Risks For The Long Run And The Real Exchange Rate

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Abstract

We propose an equilibrium model that can explain a wide range of international finance puzzles, including the high correlation of international stock markets despite the lack of correlation of fundamentals. We conduct an empirical analysis of our model, which combines cross-country-correlated long-run risk with Epstein and Zin (1989) preferences, using US and UK data and show that it successfully reconciles international prices and quantities, thereby solving the international equity premium puzzle. These results provide evidence suggesting a link between common long-run growth perspectives and exchange rate movements.

JEL classification: G12; G15; F31.


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1 Introduction

Since Backus and Smith (1993), a long strand of the literature has documented the tension between international prices and quantities. As noted among others by Backus, Foresi and Telmer (1996) and Lustig and Verdelhan (2006), if markets are complete, the growth of the log-exchange rate would equal the difference of the log-stochastic discount factors for foreign currency– and dollar–denominated assets:

$$\Delta e_{t+1} = m^*_{t+1} - m_{t+1}$$

If investors have power utilities, their log-stochastic discount factors will be proportional to log-consumption growth rates. Within this framework, it is well documented that: i) risk aversion has to be large to reconcile the low volatility of consumption growth rates with highly volatile stochastic discount factors; ii) consumption is poorly correlated across countries at annual or higher frequencies. Indeed, if we were to set the coefficient of risk aversion to a number as high as 30, exchange rates would have to fluctuate between 15 and 30 times more than what we observe in the data. Based on this insight, Brandt, Cochrane and Santa-Clara (2006) conclude that given the degree of volatility of the US dollar that we observe in the data, there should be a larger correlation of stochastic discount factors across countries.

We like to think of this as the international equity premium puzzle. In a one-country model, consumption growth does not vary enough to explain the excess return over the risk-free rate. In a two-country model, consumption growth does not co-vary enough to track movements in the exchange rate and returns. This dichotomy of prices and quantities strikes us as an important unresolved puzzle in international finance.

The real exchange rate reflects the relative value of consumption in two countries. With power utility only short-run consumption differences affect the current currency value. In this paper, we propose an equilibrium model in which exchange rates adjust to reflect differences in both current and future relative consumption across countries. We show that the marginal valuation of a unit of consumption in the two countries can be similar even when their current consumption growth rates are different. This requires that agents care about the temporal...
distribution of risk and that countries share very similar long-run growth prospects.

We assume that consumers have recursive preferences à la Epstein and Zin (1989). In this economy, agents are extremely sensitive to uncertainty about the long-run perspectives of the economy: news about future consumption growth affect their marginal utility of consumption today. If the long-run components of the endowments are highly correlated across countries, then stochastic discount factors will be too. Ultimately, exchange rates need to fluctuate much less to reflect differences in the marginal valuation of consumption in the two countries.

Following Bansal and Yaron (2004) and Hansen, Heaton and Li (2008), we refer to these components as the long-run risks. These risks enter as small, slowly moving, predictable components of consumption growth rates that are consistent with the empirical evidence showing consumption to be close to a random walk. This addition changes the joint conditional distribution of consumption growth rates quite a bit. In particular, while over a short horizon the long-run component explains only a very small share of consumption volatility, over a longer horizon the long-run risks are responsible for most of the fluctuations and the co-movements of domestic and foreign consumption. We can think of these components as capturing long-run movements in national technological frontier and we assume that technological frontiers are highly correlated across countries in the long-run. This is consistent with the analysis of Eaton and Kortum (1999), whose estimates about international technological diffusion suggest substantial, but not perfect, sharing of ideas.

Our consumption-based equilibrium model simultaneously reproduces several other findings regarding international quantities and prices: i) consumption growth rates prove to be as volatile, as autocorrelated, and as cross-country-correlated as they are in the data; ii) the equity premium puzzle is solved within each country; iii) international asset returns are highly correlated despite the almost lack of correlation of fundamentals, and iv) the contemporaneous correlation between log exchange rate growth and international returns is as low as in the data.

Our theoretical analysis suggests the following empirical question: is it possible to provide a direct estimate of the low-frequency components in consumption and make the case that long-run growth rate prospects look similar across countries? We address this question, by adopt-
ing the view that the long-run movements of consumption trend can be identified by means of predictive variables. Similarly to Bansal, Kiku and Yaron (2006), we project consumption growth on lagged values of price-dividend ratio, consumption-output ratio, and consumption growth itself, to measure long-run risks. Applying this methodology to US and UK data separately, we conclude that the predictable components of consumption growth rates are highly persistent and that their correlation increases over time, just as the volatility of exchange rate growth decreases. The latter result reinforces our conjecture regarding the link between common long-run growth perspectives and exchange rate movements.

Using this measure of long-run risks, we can also test the moment restrictions resulting from our model and estimate preference parameters. Results confirm that the intertemporal elasticity of substitution should be in excess of 1 and that a coefficient of risk aversion smaller than 10 can account for both the domestic and international version of the equity premium puzzle. By focusing only on the post-Bretton Woods sample, the typical result is that our model cannot be rejected.

One of the novelties of our empirical work is that we identify long-run consumption risk using information from domestic financial markets and better assess the impact that such news has on currency markets. This enables our model to bridge part of the gap between international prices and quantities.

The paper is organized as follows. In section 2, we propose a simple model that we can both solve and calibrate. This provides a useful instrument to show the internal transmission mechanism of the economy. We document the ability of the model to account for the degree of volatility of exchange rate movements and a large number of other international moments. The focus of section 3 is on the estimation of the model. We first document that the likelihood function obtained from consumption data only is quite uninformative about the existence and the nature of the predictive components of consumption growth in the cross section of countries. We then proceed with our predictive regressions approach and test a number of predictions of the model. Section 4 concludes the paper, with a discussion of the potential extensions of the model and a summary of the main findings.
2 Reconciling International Prices and Quantities

A Exchange Rate’s Movements

Basics. If markets are complete, the movements of the real exchange rate are pinned down by a simple no-arbitrage condition:

\[ \Delta e_t = m_t^* - m_t \]  

where \( m_t \) and \( m_t^* \) are the intertemporal log-marginal rates of substitution of consumption in the two countries. Equivalently, we shall refer to them as domestic and foreign log-pricing kernels. According to equation (1), the more correlated pricing kernels are, the smoother exchange rate’s movements should be.

Is correlation the same as Risk-Sharing? Not necessarily, as it crucially depends on the model’s assumptions. For example, consider an economy populated by two consumers with standard time-additive preferences defined over an aggregate of two tradable goods (e.g. one domestic and one foreign good). Risk-sharing impels that the intertemporal marginal rates of substitutions (IMRS) with respect to each good are equalized and therefore perfectly correlated across agents. If agents’ preferences are biased toward the consumption of opposite goods (e.g. there is consumption home bias), however, their IMRS with respect to the consumption bundle are going to be less than perfectly correlated, even if in equilibrium all risk-sharing opportunities have been exhausted. For this reason, we analyze the volatility the exchange rate’s fluctuations in terms of correlation of the log-pricing kernels, without drawing any implication on international risk sharing.

Exchange Rates as Relative Consumption Growth. The real exchange rate is the relative price of two consumption bundles. As such, it should adjust to reflect differences in both current and future relative consumption across countries. In the special case of time-additive CRRA preferences, only current consumption matters, since equation (1) reduces to:

\[ \Delta e_t \propto (\Delta c_t - \Delta c_t^*) \]
In this paper, we consider the case in which agents care about the entire sequence of future consumption streams. A more general formulation for the dynamics of exchange rates emerges, in which long- and short-run consumption growth differences are both responsible for the current currency value. The next subsection introduces the model and highlights the two sources of international consumption risk.

B The Model

Setup of the economy. We analyze an economy with two countries that we denote as home and foreign. For convenience, we characterize preferences, endowments and prices only for the home country. Identical expressions indexed by a “*” apply to the foreign country. The home representative consumer has Epstein and Zin (1989) preferences:

\[ U_t = \left\{ (1 - \delta)(C_t)^{1-1/\psi} + \delta E_t [(U_{t+1})^{1-\gamma}]^{1-1/\psi} \right\}^{1-1/\psi} \]

where \( \gamma \) is the coefficient of risk aversion and \( \psi \) is the intertemporal elasticity of substitution.

There are two country specific goods in the economy. To simplify the setup, we impose that preferences are such that there is complete home bias, meaning that the representative consumer in each country is willing to consume only the good that she is endowed with. Markets are complete. An equilibrium of this economy exists, in which each country behaves as in autarky both for consumption and asset holdings.

The logarithm of the pricing kernel, \( m_{t+1} \), is a stochastic process that depends on both log-consumption growth, \( \Delta c_t \), and the log-return on the asset that pays consumption, \( r_{c,t+1} \):

\[ m_{t+1} = \left\{ 1 - \gamma \left[ \log \delta - \frac{1 - \gamma}{\psi - 1} \Delta c_t + \frac{1}{\psi - 1} r_{c,t+1} \right] \right\}^{1-1/\psi} \]

We complete the system by specifying exogenous laws of motion for consumption and dividend.
growth rates as follows:

\[
\begin{align*}
\Delta c_t &= \mu_c + x_{t-1} + \varepsilon_{c,t} \\
\Delta d_t &= \mu_d + \lambda x_{t-1} + \varepsilon_{d,t} \\
x_t &= \rho_x x_{t-1} + \varepsilon_{x,t} \\
\xi_t &\sim i.i.d. \ N(0, \Sigma)
\end{align*}
\]  

where \( \xi_t = [\varepsilon_{c,t} \ \varepsilon_{d,t} \ \varepsilon_{x,t} \ \varepsilon^*_{c,t} \ \varepsilon^*_{d,t} \ \varepsilon^*_{x,t}] \) is the vector of shocks of the economy.

In this section, the two economies are assumed to be symmetric, having the same preference and transition laws parameters. Furthermore, we assume that the shocks \( \varepsilon_{c,t}, \varepsilon_{d,t}, \) and \( \varepsilon_{x,t} \) are mean zero i.i.d. normally distributed within each country with volatilities \( \sigma, \varphi_d \sigma, \) and \( \varphi_e \sigma, \) respectively. The shocks are allowed to be cross-country correlated. We shall denote \( \rho^{hf}_c \) as the correlation between \( \varepsilon_{c,t} \) and \( \varepsilon^*_{c,t} \); \( \rho^{hf}_d \) as the correlation between \( \varepsilon_{d,t} \) and \( \varepsilon^*_{d,t} \); and \( \rho^{hf}_x \) as the correlation between \( \varepsilon_{x,t} \) and \( \varepsilon^*_{x,t} \). All other correlations are set to zero.

The following system is obtained as a first-order linear approximation of the model:

\[
\begin{align*}
m_{t+1} &= \log \delta - \frac{1}{\psi} x_t + \delta \frac{1 - \gamma \psi}{\psi (1 - \rho_x \kappa_c)} \varepsilon_{x,t+1} - \gamma (\varepsilon_{c,t+1} - \varepsilon_{c,t}) \\
\Delta e_t &= m^*_{t+1} - m_{t+1} \\
v_{d,t} &= \overline{v}_d + \frac{\lambda - \frac{1}{\psi}}{1 - \rho_x \kappa_d} x_t, \quad v_{c,t} = \overline{v}_c + \frac{1 - \frac{1}{\psi}}{1 - \rho_x \kappa_c} x_t \\
r_{d,t+1} &= \overline{r}_d + \frac{1}{\psi} x_t + \kappa_d \frac{\lambda - \frac{1}{\psi}}{1 - \rho_x \kappa_d} \varepsilon_{x,t+1} + \varepsilon_{d,t+1} \\
r_{f,t+1} &= \overline{r}_f + \frac{1}{\psi} x_t
\end{align*}
\]

where \( v_{d,t} \) and \( v_{c,t} \) are the price-dividend and price-consumption ratios respectively, \( r_{f,t+1} \) is the log–risk-free rate, and \( \overline{r}_j \) is the average return on asset \( j \). The approximation constants \( \kappa_c \) and \( \kappa_d \) are endogenously determined as in Campbell and Shiller (1988).

The dynamics of the exchange rate growth is jointly affected by both short- and long-run news
to international consumption:

\[
\Delta e_{t+1} - E_t [\Delta e_{t+1}] = \delta \frac{1 - \gamma \psi}{\psi (1 - \rho_x \kappa_c)} (\varepsilon^x_{x,t+1} - \varepsilon_{x,t+1}) - \gamma (\varepsilon^c_{c,t+1} - \varepsilon_{c,t+1})
\]

(5)

Equation (5) highlights the key ingredients of exchange rates fluctuation in our model. First, we need to disentangle risk-aversion and intertemporal elasticity of substitution, \(\gamma \neq 1/\psi\), for long-run growth prospects to affect the current exchange rate. Second, the more persistent the long-run components are (high \(\rho_x\)), the stronger the impact of long-run news on the current exchange rate. Equivalently, news with long-lasting effects on future consumption determine large movements in exchange rates. Third, the degree of cross-country correlation of short- and long-run consumption news matters: perfectly correlated shocks do not change the relative value of consumption bundles across countries, i.e. they do not alter the exchange rate.

**Calibration.** The structure of our two parallel economies mimics the one discussed by Bansal and Yaron (2004). In table 1 we report our baseline calibration. The subjective discount factor is set to 0.998, to reflect that our model describes a monthly decision problem. We set the coefficient of risk aversion \(\gamma\) equal to 4.25, which is relatively low compared to both the number commonly found in the equity premium puzzle literature and the coefficient proposed by Brandt et al. (2006). The intertemporal elasticity of substitution-\(\psi\) is equal to 2, a number consistent with the long-run risks literature.

As far as the calibration of the parameters of the laws of motion of consumption growth is concerned, our goal is to reproduce the average behavior of consumption growth in the United States and in the United Kingdom. We motivate in detail this specific choice of countries in the next section. Here we anticipate that we need to work with countries that display international financial openness, and that have long time series for consumption and dividends. The standard deviation of consumption growth implied by our choice of parameters is approximately 2.4% in annualized terms. This falls in-between the US and UK’s average growth

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1The value of the intertemporal elasticity of substitution has been the subject of a long-lasting debate in the literature. Hall (1988) and Lustig and Van Nieuwerburgh (2006) estimate this number to be below unity. Guvenen (2006) reproduces capital and consumption fluctuations, as long as most of the wealth is held by a small fraction of the population with a high elasticity of intertemporal substitution. Attanasio and Weber (1989) documents an intertemporal elasticity of substitution in excess of one in the UK.
of per capita consumption of nondurables and services from 1970 to 1998.

The dividends processes require the calibration of four additional parameters, which we have selected with the goal of matching the first two moments of dividend growth rates, their cross-country correlation, and their leverage as the average of actual US and UK data. The variance of dividend growth explained by its predictable component is very small, in the order of 3%.

We set the correlation of long-run risks, $\rho_{x}^{hf}$, to 1. This enables to keep the volatility of exchange rate’s fluctuations to about 11%-12%. We discuss in the next section the empirical evidence supporting a large correlation of consumption growth’s predictive components. The cross-country correlation of the shocks to consumption, $\rho_{c}^{hf}$, is chosen so as to obtain a correlation of consumption growth in the order of 0.3.

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**Table 1**

**Baseline Calibration.**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Psi$</td>
<td>Intertemporal elasticity of substitution</td>
<td>2</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Risk aversion</td>
<td>4.25</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Subjective discount factor</td>
<td>0.998</td>
</tr>
<tr>
<td>$\mu_{c}$</td>
<td>Average consumption growth</td>
<td>$15 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Autoregressive coefficient of the long-run component $x_t$</td>
<td>0.987</td>
</tr>
<tr>
<td>$\varphi_c$</td>
<td>Ratio of long-run shock and short-run shock volatilities</td>
<td>0.048</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Standard deviation of the short-run shock to consumption</td>
<td>$68 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\mu_d$</td>
<td>Average dividend growth</td>
<td>0.0007</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Leverage</td>
<td>3.0</td>
</tr>
<tr>
<td>$\varphi_d$</td>
<td>Volatility ratio of short-run shocks to dividend and consumption growth</td>
<td>5.0</td>
</tr>
<tr>
<td>$\rho_{x}^{hf}$</td>
<td>Cross-country correlation of the long-run shock</td>
<td>1.0</td>
</tr>
<tr>
<td>$\rho_{c}^{hf}$</td>
<td>Cross-country correlation of the short-run shock to consumption</td>
<td>0.3</td>
</tr>
<tr>
<td>$\rho_{d}^{hf}$</td>
<td>Cross-country correlation of the short-run shock to dividends</td>
<td>-0.1</td>
</tr>
</tbody>
</table>

Notes - The countries share the same calibration. The model describes a monthly decision problem. The parameters of the laws of motion of consumption and dividend growth are set to reproduce the average behavior of consumption growth in the United States and in the United Kingdom. The cross-country correlations of the idiosyncratic shocks to consumption and shocks to the trend are chosen as to obtain a correlation of consumption growths in the order of 0.3. The coefficient $\lambda$ is set in such a way that the ratio $\sigma_{\Delta d}/\sigma_{\Delta c}$ is equal to 4.86 that is in the range (4,8) estimated by Ludvigson, Lettau and Wachter (2004). The cross-country correlation of the short-run shocks to dividends, $\rho_{d}^{hf}$, is set so as to achieve an almost zero correlation of dividend growths between the US and the UK.
Correlation of stochastic discount factors
Intertemporal Elasticity of substitution ($\psi$)
$\rho_x^{hf}=1$
$\rho_x^{hf}=0.75$

Correlation of stochastic discount factors
Intertemporal Elasticity of substitution ($\psi$)
$\rho_x=0.987$
$\rho_x=0.9$

Fig. 1 - The role of intertemporal elasticity of substitution. In both panels, the dark line reports the correlation of stochastic discount factors when $\psi$ changes. The grey line on the left panel is drawn for a smaller value of $\rho_x^{hf}$, everything else held equal; the grey line on the right panel is drawn for a lower $\rho_x$.

C Correlations and Volatilities

Correlation of stochastic discount factors. In figure 1, we report the international correlation of stochastic discount factors as a function of the coefficient of intertemporal elasticity of substitution. The plot allows us to better understand the role of the key ingredients mentioned in the previous section: i) long-run risk sensitivity, ii) high persistence, and iii) high international correlation of the long-run components.

In each of the two panels, the dark line is drawn according to our baseline calibration. The minimum of this graph falls in the vicinity of $\psi \approx 1/\gamma$, the CRRA case. By moving away from $1/\gamma$ in either direction, the increasing sensitivity to long-run risk produces a sharp increase in the correlation of stochastic discount factors. Interestingly, we do not need to take a stand on the intertemporal elasticity of substitution being greater or smaller than 1, in order to obtain highly correlated pricing kernels. Any $\psi$ larger than 0.5 would yield a correlation of the log-pricing kernels of at least 80%, provided that the remaining coefficients are calibrated as in table 1.

We are able to obtain highly correlated pricing kernels while keeping the unconditional in-


ternal correlation of consumption as low as in the data. This result is primarily driven by the high correlation of the long-run components. The left panel of figure 1 documents that by lowering the correlation of the long-run risks, the correlation of the pricing kernels decreases as well. The right panel of figure 1 documents that for the small shocks to the trend of consumption to matter, they must be long-lasting. If investors care about the temporal distribution of risk, near-permanent changes in the average growth of their consumption lead to large reactions in marginal utilities.

**Volatility of exchange rates fluctuations.** By keeping the correlation of long-run shocks to a high level, the dynamics of the exchange rates are governed mostly by short-run shocks. Indeed, in the limiting case of our benchmark calibration (in which long-run risks are perfectly correlated across countries), short-run shocks are solely responsible for exchange rate fluctuations. This means that exchange rate movements are intimately related to investors’ degree of risk aversion.

For this reason, in figure 2 we plot the volatility of exchange rate growth against the coefficient of risk aversion. The two horizontal dashed lines represent the region within which the volatility of the exchange rate growth typically falls for G-7 countries. In our baseline calibration we set $\gamma = 4.25$ which yields a volatility that is well within the region of interest, as documented by the dark lines in the two panels of figure 2. Increasing the risk aversion of the representative consumers of the two countries to the high levels required by the equity premium puzzle literature would push the volatility to counterfactual levels—similar to those that motivated Brandt et al. (2006) to postulate the existence of a puzzle. In this case, our calibration again proves itself crucial.

Reducing the correlation of the long-run shocks reduces the correlation of stochastic discount factors, as we documented before, and increases the exchange rate fluctuations. Unless agents are made less risk averse, we are left with exchange rates that are too volatile at the calibrated level of risk aversion (figure 2 left panel). The high exchange rate volatility can be offset by decreasing the persistence of the predictable component (figure 2 right panel). In this way, the volatility of the stochastic discount factors falls more than their international correlation.
Fig. 2 - The role of risk aversion. In both panels, the dark line reports the volatility of the depreciation rate when $\gamma$ changes. The grey line in the top panel is drawn for a smaller value of $\rho_{hf}^x$, everything else held equal; the grey line in the bottom panel is drawn for lower $\rho_x$ and $\rho_{hf}^x$.

D Other moments of international financial markets

In table 2 we show that our baseline calibration is able to match key features of international financial markets. In comparing our results to the data, we have assumed the United States to be the home country and the United Kingdom to be the foreign country. We have already documented the ability of the model to produce highly correlated pricing kernels and exchange rates that are as volatile as they are in the data. Here we show that our symmetric calibration implies an average growth of the US dollar vis-à-vis the British pound equal to zero, which is not too far from the 1.3% sample average obtained in the post-Bretton Woods era.

Another well documented empirical feature of the exchange rate growth is its lack of serial correlation and predictability. Our model replicates this finding. In general, system (4) suggests that exchange rate growth contains a predictable component proportional to the cross-country difference of the price-dividends ratio. When the long-run components are highly correlated, however, almost all the volatility of the exchange rate is explained by i.i.d. news to consumption. This explains why, consistently with the data, both the lagged exchange rate growth rate and the lagged difference of the price-dividend ratios are not correlated to ex-
change rate movements in our model. As we can see from equation (5), the perfect correlation of the long-run components is also responsible for the extremely high correlation between the

\[
\rho \left( m^h, m^f \right)
\]

Correlation of pricing kernels

-  -  0.931

\[
\sigma (\Delta c)
\]

Volatility of consumption growth

1.369  2.860  2.450

\[
E(\Delta c)
\]

Average consumption growth

0.040  -0.030  0.000

\[
\rho (\Delta e_{t+1}, \Delta e_t)
\]

Autocorrelation of FX growth

0.002  0.003

\[
\rho (\Delta e_{t+1}, v_{d,t} - v_{d,t}^*)
\]

Correlation of FX growth and price-dividend ratios differences

0.070  -0.003

\[
\rho (\Delta e, \Delta c - \Delta c^*)
\]

Correlation of FX growth and consumption growth differentials

0.15  0.80

\[
\rho (v_d, v_{d,t-1})
\]

Autocorrelation of price-dividend ratio

0.624  0.716  0.925

\[
\sigma (v_d)
\]

Volatility of price-dividend ratio

31.207  32.210  24.566

\[
E(v_d)
\]

Average price-dividend ratio

3.331  2.890  2.879

\[
E(\Delta r_d - \Delta r_f)
\]

Average excess return

5.504  6.501  5.346

\[
\rho (\Delta r_d - \Delta r_f)
\]

Volatility of excess return

17.130  22.830  19.132

\[
E(\Delta r_f)
\]

Average risk free rate

1.470  1.620  1.332

\[
\rho (\Delta r_f)
\]

Volatility of risk free rate

1.530  2.920  1.191

\[
\rho (\Delta r_d - \Delta r_f, r_d^* - r_f^*)
\]

Correlation of excess returns

0.670  0.603

\[
\rho (\Delta r_d - \Delta r_f, v_{d,t}^*)
\]

Correlation of price-dividend ratios

0.770  0.925

\[
\rho (\Delta d, \Delta d^*)
\]

Correlation of dividend growth

-0.03  -0.07

\[
\rho (\Delta r_f, r_f^*)
\]

Correlation of risk free rates

0.653  1.000

\[
\frac{E(\Delta r_d - \Delta r_f + \Delta e)}{\sigma (\Delta r_d - \Delta r_f + \Delta e)}
\]

Foreign Sharpe-Ratio in domestic units

0.200  0.312  0.227

\[
\rho (\Delta r_d - \Delta r_f, r_d^* - r_f^* + \Delta e)
\]

Correlation of excess returns, domestic units

0.631  0.600  0.543

\[
\rho (\Delta r_d - \Delta r_f, \Delta e)
\]

Correlation of excess returns, local units, and FX growth

0.040  -0.030  0.000

\[
\sigma (\Delta c)
\]

Volatility of consumption growth

1.369  2.860  2.450

\[
\frac{\sigma^2(x)}{\sigma^2(\Delta c)} \times 100
\]

Share of predictable cons. variance

-  -  8.180

\[
\sigma (\Delta d)
\]

Volatility of dividend growth

16.851  6.872  11.961

\[
\rho (\Delta c, \Delta c^*)
\]

Correlation of consumption growth

0.280  0.351

Notes - Data are quarterly, real per-capita and from 1971:01 to 1998:04 as in Brandt et al. (2006). Means and variances are annualized and multiplied by 100. All variables are in log-units and measured in terms of real consumption, “local units”, unless differently stated. “Domestic (foreign) units” stands for variables measured in terms of the same real domestic (foreign) consumption good. “FX” denotes log exchange rate growth. The model calibration is reported in Table 1. All simulated variables are time-aggregated to annual frequency.
log exchange rate growth and the cross-country difference of the consumption grow rates.2

Turning our attention to stock markets, our model generates persistent and volatile price-dividend ratios. When investors are concerned about the intertemporal distribution of risk, the price-dividends ratio fluctuations are considered very risky, resulting in high risk premia and low average levels for price-dividends ratios. As documented in table 2, our model produces excess returns and risk-free rates whose average and volatility are precisely in line with those observed in the US and UK data.

We push our analysis one step further, to demonstrate that the model can also match the average correlation of excess returns in the two countries, in spite of the low calibrated correlation of dividend growth rates. In the data, the high correlation of the returns is driven by the high correlation of the price-dividend ratios. This is exactly what happens in our model. We regard this finding as further validation of the existence of highly cross-country-correlated predictable components of consumption and dividend growth. When cash flow growth rates are i.i.d., the price-dividend ratios are constant and returns are as cross-country correlated as are the growth of the dividends—a counterfactual implication. By employing Epstein and Zin preferences and by allowing a small but highly persistent and highly cross-country-correlated component be also responsible for the dynamics of consumption and risky cash flows, we unveil an all-new layer of stock markets co-movements.

Our model produces excess correlation of real risk-free rates. The system of equations (4) shows that the approximate solution for the risk-free rates in this economy is a linear function of the predictable component of consumption growth alone. As a consequence, the correlation of real risk-free rates entirely reflects the perfect correlation of the $x$'s. In the next empirical section this problem is mitigated as we assume a less than perfect correlation of the long-run components.

Up to this point we have focused on returns expressed in local units, $r^{US}$ for the US and $r^{UK}$ for the UK. In table 2, however, we also report statistics for returns expressed in domestic units. For a US-based investor, foreign returns expressed in domestic units are $r^{UK}_{t+1} + \Delta e_{t+1}$. Similarly, for a UK-based investor returns equal $r^{US}_{t+1} - \Delta e_{t+1}$. Focusing on returns expressed

\[2\] The correlation $\rho(\Delta c, \Delta c - \Delta c^*)$ generated by the model is smaller than one solely because of time-aggregation.
in domestic units is a necessary step in order to study the model’s implications for the international investment opportunity sets of both a US- and a UK-based investor. Foreign assets turn out to be attractive for two crucial reasons. First, exchange rate movements appear to be uncorrelated with foreign asset price fluctuations. Second, the cross-country correlation of returns expressed in local units is about 60%. As table 2 suggests, our model succeeds in replicating both of these pieces of empirical evidence and enables us to account for foreign Sharpe ratios expressed in domestic units and for the optimal reward-to-variability ratio that may be achieved by US- and UK-based investors.

In the lower part of table 2 we report some statistics for consumption and dividend growth rates for the US and UK confirming that our calibration is consistent with the empirical evidence on international quantities.

In this paper, we abstract from modeling time-varying volatility in the dynamics of consumption and risky cash flows. As a consequence, the model produces constant risk premia and it cannot replicate the empirical failure of the uncovered interest parity. Lustig, Roussanov and Verdelhan (2008) document that time-varying exposure to a large common component is capable of accounting for the cross-section of currency risk-premia.

3 Estimating international long-run risks

The task of measuring long-run risks from consumption data is made even harder in small samples. However, the ability to estimate the predictable components of consumption growth at each point in time plays a crucial role in the testing of our model against the data. In this section we quantify the finite sample detectability issue of the slowly moving conditional means of consumption growth in the cross-section of countries and we explore one way to construct an observable time series of long-run risks using the predictive information contained in asset prices.

We restrict our attention to annual consumption, as longer term consumption growth rates are more likely to contain a better signal and less noise regarding the low frequency compo-
nent of consumption. We focus on the US and the UK for a number of reasons both practical and historical. First of all, the identification of a highly persistent time series requires as long a sample as possible. From this stand-point, US and UK are probably the best examples of countries with a long history of accurate consumption data. Second, the US and UK have had an excellent tradition of financial integration at least for a large part of the post-World War II period, as documented by Taylor (1996), Obstfeld (1998), and Quinn (1997) among others. This is important because in assessing the predictions of the model for the dynamics of international asset prices, we need to ensure that US and UK agents considered each other’s assets as part of their own investment opportunity sets.

**Description of the data.** Our common sample covers the years from 1929 to 2006. Data on US consumption of nondurables and services, gross domestic product, and population were collected from the NIPA tables of the Bureau of Economic Analysis. Value-weighted market returns, yields on 3-month T-bills, dividends, and dividend yields for the US are from CRSP. CPI inflation and the spread between BAA and AAA corporate bonds (our measure of the default premium) were obtained from the web-site of the Federal Reserve of St. Louis. The UK consumption series for the years 1963–2006 were obtained from the UK Office of National Accounts. For earlier data, we referred to Mitchell (1979). Long time series for the UK gross domestic product, FTSE returns, yields on three-month T-bills, dividend yields, population, and CPI inflation were obtained from the website of Global Financial Data. All variables have been de-meaned.

### A Consumption Only Information

**A Kalman filter approach.** We document that detecting predictive components of consumption growth with the postulated properties of the previous section is an extremely hard task also in the cross-section of countries. We start our analysis by using the Kalman filter to derive a recursive representation of the likelihood function for the state space model in the system of equations (3) and to estimate the relevant parameters governing the consumption dynamics in the US and in the UK. Our ultimate goal is to test the null of high persistence and high cross-country correlation of the predictive components through a likelihood ratio
test. Figure 3 reports the contours of the maximized likelihood function upon restricting the autoregressive coefficients and correlation parameter to take on any pair of values in the \([0, 1]\) range. According to our findings, we cannot reject that US and UK predictive components are highly persistent and perfectly cross-country correlated at conventional significance levels. Specifically, a correlation of one and a persistence of 0.965 (which is the monthly equivalent of 0.65 at annual frequency), cannot be rejected at a 5% level of significance. More generally, figure 3 reveals that the likelihood function is very uninformative about the parameters governing the consumption dynamics.

A MonteCarlo experiment. What is the minimum sample size needed to sharply identify the long-run components and reject the random walk model? How would our estimation differ if we were to increase the number of countries and include more consumption time series? We address these questions by simulating the dynamics of consumption growth in \(N\) countries.
with the following process:

\[
\begin{align*}
\Delta c_t &= x_{t-1} + \sigma \epsilon_{c,t} \\
x_t &= P x_{t-1} + \varphi \sigma R^{1/2} \epsilon_{x,t}
\end{align*}
\]

where \( P \) is a diagonal matrix that collects the persistence parameters \( \rho_x \) on the main diagonal and has zeros on the off-diagonal, and \( R \) is the correlation matrix of the shocks to the predictive components. We assume that all predictive components share the same autoregressive coefficient and that there is only one correlation coefficient, which is common to all pairs of \( x \)'s. We also impose that the country-specific shocks to consumption growth are uncorrelated across countries and that the average consumption growth rate is zero. Notice that all these restrictions make the estimation exercise simpler than it would be with actual data. In a small sample environment, these simplifying assumptions are important: our results should be interpreted as providing an upper bound on the ability of an econometrician to identify the long-run risk components in consumption growth.

We set the autoregressive coefficient of the predictive components to 0.8547 and the volatility parameter \( \sigma \) to 0.2355. These choices correspond to the annualized monthly frequency persistence of 0.987 and volatility of 0.0068 that we assumed in our calibrations. We set the identification parameter \( \varphi = 0.34 \). This correspond to an \( R^2 \) of about 40%, which is an upper bound on the predictability of consumption growth. We simulate 40,000 independent samples of size ranging from 50 to 200 years. We let the number of simulated consumption series be 1, 2, and 5 and the correlations of the predictive components be 0.9, 0.95, and 1. In each sample we estimate (6).

The results on the estimate of \( \varphi \) are reported in table 3. A number of interesting facts seem to emerge. First, with only one consumption series it is virtually impossible to reject the pure random walk hypothesis. This is in essence the message contained in Bansal and Yaron (2004). Second, a sample of size 80 appears to be enough to reject the pure random walk hypothesis with two consumption growth series, affected by the same predictive component. Third, even a small departure from perfect correlation of the predictive components
TABLE 3
FILTERING PREDICTIVE COMPONENTS

<table>
<thead>
<tr>
<th>$\rho(x, x^*)$</th>
<th># of $\Delta c$'s</th>
<th># of $x$'s</th>
<th>50</th>
<th>80</th>
<th>100</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>(0.000, 2.107)</td>
<td>(0.000, 1.478)</td>
<td>(0.000, 1.234)</td>
<td>(0.000, 0.746)</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>(0.000, 0.695)</td>
<td>(0.066, 0.667)</td>
<td>(0.093, 0.637)</td>
<td>(0.178, 0.527)</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
<td>(0.135, 0.553)</td>
<td>(0.182, 0.500)</td>
<td>(0.204, 0.476)</td>
<td>(0.242, 0.435)</td>
</tr>
<tr>
<td>0.95</td>
<td>2</td>
<td>2</td>
<td>(0.000, 1.108)</td>
<td>(0.000, 0.740)</td>
<td>(0.000, 0.672)</td>
<td>(0.099, 0.552)</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>5</td>
<td>(0.000, 0.583)</td>
<td>(0.115, 0.559)</td>
<td>(0.149, 0.525)</td>
<td>(0.218, 0.448)</td>
</tr>
<tr>
<td>0.9</td>
<td>2</td>
<td>2</td>
<td>(0.000, 1.159)</td>
<td>(0.000, 0.780)</td>
<td>(0.000, 0.683)</td>
<td>(0.138, 0.580)</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>5</td>
<td>(0.000, 0.610)</td>
<td>(0.126, 0.567)</td>
<td>(0.158, 0.531)</td>
<td>(0.222, 0.446)</td>
</tr>
</tbody>
</table>

Notes - Each column reports the 95% confidence interval for the estimated $\varphi_e$ parameter for simulated samples of increasing size. The true value of $\varphi_e$ is 0.34.

significantly hampers the task of rejecting the null.

The results of table 3 have important consequences for our empirical investigation. The only countries for which we have relatively stable long samples of about 80 years of consumption data are the United States and the United Kingdom. For any other developed countries the number of reliable annual data points shrinks to about 50. Unless the predictive components are perfectly correlated, consumption data alone are not enough to identify the low frequency dynamics of international consumption movements.

B Consumption and Price Information

Predictive regressions approach. Bansal et al. (2006) document that variables, such as lagged price-dividend, consumption-output ratios, risk-free rates, lagged consumption growth, and default premia, should contain a direct measure of long-run risk at each date and state. Based on this prediction, we take the projection of consumption growth on the aforementioned set of variables to be our measure of long-run risks. In principle, there are multiple ways of constructing the conditioning information set for the predictive regressions. In our empirical
### Table 4
**Predictive Regressions: The Key Ingredients**

<table>
<thead>
<tr>
<th></th>
<th>F-stat US</th>
<th>F-stat UK</th>
<th>Adj. $R^2$ US</th>
<th>Adj. $R^2$ UK</th>
<th>$\rho_x$ US</th>
<th>$\rho_x$ UK</th>
<th>corr $(x^US, x^UK)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pd and risk-free</td>
<td>6.641</td>
<td>3.476</td>
<td>0.145</td>
<td>0.086</td>
<td>0.768</td>
<td>0.787</td>
<td>0.568</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.036)</td>
<td></td>
<td></td>
<td>(0.076)</td>
<td>(0.074)</td>
<td></td>
</tr>
<tr>
<td>All predictive variables</td>
<td>6.585</td>
<td>6.852</td>
<td>0.315</td>
<td>0.278</td>
<td>0.672</td>
<td>0.759</td>
<td>0.780</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td></td>
<td></td>
<td>(0.074)</td>
<td>(0.074)</td>
<td></td>
</tr>
<tr>
<td>Pd only</td>
<td>7.299</td>
<td>6.013</td>
<td>0.086</td>
<td>0.076</td>
<td>0.885</td>
<td>0.726</td>
<td>0.879</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.016)</td>
<td></td>
<td></td>
<td>(0.065)</td>
<td>(0.099)</td>
<td></td>
</tr>
<tr>
<td>VAR</td>
<td>11.719</td>
<td>4.204</td>
<td>0.325</td>
<td>0.149</td>
<td>0.869</td>
<td>0.765</td>
<td>0.817</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.008)</td>
<td></td>
<td></td>
<td>(0.058)</td>
<td>(0.075)</td>
<td></td>
</tr>
</tbody>
</table>

Notes - Columns 2 and 3 report the F-statistic and the associated p-value for the test that all the coefficients of the predictive regressions are equal to zero for the corresponding row model for the US and the UK. Columns 4 and 5 report the $R^2$ for the predictive regressions for the US and the UK. Columns 6 and 7 report the estimated autoregressive coefficients of the predictive components of US and UK consumption growth. Column 8 reports the cross-country correlations of the estimated predictive components in the 1971-2006 sample.

In analysis, we consider the following alternatives:

\[
\Delta c^i_t = pd^i_{t-1}\beta_1^i + \varepsilon^i_{c,3,t} \quad (7)
\]

\[
\Delta c^i_t = [pd^i_{t-1}, r^i_{f,t-1}] \beta_2^i + \varepsilon^i_{c,1,t} \quad (8)
\]

\[
\Delta c^i_t = [pd^i_{t-1}, r^i_{f,t-1}, \Delta c^i_{t-1}, \Delta c_y^i_{t-1}, \text{default}^i_{t-1}] \beta_3^i + \varepsilon^i_{c,2,t} \quad (9)
\]

\forall i \in \{US, UK\}. Regression (7) focuses on the explanatory ability of price-dividend ratios only. Regression (8) incorporates also the risk-free rate, an additional measure of long-run risk according to our model (see system (4)). Regression (9) provides a more accurate fit, by including additional control variables that are shown to have predictive power for consumption.\(^3\)

In table 4 we report summary statistics supporting the key features of the long-run components highlighted in our theoretical model. First, the long-run components exist, as the F-tests consistently reject the null that consumption growth rates are *i.i.d.*. Second, the long-run components are small and explain between 7% and 32% of annual consumption growth.

\(^3\)The default premium was not included in the UK regression, because of the unavailability of a long time series.
variance. Third, the long-run components are very persistent: the monthly equivalents of our estimates of \( \rho_{x}^{US} \) and \( \rho_{x}^{UK} \) range between 0.967 and 0.989. Fourth, their international correlation can be as high as 0.879 in the post–Bretton-Woods sample.

An additional way of constructing the time series of observable long-run risks in the two countries consists in deriving a measure for the price-consumption ratios, \( v_c \) and \( v^*_c \). According to system (4), they should contain direct information about the predictive components of consumption growth. As a way of building these series, we consider the expected discounted value of consumption growth over the infinite horizon, \( \sum_{j=0}^{\infty} E_t \kappa_1^j \Delta c_{i,t+j} \). The discount factor \( \kappa_1 \) is related to the unconditional mean of the price-consumption ratio as in Campbell and Shiller (1988). We set \( \kappa_1 = 0.965 \), a value consistent with the average wealth-consumption ratio estimates in Lustig, Van Nieuwerburgh and Verdelhan (2009). The results are basically unaltered for a wide range of alternative values. Expectations are computed recursively from an unrestricted VAR(1) on the vector \( z_i^t = [\Delta c_i^t, pd_i^t, rf_i^t] \). Given this assumption, it follows that

\[
\sum_{j=0}^{\infty} E_t \kappa_1^j \Delta c_{i,t+j} = A^i (I - \kappa_1 A^i)^{-1} z_i^t, \quad \forall i \in \{US, UK\}
\]

where \( A^i \) is the estimated matrix of VAR coefficients. The last row of table 4 reports the summary statistics for this approach, providing further evidence in favor of the presence of highly cross-country correlated slowly moving predictive components of consumption growth rates.

**Correlations and Volatilities.** In figure 4 we plot the time series of \( \hat{x}_{i,t}^{US} \) and \( \hat{x}_{i,t}^{UK} \) from regression (9). The other regressions produce similar results. A number of interesting facts emerge from this picture. First, the correlation between the predictable components of consumption growth seems to change over time. Indeed, it ranges from a historical low value of 0.22 from the beginning of the sample until 1941, to an all-time high level of 0.90 in the last twenty years. Second, there appears to be a negative relationship between the correlation of the predictable components of consumption and the volatility of the growth of the dollar-pound exchange rate. In particular, if we divide the post–Bretton Woods sample into two parts, the lowest exchange rate volatility occurred in the years in which the correlation
To further investigate this negative relationship, we construct a time-varying measure of annual correlations between $x_{LH,t}^{US}$ and $x_{LH,t}^{UK}$ and time-varying volatilities of exchange rate’s movements. The appendix reports the details on how the second moments were constructed. Table 5 reports the time series regressions of exchange rate volatilities on a constant and on the measures of cross-country correlations of long-run risks obtained from the four predictive regression approaches described above. The negative relationship between volatilities and correlations is a robust finding across all the alternative ways of measuring long-run risks.

This is even more apparent from looking at figure 5. The fluctuations in the British Pound–US Dollar exchange rate have become less and less volatile over time, just as the correlation of long-run growth perspectives has steadily increased. This finding provides further empirical validation of our theoretical model.

The last column of table 5 reports the variance of the difference of the predictive components
FIG. 5 - Correlations of long-run risks and volatilities of exchange rate’s movements. The correlations of long-run risks are measured as the expected discounted sum of consumption growth rates obtained from the VAR against the volatility of the growth of the exchange rate between US Dollars and British Pounds.

as a fraction of the overall exchange rate volatility. This should give a sense of the amount of predictability of exchange rate’s fluctuations that is contained in the long-run risks. These numbers are typically very low, confirming that log exchange rate growth is difficult to forecast.

C  Long-run risk, consumption and Euler equations

In this section we use the set of predictive variables for consumption outlined above to test a set of moment restrictions resulting from our model (equations (2)-(3)). In particular, we focus on the Euler equations for domestic and foreign stock market returns and risk-free rates for US- and UK-based investors and on the first two moments of exchange rate growth. We estimate a common set of preference parameters, by forcing US and UK agents to share the same risk aversion coefficient and elasticity of intertemporal substitution.

We adopt a continuous GMM estimator in which the optimal weighting matrix is continuously
<table>
<thead>
<tr>
<th></th>
<th>$R^2$</th>
<th>$\beta$</th>
<th>$\frac{\text{Var} \left( x^{\text{US}} - x^{\text{UK}} \right)}{\text{Var}(\Delta e)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pd and risk-free</td>
<td>0.929</td>
<td>-2.456</td>
<td>0.007</td>
</tr>
<tr>
<td></td>
<td>(0.128)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>All predictive variables</td>
<td>0.610</td>
<td>-9.809</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>(2.184)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pd only</td>
<td>0.792</td>
<td>-5.996</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>(0.773)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VAR</td>
<td>0.924</td>
<td>-7.419</td>
<td>0.061</td>
</tr>
<tr>
<td></td>
<td>(0.401)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes - The second and third columns report the $R^2$ and the slope coefficient in the regression of exchange rate’s volatility on the cross-country correlation of long-run risks, respectively. The numbers in parenthesis are Newey-West adjusted standard errors. The last column displays the faction of exchange rate growth variance that is accounted for by the variance of the difference of the predictive components of consumption growth.

Focusing on the entire sample may be inappropriate when it comes to testing the ability of our model to reproduce the average volatility of the exchange rate and the validity of some of the Euler equations. Exchange rates have abandoned the nominal rigidities regime imposed by the Bretton Woods system in the early 1970's, and, as a consequence of that, the degree of volatility of this series changes dramatically. Furthermore, an extensive literature (reviewed by Quinn (1997)) documents that the degree of financial markets’ openness between the US...
### Table 6
**Euler equations’ GMM**

<table>
<thead>
<tr>
<th></th>
<th>Conditional Estimation</th>
<th>Joint Estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P/D$</td>
<td>$P/D$ and $R_f$</td>
</tr>
<tr>
<td>$\psi$</td>
<td>5.526</td>
<td>3.881</td>
</tr>
<tr>
<td></td>
<td>( -38.864 )</td>
<td>( -31.361 )</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>13.537</td>
<td>13.915</td>
</tr>
<tr>
<td></td>
<td>( -1.053 )</td>
<td>( -2.182 )</td>
</tr>
<tr>
<td>$\rho_x$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Wald-stat p-value</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\gamma - 1/\psi$ t-stat</td>
<td>5.872</td>
<td>16.644</td>
</tr>
<tr>
<td></td>
<td>15.589</td>
<td>5.022</td>
</tr>
<tr>
<td>J-stat</td>
<td>21.801</td>
<td>26.279</td>
</tr>
<tr>
<td></td>
<td>30.526</td>
<td>32.969</td>
</tr>
<tr>
<td></td>
<td>14.985</td>
<td>14.643</td>
</tr>
<tr>
<td></td>
<td>( 0.005 )</td>
<td>( 0.001 )</td>
</tr>
<tr>
<td></td>
<td>( 0.000 )</td>
<td>( 0.001 )</td>
</tr>
<tr>
<td></td>
<td>( 0.000 )</td>
<td>( 0.001 )</td>
</tr>
<tr>
<td></td>
<td>( 0.101 )</td>
<td>( 0.101 )</td>
</tr>
</tbody>
</table>

Notes - Each column reports the parameters estimated using GMM on the 1971-2006 sample. All GMMs have the following set of moments conditions in common: Euler equations for domestic stock market returns and risk free rates (4), Euler equations for foreign stock market returns and risk free rates expressed in local units (4), and first four moments of the exchange rates growth (4). The GMMs in the second, third and fourth columns are conditional on the OLS estimates of the regression of consumption growth of the previous section. For the results reported in columns 5-7 all models’ parameters are estimated jointly by adding the appropriate set of orthogonality restrictions. The numbers in parenthesis below each estimate are Newey-West adjusted standard errors. Wald statistics test the null of no predictability in consumption growth rates in both countries. The row $\gamma - 1/\psi$ t-stat reports the t-stat for the one-tailed null $H_0 : \gamma \leq 1/\psi$. The numbers in parenthesis below each J-stat are p-values.

and the UK increased only after the late 1960’s. For all these reasons, we find it appropriate to restrict our attention on the 1971-2006 sample.

In tables 6 we report the results of two alternative approaches. In the first case, we take the time series of long-run risks as exogenously given and use the moment conditions to estimate the preference parameters of the two countries’ investors (see columns 1–3). Secondly, we test the entire model by jointly estimating the set of preference parameters and coefficients governing the dynamics of the predictable components of consumption growth in the US and the UK (see columns 4–6). In the latter case, we also include the set of least-squares orthogonality conditions to identify the additional parameters of the model. We further assume that the predictive components of consumption growth rates have the exact same persistence.
The first three columns of table 6 are two-country counterparts of the exercise conducted by Bansal et al. (2006), whose focus is on US Euler equation restrictions alone. Compared to their results, our estimated risk aversion coefficient is about half as large and the intertemporal elasticity of substitution is in excess of one. We regard this as an important result, which is driven primarily by the information contained in exchange rate movements. The explanation is as follows. Risk aversion and persistence of the predictive component of consumption growth play complementary roles in contributing to risk premia. At annual frequency, the persistence parameter suffers a downward bias from its monthly counterpart, due to temporal aggregation issues. In a single-country setting, risk aversion has to increase to generate large risk premia. To prevent the risk-free rate from being excessively low, the intertemporal elasticity of substitution drops below one. In a multi-country setting, the modest degree of volatility of exchange rate’s movements prevents risk aversion from getting too large. This limits the downward pressure on the estimated intertemporal elasticity of substitution.

When we turn our attention to the joint estimation of the predictive components of consumption growth and the preference parameters, the results are significantly improved. The estimated preference parameters are in line with the numbers that we suggested in the earlier sections. The estimated risk aversion coefficient is further reduced to a level of about 3. As we jointly estimate all model’s parameters, the persistence of the predictive components increases. This improves the ability of the model to satisfy the Euler equation restrictions. The model is not rejected by the data at typical levels of significance, with the only exception for the case in which the set of predictors is limited to the price-dividend ratio only. A Wald test on the null that consumption growth rates are jointly i.i.d. processes is overwhelmingly rejected. This provides further validation of the presence of long-run risks in consumption dynamics.

In table 6 we also report the t-statistics for the null hypothesis that consumers have a preference for late resolution of uncertainty, i.e. \( \gamma \leq \frac{1}{\psi} \). We typically reject the null at conventional levels of significance. According to equation (5), this implies that a currency should depreciate not only in response to positive short-run news, but also following good news for the long-run.
D Long-run risk, dividends and returns

In this section we implement a GMM estimation that directly compares the time series of returns and exchange rates implied by our model, (3)-(4), with those observed in the data. In order to estimate the dividends process reported in (3), we proceed as follows. We jointly estimate the following two equations on the entire sample:

\[
\Delta c_i = \beta_0 \Delta c_{i-1} + \beta_1 p_{i-1} + \beta_2 \Delta cy_{i-1} + \beta_3 \text{default}_{i-1} + \beta_4 r_{f,t-1} + \epsilon_{c,t} \\
\Delta d_i = \lambda (\beta_0 \Delta c_{i-1} + \beta_1 p_{i-1} + \beta_2 \Delta cy_{i-1} + \beta_3 \text{default}_{i-1} + \beta_4 r_{f,t-1}) + \epsilon_{d,t},
\]

for all countries \(i = \{US, UK\}\). The estimates are reported in the first six columns of table 7 for three different combinations of the predictive variables. The results suggest that there is predictability of dividends growth both in the US and the UK, regardless of the set of predictors. The \(R^2\)’s are lower than those reported in table 4, but most of the estimated coefficients are significantly different from zero.

We regard system (10) as the restricted version of the following system of equations:

\[
\Delta c_i = \beta_0 \Delta c_{i-1} + \beta_1 p_{i-1} + \beta_2 \Delta cy_{i-1} + \beta_3 \text{default}_{i-1} + \beta_4 r_{f,t-1} + \epsilon_{c,t} \\
\Delta d_i = \phi_0 \Delta c_{i-1} + \phi_1 p_{i-1} + \phi_2 \Delta cy_{i-1} + \phi_3 \text{default}_{i-1} + \phi_4 r_{f,t-1} + \epsilon_{d,t}
\]

The results of the LR tests for the null that \(\phi_j = \lambda \beta_j, \forall i = \{US, UK\}\) and \(\forall j = \{0, 1, 2, 3\}\) (see table 7) indicate that in both countries the null cannot be rejected at least at the 5% level of significance, which appears to be consistent with our theoretical assumption of a common long-run component in the dynamics of consumption and dividends. On the grounds of this evidence, we assume henceforth that the consumption and cash flows of each of the two countries can be predicted by the same linear combination of variables. This will enable us both to keep our econometric specification close to the theoretical model and to deal with a smaller set of parameters in the estimation.
Table 7

Predictive Regressions on Dividends

<table>
<thead>
<tr>
<th></th>
<th>$\Delta c_{t-1}$</th>
<th>$pd_{t-1}$</th>
<th>$cy_{t-1}$</th>
<th>$default_{t-1}$</th>
<th>$r_{f,t-1}$</th>
<th>$\lambda$</th>
<th>$R^2_{\Delta d}$</th>
<th>LR-test</th>
<th>$\rho_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>0.470</td>
<td>0.011</td>
<td>-0.063</td>
<td>1.289</td>
<td>-0.045</td>
<td>2.653</td>
<td>0.071</td>
<td>6.642</td>
<td>0.673</td>
</tr>
<tr>
<td></td>
<td>(0.136)</td>
<td>(0.006)</td>
<td>(0.043)</td>
<td>(0.739)</td>
<td>(0.134)</td>
<td>(1.052)</td>
<td>(.933)</td>
<td>(.074)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>0.016</td>
<td>-</td>
<td>-</td>
<td>-0.299</td>
<td>5.492</td>
<td>0.176</td>
<td>0.144</td>
<td>0.768</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.095)</td>
<td>(1.720)</td>
<td>(0.095)</td>
<td>(1.720)</td>
<td>(0.704)</td>
<td>(.074)</td>
<td>(.076)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>0.014</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>5.965</td>
<td>0.113</td>
<td>-</td>
<td>0.887</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(2.252)</td>
<td></td>
<td>(2.252)</td>
<td></td>
<td>(.056)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UK</td>
<td>0.268</td>
<td>0.032</td>
<td>-0.088</td>
<td>-</td>
<td>-0.226</td>
<td>1.666</td>
<td>0.129</td>
<td>0.734</td>
<td>0.761</td>
</tr>
<tr>
<td></td>
<td>(0.128)</td>
<td>(0.020)</td>
<td>(0.030)</td>
<td>(0.531)</td>
<td>(0.531)</td>
<td>(0.580)</td>
<td>(.722)</td>
<td>(.074)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>0.031</td>
<td>-</td>
<td>-</td>
<td>0.183</td>
<td>2.097</td>
<td>0.063</td>
<td>3.867</td>
<td>0.787</td>
</tr>
<tr>
<td></td>
<td>(0.024)</td>
<td>(0.254)</td>
<td>(1.249)</td>
<td>(0.254)</td>
<td>(1.249)</td>
<td>(0.050)</td>
<td>(0.073)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>0.033</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2.297</td>
<td>0.067</td>
<td>-</td>
<td>0.737</td>
</tr>
<tr>
<td></td>
<td>(0.024)</td>
<td>(1.360)</td>
<td></td>
<td>(1.360)</td>
<td></td>
<td>(.081)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes - In Columns 2-7, we report the maximum likelihood estimates of the parameters in (10). Newey-West adjusted standard errors are reported in parenthesis. The next column shows the implied $R^2$ for dividend growth. The ninth column reports the Likelihood Ratio tests (p-values in parenthesis) with respect to the unrestricted system (11). The last column reports the estimates of the persistence of the predictable component. Data are annual, demeaned. The sample ranges from 1929 to 2006.

We estimate our pricing model focusing only on the post–Bretton Woods sample. All GMMs have the following set of moment conditions in common: the average residual between the log excess returns, log risk-free rate, log exchange rate growth and the squared log exchange rate growth implied by the model and those observed in the data are zero (6 restrictions overall).

Our main results are summarized in table 8. As in the previous section, we first estimate the preference parameters taking as given the dynamics of quantities in each country. We report our conditional estimates of the preference parameters in column 2-4 so that each column refers to a specific set of predictive variables. Through this empirical methodology we obtain two relevant results: i) our point estimates are even closer to our benchmark calibration; ii) the uncertainty about each estimate is significantly reduced. This result is particularly evident for the intertemporal elasticity of substitution and the composite parameter $\gamma - 1/\psi$.

According to our results, there is a statistically significant preference for early resolution of uncertainty.

In column 5-7 of table 8 we report the results of the joint estimation of (10) and (4) over the
Table 8
GMM Estimation with Dividends

<table>
<thead>
<tr>
<th></th>
<th>Conditional Estimation</th>
<th>Joint Estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P/D$</td>
<td>$P/D$ and $R_f$</td>
</tr>
<tr>
<td>$\psi$</td>
<td>2.139</td>
<td>1.538</td>
</tr>
<tr>
<td>(0.331)</td>
<td>(0.043)</td>
<td>(0.058)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>6.254</td>
<td>5.692</td>
</tr>
<tr>
<td>(0.542)</td>
<td>(0.439)</td>
<td>(0.279)</td>
</tr>
<tr>
<td>$\gamma - 1/\psi$</td>
<td>5.787</td>
<td>5.042</td>
</tr>
<tr>
<td>(0.604)</td>
<td>(0.438)</td>
<td>(0.280)</td>
</tr>
<tr>
<td>$\rho_x$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wald-stat p-value</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>J-stat</td>
<td>41.58</td>
<td>29.96</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
</tbody>
</table>

Notes - Each column reports the parameters estimated using GMM with continuous updating on the 1971-2006 sample. Data are annual. The first three GMMs are conditional on the OLS estimates reported in table7. In the last three columns we report results of the joint GMM estimation of (4) and (10). The appropriate orthogonality restrictions are added. The numbers in parenthesis below each estimate are Newey-West adjusted standard errors. The numbers in parenthesis below each J-stat are p-values. Our Wald statistics is computed imposing all the restrictions required to test the null of no predictability in consumption and dividends growth rates in both countries.

1971-2006 sample. Two things need to be noted. First, the confidence interval of the IES is still tight around 2, consistent with our the results of column 2-5. Second, the relative risk aversion coefficient is very moderate, except for the case in which the price-dividend ratio is the only predictor. This confirms our intuition that introducing exchange rates in the estimation procedure can mitigate the bias issues noted by Bansal et al. (2006).

According to the Wald-test, the assumption that all the cash-flows growth rates are i.i.d. is rejected also in the post–Bretton Woods sample. The point estimates of the persistence of the long-run components, $\rho_x$, are in line with those obtained in the rest of our empirical exercises. Furthermore, all standard errors are small, suggesting a tight identification of our parameters. Turning our attention to the J-stat, we document a significant improvement when we jointly estimate preference and quantity parameters. In two cases out of three, the model cannot be rejected at conventional levels of significance.
As seen in figure 6, the model is also able to reproduce the time series of excess returns and risk-free rates in the post–Bretton Woods era. When the larger set of predictive variables for consumption is used, the model does a good job of tracking the time behavior of returns and exchange rate movements. Indeed, at our point estimates, the volatility of exchange rate growth is about 18% and hence very close to what we observe in the data. Furthermore, the ability of the model of producing realistic path of international prices and currencies seems to increase over time, especially after the 1980-82 crises.

4 Concluding remarks

By disentangling the intertemporal elasticity of substitution from the reciprocal of the coefficient of risk aversion and by allowing for a persistent and highly cross-country-correlated forecastable component of consumption growth, it is possible to explain a large set of international finance puzzles using a frictionless consumption-based asset pricing model. In our
analysis, the dichotomy between international quantities and prices documented by the international finance literature disappears after accounting for the intertemporal composition of consumption risk. Consumption and cash flow dynamics are in line with the data, and at the same time the equity premium puzzle is solved within each country. Exchange rate fluctuations resemble those that we observe for major industrialized countries, and the international investment opportunity set reflects the Sharpe ratios and the degree of correlation suggested by the data.

Future developments of this line of research should address the deeper economic question of the origin of the predictable components of consumption growth in the context of a fully specified production economy. In particular, it will be important to study the predictions of the model for the dynamics of current accounts, portfolio allocations, and international investment flows. Furthermore, the introduction of stochastic volatility could allow the model to replicate the empirical evidence on currency excess returns, studied by Lustig and Verdelhan (2006) and Lustig et al. (2008).
Appendix

**Exponential Smoothing.** For the construction of time-varying second moments in the empirical section, we employ exponential smoothing. Specifically, the date \( t \) correlation, \( \rho^h_t \), is obtained as it follows

\[
q_{12,t} = \rho q_{12,t-1} + (1 - \rho) \bar{x}_{US,t} LH \bar{x}_{UK,t} LH,
\]

\[
q_{1,t} = \rho q_{1,t-1} + (1 - \rho) \left( \bar{x}_{US,t} LH \right)^2,
\]

\[
q_{2,t} = \rho q_{2,t-1} + (1 - \rho) \left( \bar{x}_{UK,t} LH \right)^2,
\]

\[
\rho^h_t = \frac{q_{12,t}}{\sqrt{q_{1,t} q_{2,t}}}
\]

where \( \bar{x}_{i,t} LH \) refers to demeaned variables and \( q_{1,0}, q_{2,0}, q_{12,0} \) are variances and covariance computed on a pre-sample of 15 years. The parameter \( \rho \) is set to 0.95. Similarly dynamic variances of exchange rate’s movements can be computed via exponential smoothing as:

\[
\sigma^2_t = \rho \sigma^2_{t-1} + (1 - \rho) \left( \bar{\Delta e}_t \right)^2
\]

**GMM on Euler Equations restrictions.** The predictable components of consumption growth rates are constructed according to the regressions (7)–(9). Innovations to consumption growth and its low frequency components are computed as follows:

\[
\epsilon^i_{c,t+1} = \Delta c^i_{t+1} - \bar{x}_t^i
\]

\[
\epsilon^i_{x,t} = x_t^i - \rho^i x_{t-1}^i
\]

We then use a log-linearized version of the model (equations (2)-(3)) in order to recover time series of stochastic discount factors and exchange rates. We calibrate the mean of consumption growth to \( \mu = .02 \) and the subjective discount factor to \( \delta = .99 \) for both US and UK. The continuous updating GMM procedure of Hansen et al. (1996) is applied to the sample counterparts of the following set of Euler equations

- \( E \left[ M_{t+1}^i R_{j,t+1}^i \right] - 1 = 0, \forall i = \{US, UK\} \) and \( \forall j = \{m, f\} \)
- \( E \left[ M_{t+1}^{US} R_{j,t+1}^{UK} \exp^{\Delta e_{t+1}} \right] - 1 = 0, \forall j = \{m, f\} \)
• \( E \left[ M_{t+1}^{UK} / M_{t+1}^{US} \right] (\exp^{\Delta e_{t+1}})^{-1} \right] - 1 = 0, \forall j = \{m, f\} \)

and of the following exchange rate moment conditions

• \( E \left[ M_{t+1}^{US} / M_{t+1}^{UK} - e^{\Delta e_{t+1}} \right] = 0 \)
• \( E \left[ (M_{t+1}^{US} / M_{t+1}^{UK})^2 - (e^{\Delta e_{t+1}})^2 \right] = 0 \)

When preference parameters and coefficients governing the dynamics of the predictable components of consumption growth are jointly estimated, we introduce also the appropriate set of orthogonality conditions.

**GMM on the reduced form of the model.** The innovations to consumption growth and its predictable component can be computed as in the previous section. The innovation to cash flows is

\[
\varepsilon_{d,t+1} = \Delta d_{t+1} - \lambda^i x^i_t, \forall i \in \{US, UK\}
\]

The log-linearized version of the model (equations (3)–(4)) is then used to construct the sample counterparts of the following moments conditions:

• \( E \left[ (\bar{r}_{m,t+1}^i - \bar{r}_{f,t}^i) - (r_{m,t+1}^i - r_{f,t+1}^i) \right] = 0, \forall i \in \{US, UK\} \)
• \( E \left[ \bar{r}_{f,t}^i - r_{f,t+1}^i \right] = 0, \forall i \in \{US, UK\} \)
• \( E \left[ M_{t+1}^{US} / M_{t+1}^{UK} - \exp \{\Delta e_{t+1}\} \right] = 0 \)
• \( E \left[ (M_{t+1}^{US} / M_{t+1}^{UK})^2 - (\exp \{\Delta e_{t+1}\})^2 \right] = 0 \)

Where \( \bar{r}_{m,t+1}^i \) and \( \bar{r}_{f,t}^i \) are the model predictions for the market returns and the risk free rate. When preference parameters and the coefficients governing the dynamics of the predictable components of consumption growth are jointly estimated, we introduce also the appropriate set of orthogonality conditions.
References


