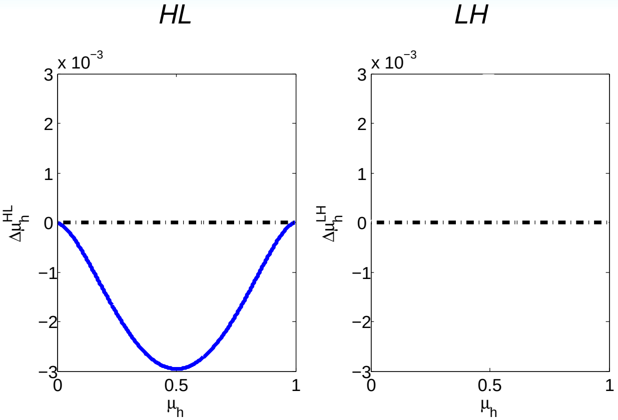


→ **Abundant** X, scarce Y:

→ Good news for home

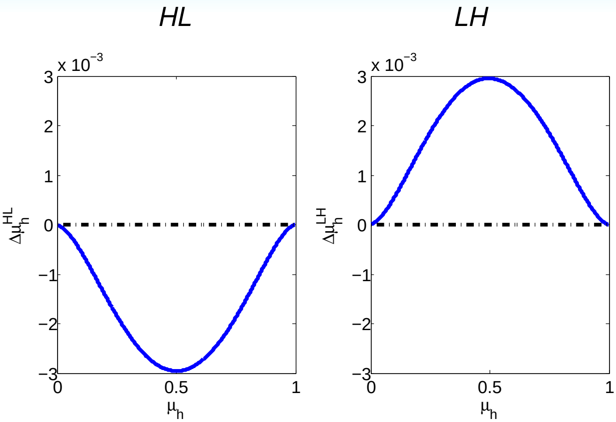
How does S_t move?



$\longrightarrow$ **Abundant** X, scarce Y:

- $\longrightarrow$ Good news for home
- $\longrightarrow$ Home Pareto weight $\downarrow$

How does S_t move?



$\longrightarrow$ **Scarce** X, abundant Y:

$\longrightarrow$ Bad news for home

$\longrightarrow$ Home Pareto weight $\uparrow$

Why does S_t move?

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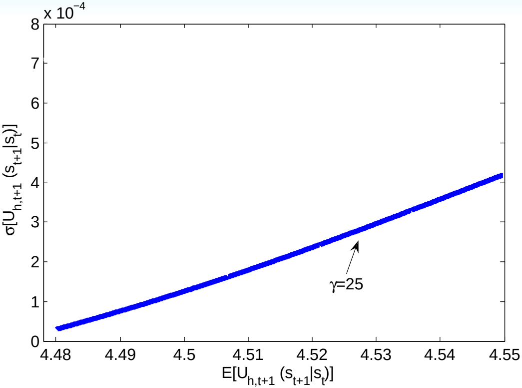
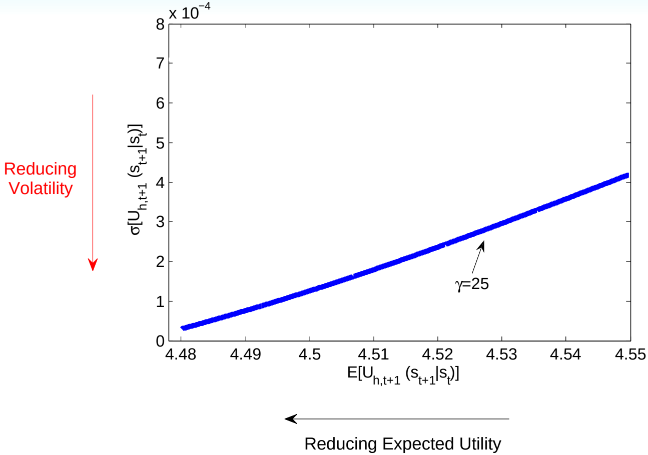


Reducing Expected Utility

Why does S_t move?



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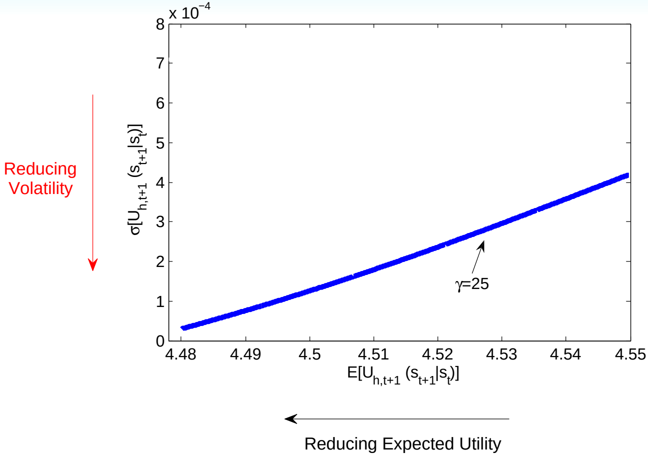
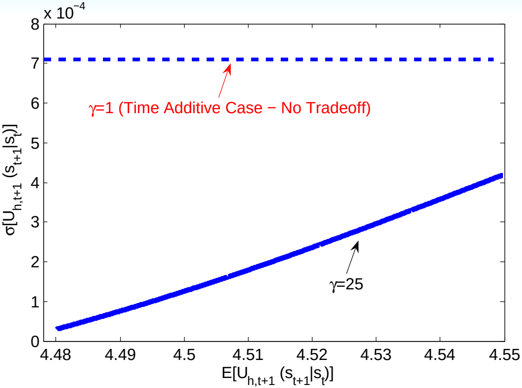
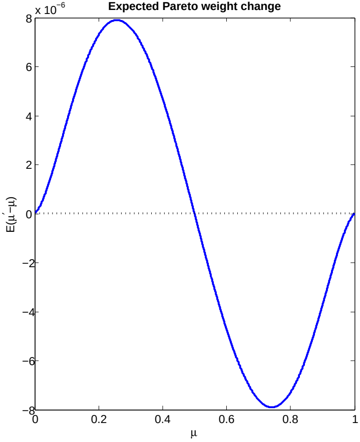


Trade-off between Expected Utility and Utility Variance

Why does S_t move?

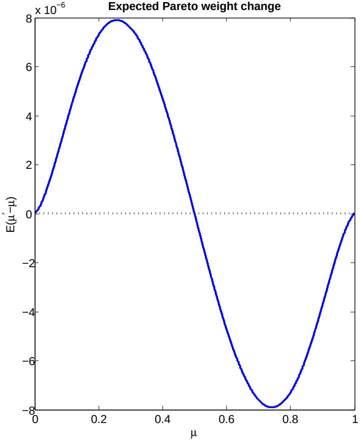


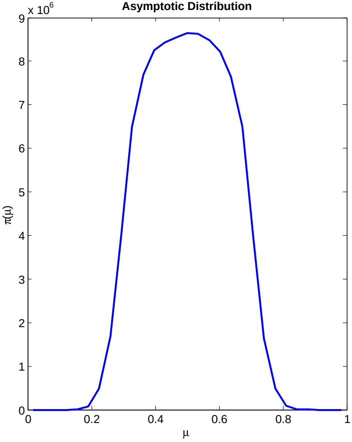
Pareto weights: expected growth and distribution



$\longrightarrow$ Mean Reversion

Pareto weights: expected growth and distribution





$\longrightarrow$ Mean Reversion $\qquad$ $\longrightarrow$ Symmetric Distribution

Risk Sharing Scheme:
State Prices

Decentralized equilibrium

Decentralized equilibrium

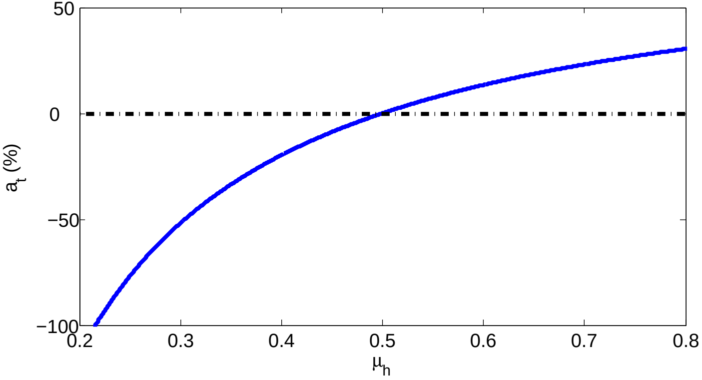
- *Home* is endowed with good X;
- *Foreign* is endowed with good Y;
- Complete set of one period ahead state contingent securities;
- Budget constraints:

$$x_{h,t} + p_t y_{h,t} + \sum_{s_{t+1}} Q_{t+1}(s_{t+1}) A_{t+1}(s_{t+1}) \leq X_t + A_t \qquad [\textit{Home}]$$

$$x_{f,t} + p_t y_{f,t} - \sum_{s_{t+1}} Q_{t+1}(s_{t+1}) A_{t+1}(s_{t+1}) \leq p_t Y_t - p_t A_t \qquad [\textit{Foreign}]$$

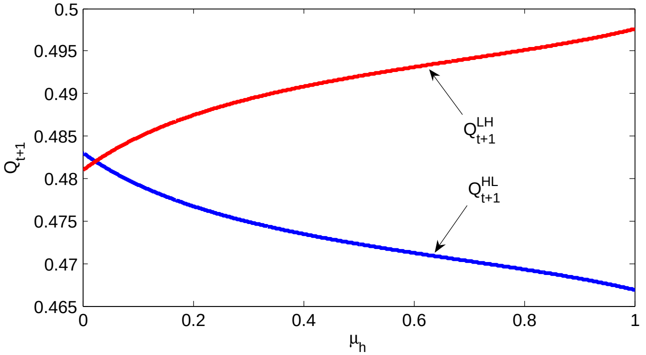
- Use welfare theorems to find state prices and asset holdings.

Contingent securities

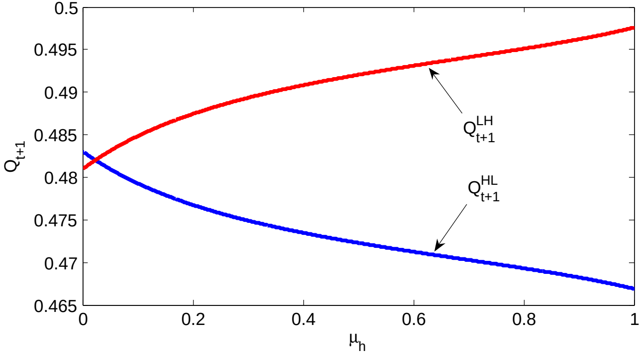


$\longrightarrow$ Savings are increasing in Pareto weight

State Prices



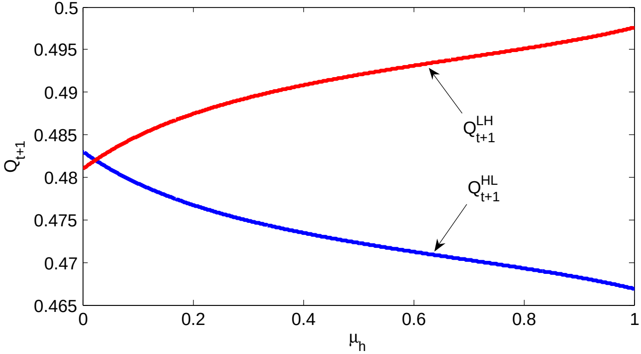
State Prices



→ State prices reflect continuation utilities:

$$\frac{Q^{LH}_{t+1}}{Q^{HL}_{t+1}} = \frac{x^{HL}_{h,t+1}}{x^{LH}_{h,t+1}} \exp\left\{ \frac{U^{LH}_{h,t+1} - U^{HL}_{h,t+1}}{\theta} \right\}$$

State Prices



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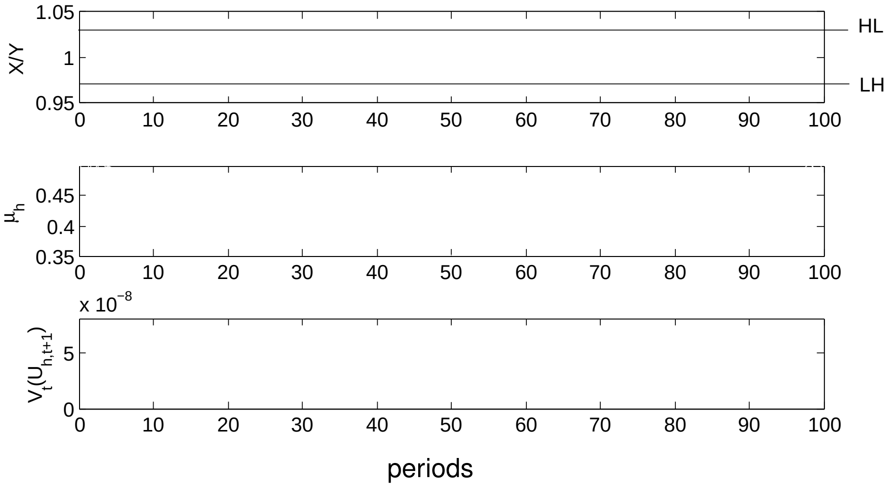
→ Time varying volatility of utilities

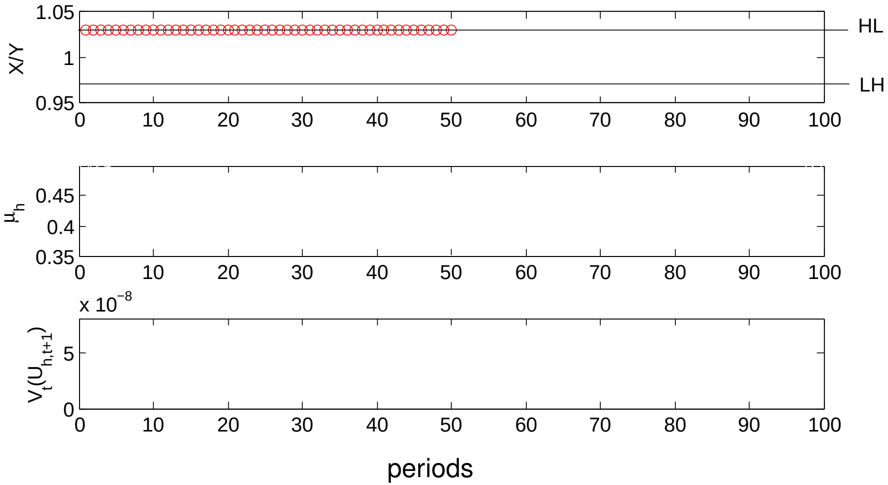
Risk Sharing Scheme:

Endogenously time-varying volatilities and correlations

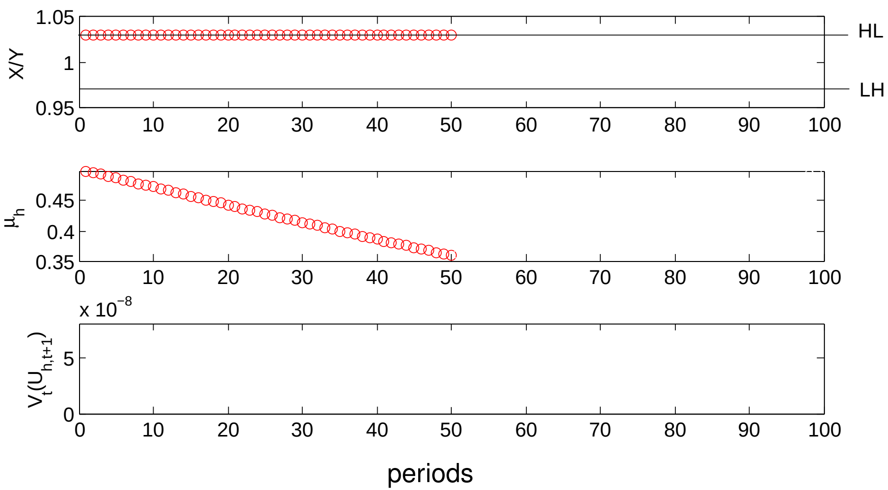
Conditional Volatilities



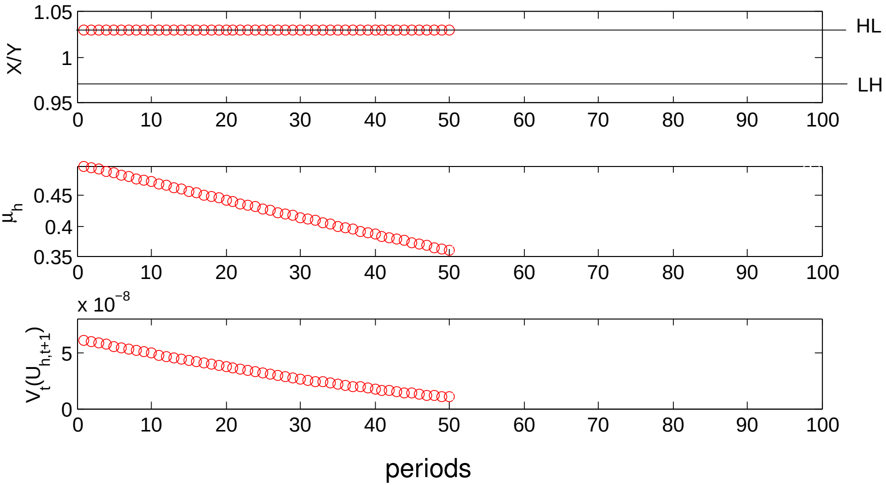
Conditional Volatilities



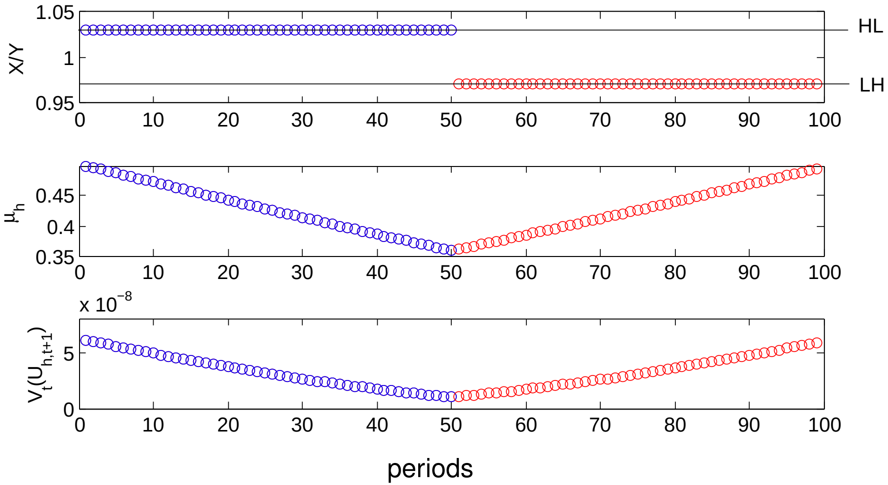
Conditional Volatilities



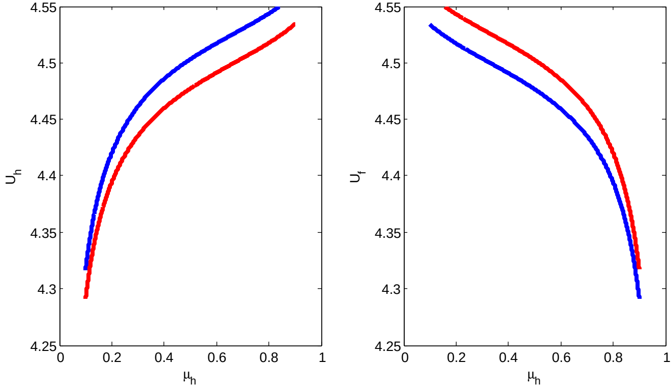
Conditional Volatilities



Conditional Volatilities



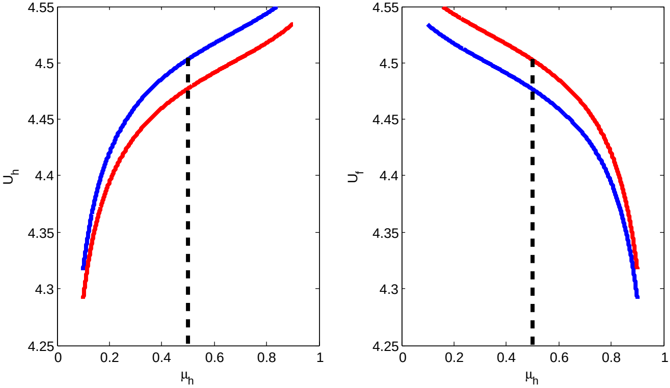
Utilities' correlations



$\longrightarrow$ Blue curves: utilities when supply of good X is high

$\longrightarrow$ Red curves: utilities when supply of good Y is high

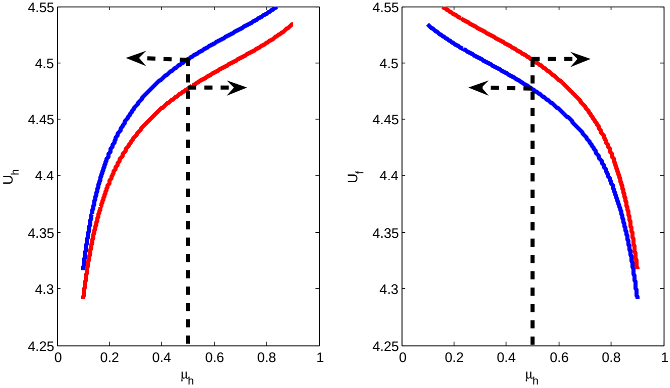
Utilities' correlations



→ **Time additive** preferences:

→ Home utility is high (low) when foreign utility is low (high)

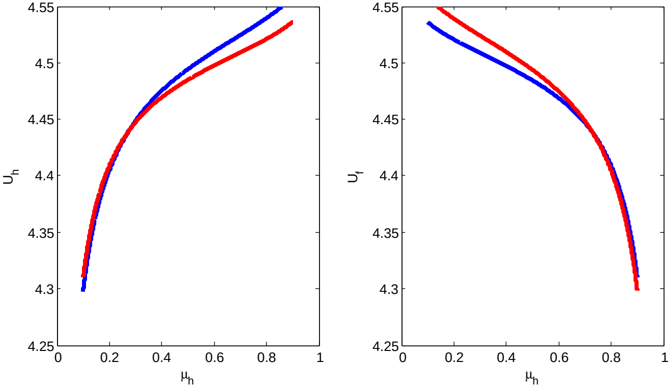
Utilities' correlations



$\longrightarrow$ **Risk-sensitive** preferences:

- $\longrightarrow$ Must take into account international redistribution of wealth

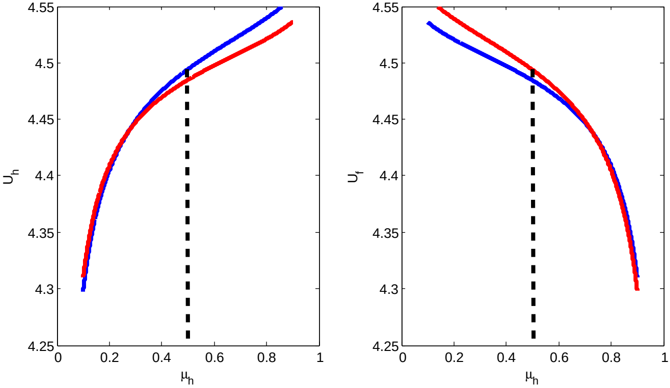
Utilities' correlations



$\longrightarrow$ Continuation utilities including redistribution of wealth

$\longrightarrow$ (i.e. U_{t+1} as a function of μ_t)

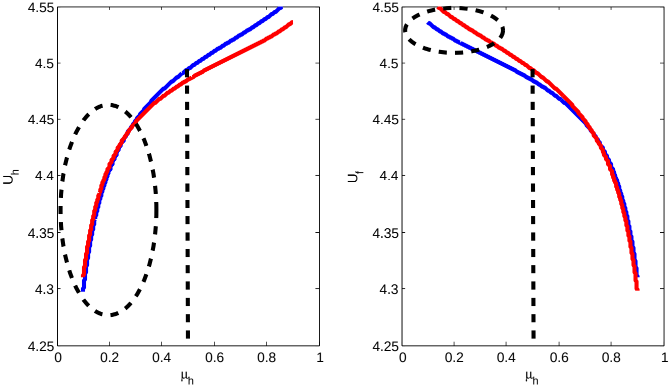
Utilities' correlations



$\longrightarrow$ If wealth is similar:

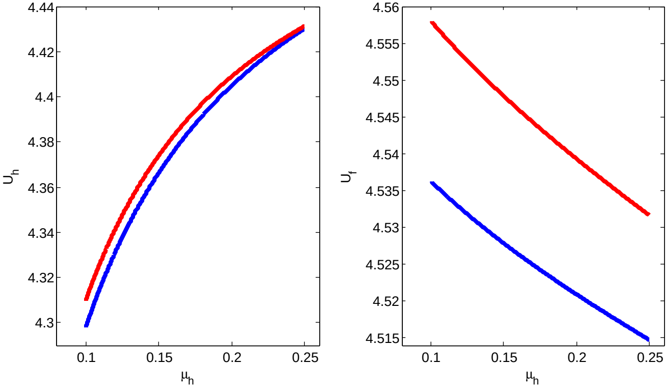
$\longrightarrow$ Home utility is high (low) when foreign utility is low (high)

Utilities' correlations



→ What if one country is more wealthy than the other?

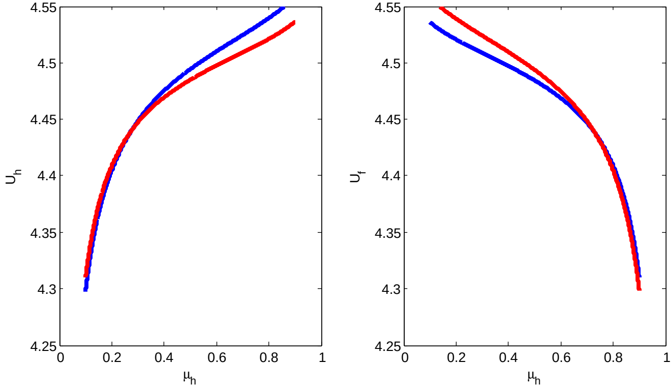
Utilities' correlations



$\longrightarrow$ If wealth is **not** similar:

- $\longrightarrow$ Home utility is high (low) when foreign utility is high (low)

Utilities' correlations



Correlation of utilities increases with wealth inequality

International Asset Pricing

Introducing Rare Events

X	Y	π
103	103	0.2375
103	100	0.2375
100	103	0.2375
100	100	0.2375
103	60	0.0100
100	60	0.0100
60	60	0.0100
60	103	0.0100
60	100	0.0100

Introducing Rare Events

Four equally likely no-disaster events

X	Y	π
103	103	0.2375
103	100	0.2375
100	103	0.2375
100	100	0.2375
103	60	0.0100
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60	103	0.0100
60	100	0.0100

Five equally likely disaster events

Stochastic Discount Factors

- International Stochastic Discount Factors:

$$\log M_{i,t+1} = \log \delta + \log \frac{C_{i,t}}{C_{i,t+1}} + \log \frac{\exp\{U_{i,t+1}/\theta\}}{E_t \exp\{U_{i,t+1}/\theta\}}, \quad \forall i \in \{h, f\}$$

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- Properties:

1. Volatility is high
2. Volatility is time-varying
3. Correlation is high
4. Correlation is time-varying

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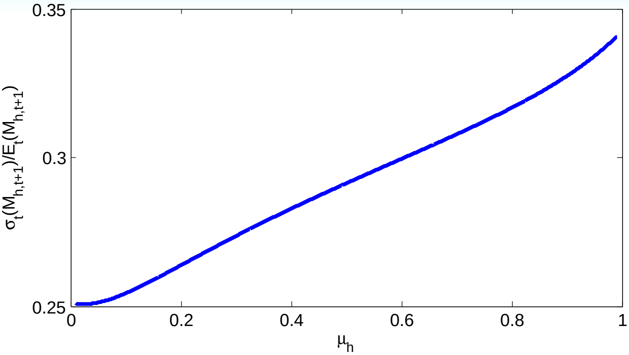
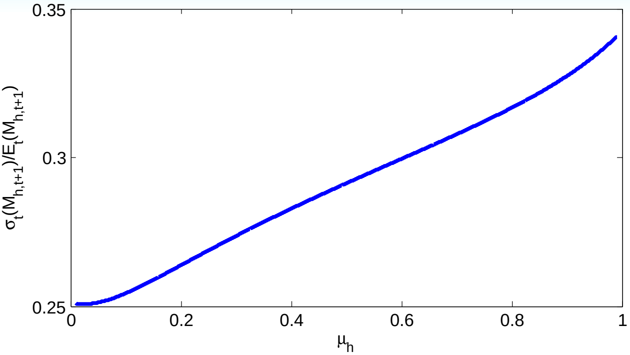
- Properties:
  1. Volatility is high $\Rightarrow$ Equity Sharpe ratios are high
  2. Volatility is time-varying $\Rightarrow$ Equity risk-premia are time-varying
  3. Correlation is high $\Rightarrow$ Volatility of FX growth is "low"
  4. Correlation is time-varying $\Rightarrow$ Volatility of FX growth is time-varying

Conditional volatility of SDF



$\longrightarrow$ Average Volatility $\approx 30\%$

Conditional volatility of SDF

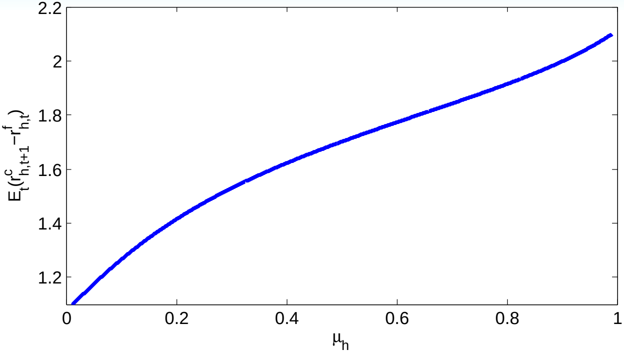


$\longrightarrow$ Average Volatility $\approx$ 30%

$\longrightarrow$ Equity risk-premia

$$E_t\left[r^c_{h,t+1} - r^f_{h,t}\right] = -\rho_t\left(\Delta c_{h,t+1}, M_{h,t+1}\right)\sigma_t\left(\Delta c_{h,t+1}\right)\frac{\sigma_t\left(M_{h,t+1}\right)}{E_t\left(M_{h,t+1}\right)}$$

Conditional volatility of SDF



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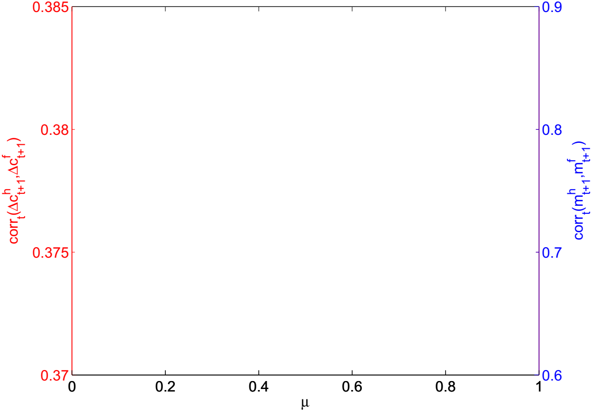
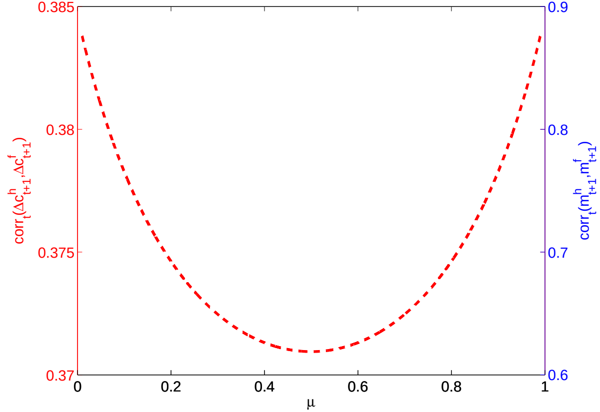
are time varying and counter-cyclical.

Conditional Correlations

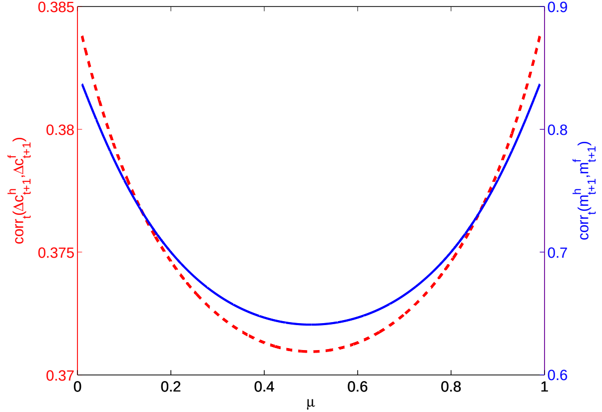


Conditional Correlations



$\longrightarrow$ Low, time-varying correlation of consumption

Conditional Correlations



$\longrightarrow$ Low, time-varying correlation of consumption

$\longrightarrow$ High, time-varying correlation of marginal utilities

Conditional volatility of FX growth

$$\boxed{\Delta e_{t+1} = m_{f,t+1} - m_{h,t+1}}$$

Conditional volatility of FX growth

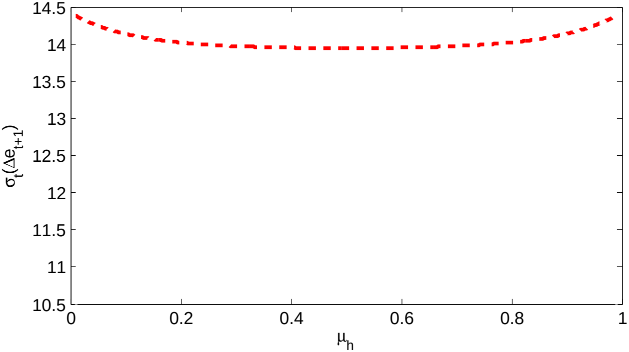
$$\boxed{V_t[\Delta e_{t+1}] = V_t[m_{f,t+1} - m_{h,t+1}]}$$

Conditional volatility of FX growth

$$\boxed{V_t[\Delta e_{t+1}] = V_t[m_{f,t+1}] + V_t[m_{h,t+1}] - 2\rho_t \cdot \sqrt{V_t[m_{f,t+1}]} \cdot \sqrt{V_t[m_{h,t+1}]}}$$

Conditional volatility of FX growth

$$\boxed{V_t[\Delta e_{t+1}] = V_t[m_{f,t+1}] + V_t[m_{h,t+1}] - 2\overline{\rho} \cdot \sqrt{V_t[m_{f,t+1}]} \cdot \sqrt{V_t[m_{h,t+1}]}}$$



$\longrightarrow$ Average Volatility $\approx$ 14%

Conditional volatility of FX growth

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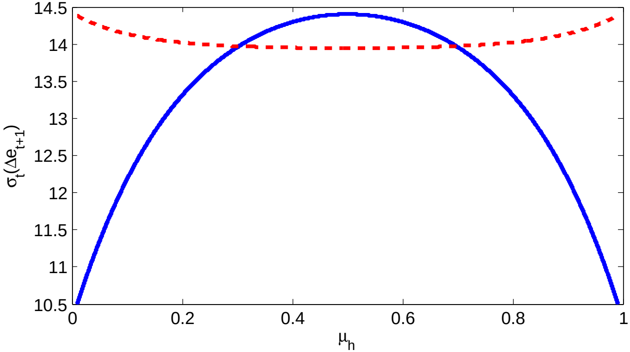


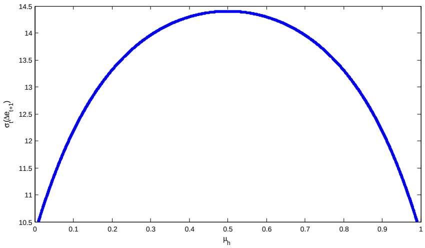
→ Average Volatility ≈ 14%

→ Time-varying exchange rate volatility

Qualitative implications

Qualitative implications

1. Inverse relationship between
   - Volatility of exchange rate
   - Absolute level of savings



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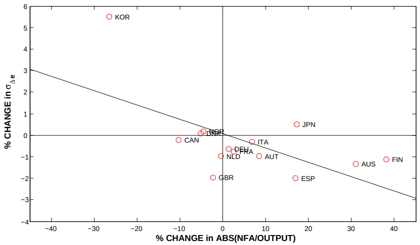

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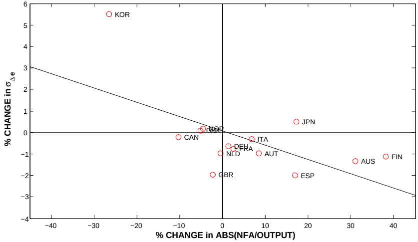

2. Positive relationship between
   - Volatility of consumption
   - Level of savings

Qualitative implications

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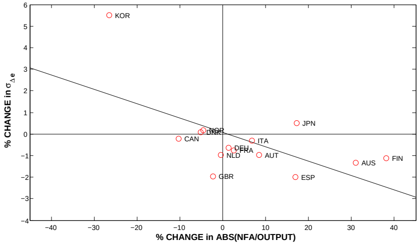

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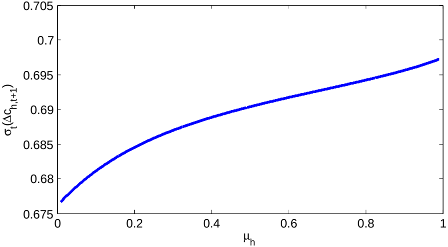

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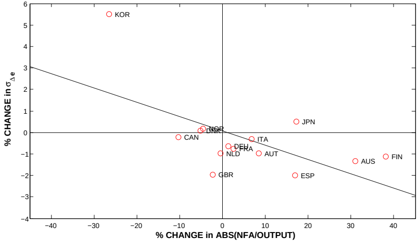

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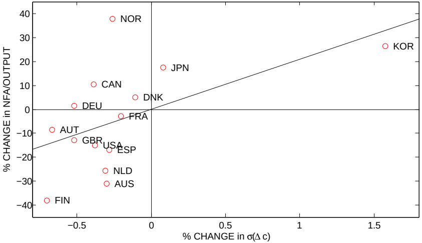
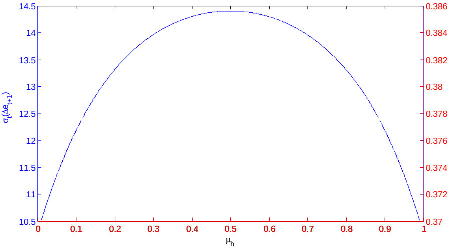


Qualitative implications (cont'd)

3. Inverse relationship between

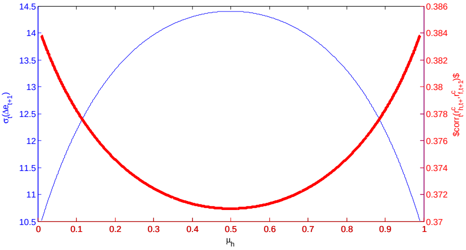
Qualitative implications (cont'd)

3. Inverse relationship between
   - Volatility of exchange rate



Qualitative implications (cont'd)

3. Inverse relationship between
   - Volatility of exchange rate
   - Correlation of returns



Qualitative implications (cont'd)

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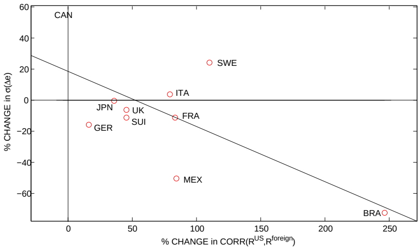

Concluding remarks

A two-countries model with:

- complete markets
- two goods
- i.i.d. endowments
- risk-sensitive preferences

1. generates
   - dynamic risk-sharing scheme
   - endogenously time varying second moments
2. replicates a number of international finance facts
3. introduce frictions and investments