RISKS FOR THE LONG RUN:
A POTENTIAL RESOLUTION OF
ASSET PRICING PUZZLES

Ravi Bansal and Amir Yaron (2004)
The Journal of Finance
Recursive Preferences

1)  
\[
\begin{array}{c}
1/2 \\
2 \\
1/2 \\
1 \\
\end{array}
\]

Permanent shock

2)  
\[
\begin{array}{c}
1/2 \\
2 \\
1/2 \\
1 \\
\end{array}
\]

i.i.d. every period
Roadmap

- **Setup of the economy**
  1) Endowment: consumption and dividend
    - Two Channels of Economic Risks
      - Long-Run Expected Growth
      - Time-Varying Uncertainty
  2) Preference: Recursive Preference
    - Epstein and Zin (1989)

- **Equilibrium**
  - Euler Equation
  - IMRS (Endogenous)
  - Equity Premium
Recursive Preferences

- Epstein and Zin (1989)

\[
U_t = \left[ (1 - \delta) C_t^{\frac{1-\gamma}{\theta}} + \delta \left( E_t[U_{t+1}^{1-\gamma}] \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}}
\]

where \( \theta = \frac{1 - \gamma}{1 - \frac{1}{\psi}} \), \( \gamma : \text{Risk Aversion} \), \( \psi : \text{IES} \)

1) Time-Inseparable Utility
2) Utility Smoothing
3) Separation of risk aversion and IES
4) The timing of resolution of uncertainty

\( \gamma > \frac{1}{\psi} : \text{Early resolution of uncertainty is preferred} \)

\( \gamma = \frac{1}{\psi} : \text{Indifferent to the timing of resolution of uncertainty} \)

\( \gamma < \frac{1}{\psi} : \text{Late Resolution of Uncertainty is preferred} \)
Recursive Preferences

Example

\[ U_t = (1 - \delta) \log C_t + \delta \theta \log E_t [\exp \left( \frac{U_{t+1}}{\theta} \right)] \]

where \[ \theta = \frac{1}{1 - \gamma} \]

Assume \[ U_t \sim \text{Normal} \]

\[ U_t = (1 - \delta) \log C_t + \delta E_t(U_{t+1}) + \frac{1}{2\theta} \text{Var}_t(U_{t+1}) \]

Time additive case

Epstein and Zin

Conditional variance of utility matters
Two Channels of Economic Risks

- Small persistent predictable component in consumption growth and dividend growth

\[
x_{t+1} = \rho x_t + \varphi \sigma e_{t+1} \\
g_{t+1} = \mu + x_t + \sigma \eta_{t+1} \\
g_{d,t+1} = \mu_d + \phi x_t + \varphi_d \sigma u_{t+1}
\]

\[e_{t+1}, u_{t+1}, \eta_{t+1} \sim N.i.i.d.(0, 1),\]

- \(\rho\): The persistence of expected growth rate

- Time-Varying Uncertainty

\[
\sigma_{t+1}^2 = \sigma^2 + v_1 (\sigma_t^2 - \sigma^2) + \sigma_w w_{t+1}
\]

\[e_{t+1}, u_{t+1}, \eta_{t+1}, w_{t+1} \sim N.i.i.d.(0, 1),\]

- \(v_1\): The persistence of economics uncertainty
Long-Run Risks in Consumption

- In Macroeconomics literature, i.i.d. Gaussian innovations model consumption growth very well.

- But it is difficult to distinguish i.d.d. process from the specification in which the drift in log consumption growth is a highly persistent with low conditional volatility but high unconditional volatility.

- The asset pricing implications of those two processes are tremendous.
Asset Pricing Model

- **Euler Equation**

\[
E_t \left[ \delta^\theta G_{t+1}^\theta R_{a,t+1}^{-(1-\theta)} R_{i,t+1} \right] = 1
\]
\[
= M_{t+1}
\]

- **Stochastic Discount Factor (IMRS)**
  - Take log of IMRS and asset returns

\[
\ln M_{t+1} = \theta \ln \delta - \frac{\theta}{\psi} g_{t+1} + (\theta - 1)r_{a,t+1}
\]

\[
E_t \left[ \exp \left( \theta \ln \delta - \frac{\theta}{\psi} g_{t+1} + (\theta - 1)r_{a,t+1} + r_{i,t+1} \right) \right] = 1
\]

- Need to solve for \( r_{a,t+1} \)
Asset Pricing Model

- Return Approximation (Campbell and Shiller (1998))

\[ r_{a,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + g_{t+1} \]

\[ z_t : \log \text{price} - \text{dividend ratio} \left( \log \frac{P_t}{C_t} \right) \]

- Guess \( z_t \) substitute this into Euler Equation

\[ z_t = A_0 + A_1 x_t + A_2 \sigma_t^2 \]

\[ A_1 = \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho} \quad A_2 = \frac{0.5 \left( \frac{\theta - \frac{\theta}{\psi}}{\theta(1 - \kappa_1 \nu_1)} + (\theta A_1 \kappa_1 \varphi_c)^2 \right)}{\theta(1 - \kappa_1 \nu_1)} \]

- If \( IES > 1 \), the substitution effect dominates the wealth effect and then buy more assets and wealth-consumption increase

- If \( IES < 1 \), the wealth effect dominates the substitution effect and then sell more assets and wealth-consumption decrease
Asset Pricing Model

- **Innovation in Pricing Kernel**

\[ m_{t+1} - E_t(m_{t+1}) = \lambda_{m,\eta} \sigma_t \eta_{t+1} - \lambda_{m,e} \sigma_t e_{t+1} - \lambda_{m,w} \sigma_w \omega_{t+1} \]

\[ \lambda_{m,\eta} \equiv \left[ -\frac{\theta}{\psi} + (\theta - 1) \right] = -\gamma \]

\[ \lambda_{m,e} = (1 - \theta) \kappa_1 (1 - \frac{1}{\psi}) \frac{\psi_e}{1 - \kappa_1 \rho} \]

\[ \lambda_{m,w} \equiv (1 - \theta) A_2 \kappa_1. \]

*As \( \rho \) and \( \nu_1 \) increase, \( \lambda_{m,e} \) and \( \lambda_{m,w} \)*

- **Asset Pricing Relation**

\[ E_t(r_{m,t+1} - r_{f,t}) = -\text{cov}_t[m_{t+1} - E_t(m_{t+1}), r_{m,t+1} - E_t(r_{m,t+1})] - 0.5 \text{var}_t(r_{m,t+1}) \]

- **Equity Premium**

\[ E_t(r_{m,t+1} - r_{f,t}) = \beta_{m,e} \lambda_{m,e} \sigma_t^2 + \beta_{m,w} \lambda_{m,w} \sigma_w^2 - 0.5 \text{var}_t(r_{m,t+1}) \]

\[ \beta_{m,e} \equiv \kappa_1 m A_{1,m} \psi_e \quad \beta_{m,w} \equiv \kappa_1 A_{2,m} \]

*As \( \rho \) and \( \nu_1 \) increase, \( \beta_{m,e} \) and \( \beta_{m,w} \)*
Calibration

- Calibration of model parameters
  - The Model is at monthly frequency
  - Time-aggregated annual growth rates of consumption and dividend match the annual data
  - Assume constant economic uncertainty to isolate the effects of persistent expected growth
  - 1,000 Monte Carlo experiment with 840 monthly observations
### Table I

**Annualized Time-Averaged Growth Rates**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>SE</th>
<th>Data</th>
<th></th>
<th>95%</th>
<th>5%</th>
<th>p-Val</th>
<th>Pop</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(g)$</td>
<td>2.93</td>
<td>(0.69)</td>
<td>2.72</td>
<td>3.80</td>
<td>2.01</td>
<td>0.37</td>
<td>2.88</td>
<td></td>
</tr>
<tr>
<td>$AC(1)$</td>
<td>0.49</td>
<td>(0.14)</td>
<td>0.48</td>
<td>0.65</td>
<td>0.21</td>
<td>0.53</td>
<td>0.53</td>
<td></td>
</tr>
<tr>
<td>$AC(2)$</td>
<td>0.15</td>
<td>(0.22)</td>
<td>0.23</td>
<td>0.50</td>
<td>−0.17</td>
<td>0.70</td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td>$AC(5)$</td>
<td>−0.08</td>
<td>(0.10)</td>
<td>0.13</td>
<td>0.46</td>
<td>−0.13</td>
<td>0.93</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td>$AC(10)$</td>
<td>0.05</td>
<td>(0.09)</td>
<td>0.01</td>
<td>0.32</td>
<td>−0.24</td>
<td>0.80</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>$VR(2)$</td>
<td>1.61</td>
<td>(0.34)</td>
<td>1.47</td>
<td>1.69</td>
<td>1.22</td>
<td>0.17</td>
<td>1.53</td>
<td></td>
</tr>
<tr>
<td>$VR(5)$</td>
<td>2.01</td>
<td>(1.23)</td>
<td>2.26</td>
<td>3.78</td>
<td>0.79</td>
<td>0.63</td>
<td>2.36</td>
<td></td>
</tr>
<tr>
<td>$VR(10)$</td>
<td>1.57</td>
<td>(2.07)</td>
<td>3.00</td>
<td>6.51</td>
<td>0.76</td>
<td>0.77</td>
<td>2.96</td>
<td></td>
</tr>
<tr>
<td>$\sigma(g_d)$</td>
<td>11.49</td>
<td>(1.98)</td>
<td>10.96</td>
<td>15.47</td>
<td>7.79</td>
<td>0.43</td>
<td>11.27</td>
<td></td>
</tr>
<tr>
<td>$AC(1)$</td>
<td>0.21</td>
<td>(0.13)</td>
<td>0.33</td>
<td>0.57</td>
<td>0.09</td>
<td>0.53</td>
<td>0.39</td>
<td></td>
</tr>
<tr>
<td>$corr (g, g_d)$</td>
<td>0.55</td>
<td>(0.34)</td>
<td>0.31</td>
<td>0.60</td>
<td>−0.03</td>
<td>0.07</td>
<td>0.35</td>
<td></td>
</tr>
</tbody>
</table>
## Asset Pricing Implications - Case 1

### Without Fluctuating Economic Uncertainty

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$\psi$</th>
<th>$E(R_m - R_f)$</th>
<th>$E(R_f)$</th>
<th>$\sigma(R_m)$</th>
<th>$\sigma(R_f)$</th>
<th>$\sigma(p - d)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.5</td>
<td>0.5</td>
<td>0.55</td>
<td>4.80</td>
<td>13.11</td>
<td>1.17</td>
<td>0.07</td>
</tr>
<tr>
<td>7.5</td>
<td>1.5</td>
<td>2.71</td>
<td>1.61</td>
<td>16.21</td>
<td>0.39</td>
<td>0.16</td>
</tr>
<tr>
<td>10.0</td>
<td>0.5</td>
<td>1.19</td>
<td>4.89</td>
<td>13.11</td>
<td>1.17</td>
<td>0.07</td>
</tr>
<tr>
<td>10.0</td>
<td>1.5</td>
<td>4.20</td>
<td>1.34</td>
<td>16.21</td>
<td>0.39</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Panel A: $\phi = 3.0$, $\rho = 0.979$

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$\psi$</th>
<th>$E(R_m - R_f)$</th>
<th>$E(R_f)$</th>
<th>$\sigma(R_m)$</th>
<th>$\sigma(R_f)$</th>
<th>$\sigma(p - d)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.5</td>
<td>0.5</td>
<td>1.11</td>
<td>4.80</td>
<td>14.17</td>
<td>1.17</td>
<td>0.10</td>
</tr>
<tr>
<td>7.5</td>
<td>1.5</td>
<td>3.29</td>
<td>1.61</td>
<td>18.23</td>
<td>0.39</td>
<td>0.19</td>
</tr>
<tr>
<td>10.0</td>
<td>0.5</td>
<td>2.07</td>
<td>4.89</td>
<td>14.17</td>
<td>1.17</td>
<td>0.10</td>
</tr>
<tr>
<td>10.0</td>
<td>1.5</td>
<td>5.10</td>
<td>1.34</td>
<td>18.23</td>
<td>0.39</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Panel B: $\phi = 3.5$, $\rho = 0.979$

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$\psi$</th>
<th>$E(R_m - R_f)$</th>
<th>$E(R_f)$</th>
<th>$\sigma(R_m)$</th>
<th>$\sigma(R_f)$</th>
<th>$\sigma(p - d)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.5</td>
<td>0.5</td>
<td>-0.74</td>
<td>4.02</td>
<td>12.15</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>7.5</td>
<td>1.5</td>
<td>-0.74</td>
<td>1.93</td>
<td>12.15</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>10.0</td>
<td>0.5</td>
<td>-0.74</td>
<td>3.75</td>
<td>12.15</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>10.0</td>
<td>1.5</td>
<td>-0.74</td>
<td>1.78</td>
<td>12.15</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Panel C: $\phi = 3.0$, $\rho = \varphi_e = 0$
Asset Pricing Implications-Case II

- With Fluctuating Economic Uncertainty  \((\psi = 1.5)\)

| Variable       | Data | Model  
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>SE</td>
<td>(\gamma = 7.5)</td>
<td>(\gamma = 10)</td>
<td>(\gamma = 7.5)</td>
<td>(\gamma = 10)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(E(r_m - r_f))</td>
<td>6.33</td>
<td>(2.15)</td>
<td>4.01</td>
<td>6.84</td>
<td>4.01</td>
<td>6.84</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(E(r_f))</td>
<td>0.86</td>
<td>(0.42)</td>
<td>1.44</td>
<td>0.93</td>
<td>1.44</td>
<td>0.93</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma(r_m))</td>
<td>19.42</td>
<td>(3.07)</td>
<td>17.81</td>
<td>18.65</td>
<td>17.81</td>
<td>18.65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma(r_f))</td>
<td>0.97</td>
<td>(0.28)</td>
<td>0.44</td>
<td>0.57</td>
<td>0.44</td>
<td>0.57</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(E(\exp(p - d)))</td>
<td>26.56</td>
<td>(2.53)</td>
<td>25.02</td>
<td>19.98</td>
<td>25.02</td>
<td>19.98</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma(p - d))</td>
<td>0.29</td>
<td>(0.04)</td>
<td>0.18</td>
<td>0.21</td>
<td>0.18</td>
<td>0.21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(AC1(p - d))</td>
<td>0.81</td>
<td>(0.09)</td>
<td>0.80</td>
<td>0.82</td>
<td>0.80</td>
<td>0.82</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(AC2(p - d))</td>
<td>0.64</td>
<td>(0.15)</td>
<td>0.65</td>
<td>0.67</td>
<td>0.65</td>
<td>0.67</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Conclusion

- **Two key features that explains asset pricing puzzles**
  - long-run component in consumptions growth and economic uncertainty
  - recursive preferences

- **Debate**
  - IES >1
  - Economic explanation for long-run component in consumption