By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior

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Background

- Habit formation captures a fundamental feature of psychology: repetition of a stimulus diminishes the perception of the stimulus and responses to it.
- Habit formation is external
  - Habit depends on history of aggregate consumption, not individual’s own past
- Habit moves slowly in response to consumption
  - Produces slow mean reversion in the price/dividend ratio, long-horizon return forecastability, and persistent movements in volatility
- Habit adapts nonlinearly to the history of consumption
  - Nonlinearity keeps habit always below consumption and keeps marginal utility always finite and positive even in an endowment economy.
Preference and Technology

- Identical agents maximizes the utility function:
  \[
  E \sum_{t=0}^{\infty} \delta_t \frac{(C_t - X_t)^{1-\gamma} - 1}{1 - \gamma}.
  \]
  \(X_t\) is the level of habit, and \(\delta\) is the subjective time discount factor.

- Surplus consumption ratio: \(S_t = (C_t - X_t)/C_t\)

- Local curvature of the utility function, which we write as \(\eta_t\), is related to the surplus consumption ratio:
  \[
  \eta_t \equiv - \frac{C_t u_{cc}(C_t, X_t)}{u_c(C_t, X_t)} = \frac{\gamma}{S_t}
  \]
  low consumption relative to habit, or a low surplus consumption ratio, implies a high local curvature of the utility function.
Intuition

- Consumers do not fear stocks because of the resulting risk to wealth or to consumption per se; they fear stocks primarily because stocks are likely to do poorly in recessions, times of low surplus consumption ratios.
- The volatility of \((C_{t+1}/C_t)^{-\gamma}\) is so low that it accounts for essentially no risk premia. The volatility of \((S_{t+1}/S_t)^{-\gamma}\) is much larger and accounts for nearly all risk premia.
- One can regard S as an amplification mechanism for consumption risks in marginal utility.
- Variation across assets in expected returns is driven by variation across assets in covariances with recessions rather than covariance with consumption growth.
External Habit Formation

- superscript $\alpha$ denotes population average
- lower case variables denote log
- $s^\alpha_t = \ln\left(\frac{C^\alpha_t - X_t}{C^\alpha_t}\right)$ follows an heteroskedastic AR(1) process
- $s^\alpha_{t+1} = (1 - \phi)\bar{s} + \phi s^\alpha_t + \lambda(s^\alpha_t)(c^\alpha_{t+1} - c^\alpha_t - g)$
- $\lambda(s^\alpha_t)$ is the sensitivity function
- habit $x_t$ itself adjusts slowly and geometrically to consumption $c^\alpha_t$ with coefficient $\phi$
- In equilibrium, identical individuals choose the same level of consumption. Therefore, we drop the superscript $\alpha$

Model consumption growth as an i.i.d. lognormal process

$$\Delta c_{t+1} = g + \nu_{t+1}, \quad \nu_{t+1} \sim \text{i.i.d. } \mathcal{N}(0, \sigma^2)$$
Marginal Utility

- Habit is external therefore

\[ u_c(C_t, X_t) = (C_t - X_t)^{-\gamma} = S_t^{-\gamma} C_t^{-\gamma}. \]

- Intertemporal marginal rate of substitution

\[ M_{t+1} \equiv \delta \frac{u_c(C_{t+1}, X_{t+1})}{u_c(C_t, X_t)} = \delta \left( \frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t} \right)^{-\gamma} \]

\[ M_{t+1} = \delta G^{-\gamma} e^{-\gamma(s_{t+1} - s_t + \nu_{t+1})} = \delta G^{-\gamma} e^{-\gamma((\phi - 1)(s_t - \bar{s}) + [1 + \lambda(s_t)] \nu_{t+1})}. \]
Slope of the Mean-Standard Deviation Frontier

- Hansen-Jagannathan Bound

\[
\frac{E_t(R_{t+1}^e)}{\sigma_t(R_{t+1}^e)} = -\rho_t(M_{t+1}, R_{t+1}^e) \frac{\sigma_t(M_{t+1})}{E_t(M_{t+1})} \leq \frac{\sigma_t(M_{t+1})}{E_t(M_{t+1})}
\]

- \( M \) is conditionally lognormal

\[
\frac{\sigma(M)}{E(M)} = \frac{\sqrt{E(M^2) - E(M)^2}}{E(M)} = \frac{\sqrt{e^{2\mu + 2\sigma^2} - e^{2\mu + \sigma^2}}}{e^{\mu + (\sigma^2/2)}} = \sqrt{e^{\sigma^2} - 1}
\]

- Therefore the largest possible Sharpe ratio is

\[
\max_{\text{[all assets]}} \frac{E_t(R_{t+1}^e)}{\sigma_t(R_{t+1}^e)} = \left( e^{r^2\sigma^2[1+\lambda(s_t)]^2} - 1 \right)^{1/2} \approx \gamma \sigma \left[ 1 + \lambda(s_t) \right]
\]

- Thus, to produce a time varying Sharpe ratio, \( \lambda(s) \) must vary with \( s \). To produce risk prices that are higher in bad times, when \( s \) is low, \( \lambda(s) \) and hence the volatility of \( s \) must increase as \( s \) declines.
Risk-Free Rate

Therefore, the log risk-free rate is

\[
r_f^t = -\ln(\delta) + \gamma g - \gamma (1 - \phi) (s_t - \bar{s}) - \frac{\gamma^2 \sigma^2}{2} \left[ 1 + \lambda (s_t) \right]^2.
\]

Last term is precautionary savings term. As uncertainty increases, consumers are more willing to save, and this willingness drives down the equilibrium risk-free interest rate.

The second last term reflects intertemporal substitution. If the surplus consumption ratio is low, the marginal utility of consumption is high. The consumer would then like to borrow, which would drive up the equilibrium risk-free interest rate.

Empirically, little variation in risk-free rates. Therefore:

- Either, serial correlation parameter \( \phi \) is near one
- Or, \( \lambda(s_t) \) must decline with \( s_t \) so that uncertainty is high when \( s \) is low and the precautionary saving term offsets the intertemporal substitution term.
Choosing the Sensitivity Function $\lambda(s_t)$

Choose $\lambda(s_t)$ to satisfy three conditions:

- The risk-free interest rate is constant
- Habit is predetermined at the steady state ($dx/dc=0$ at s.s.)
- Habit is predetermined near the steady state or, equivalently, habit moves non-negatively with consumption everywhere

Lead us to a specification of the sensitivity function

\[
\begin{align*}
\bar{S} &= \sigma \sqrt{\frac{\gamma}{1 - \phi}}, \\
\lambda(s_t) &= \begin{cases} 
\frac{1}{S} \sqrt{1 - 2(s_t - \bar{S}) - 1}, & s_t \leq s_{\text{max}} \\
0, & s_t \geq s_{\text{max}}
\end{cases}
\end{align*}
\]

\[
s_{\text{max}} \equiv \bar{S} + \frac{1}{2} (1 - \bar{S}^2).
\]
Plot of the Sensitivity Function $\lambda(s_t)$

Negative relationship between $\lambda(s_t)$ and $s_t$ needed to produce:
1. constant risk-free rate
2. countercyclical price of risk.

Habit responds positively to consumption, $dx/dc \geq 0$ everywhere.

Habit does not move contemporaneously with consumption at and near the steady state, marked by a vertical line.
Pricing Long-lived Assets

- Stocks is a claim to the consumption stream, the price/dividend or, equivalently, the price/consumption ratio for a consumption claim satisfies:

\[ 1 = E_t[M_{t+1}R_{t+1}], \quad R_{t+1} \equiv \frac{P_{t+1} + D_{t+1}}{P_t}, \quad \frac{P_t(s_t)}{C_t} = E_t \left[ M_{t+1} \frac{C_{t+1}}{C_t} \left( 1 + \frac{P_{t+1}(s_{t+1})}{C_{t+1}} \right) \right]. \]

- Substitute in \( M_{t+1} \) and consumption growth and then solve this functional equation numerically on a grid for the state variable \( s_t \), using numerical integration over the normally distributed shock \( v_{t+1} \) to evaluate the conditional expectation.

- Given the price/consumption ratio as a function of state variable \( (s_t) \), calculate expected returns, the conditional standard deviation of returns, and other interesting quantities.
Choosing Parameters

1. Postwar NYSE data (1947-95), 3-month T-bill rate, per capital nondurables and services consumption data
2. Century long data on S&P 500 and per capita consumption
3. Take the mean and standard deviation of log consumption growth, \( g \) and \( \sigma \), to match the consumption data; the serial correlation parameter \( \phi \) to match the serial correlation of log price/dividend ratios; choose the

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### Parameter Choices

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assumed: Mean consumption growth (%)*</td>
<td>( g )</td>
<td>1.89</td>
</tr>
<tr>
<td>Standard deviation of consumption growth (%)*</td>
<td>( \sigma )</td>
<td>1.50</td>
</tr>
<tr>
<td>Log risk-free rate (%)*</td>
<td>( r^f )</td>
<td>.94</td>
</tr>
<tr>
<td>Persistence coefficient*</td>
<td>( \phi )</td>
<td>.87</td>
</tr>
<tr>
<td>Utility curvature</td>
<td>( \gamma )</td>
<td>2.00</td>
</tr>
<tr>
<td>Standard deviation of dividend growth (%)*</td>
<td>( \sigma_w )</td>
<td>11.2</td>
</tr>
<tr>
<td>Correlation between ( \Delta d ) and ( \Delta c )</td>
<td>( \rho )</td>
<td>.2</td>
</tr>
<tr>
<td>Implied: Subjective discount factor*</td>
<td>( \delta )</td>
<td>.89</td>
</tr>
<tr>
<td>Steady-state surplus consumption ratio</td>
<td>( S )</td>
<td>.057</td>
</tr>
<tr>
<td>Maximum surplus consumption ratio</td>
<td>( S_{\text{max}} )</td>
<td>.094</td>
</tr>
</tbody>
</table>

* Annualized values, e.g., \( 12g, \sqrt{12\sigma}, 12r^f, \phi^{12}, \) and \( \delta^{12} \), since the model is simulated at a monthly frequency.
Solution and Evaluation

- solve the model numerically
- simulate 500,000 months of artificial data by drawing shocks from a random number generator, and show how the simulated data replicate many interesting statistics found in actual data.
The surplus consumption ratio spends most of its time above the steady state value, but there is a fat tail of low surplus consumption ratio. The model predicts the distribution of surplus consumption ratios to be negatively skewed.
The price/dividend ratios increase with the surplus consumption.

When consumption is low relative to habit in a recession, the curvature of the utility function is high, and prices are depressed relative to dividends.
As consumption declines toward habit, expected returns rise dramatically over the constant risk-free rate.
As consumption declines toward habit, conditional variance of returns increases.

Matches prediction of ARCH literature: price declines increase volatility, and countercyclical variation in volatility.
## Simulation results

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Consumption Claim</th>
<th>Dividend Claim</th>
<th>Postwar Sample</th>
<th>Long Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(\Delta c)$</td>
<td>1.89*</td>
<td></td>
<td>1.89</td>
<td>1.72</td>
</tr>
<tr>
<td>$\sigma(\Delta c)$</td>
<td>1.22*</td>
<td></td>
<td>1.22</td>
<td>3.32</td>
</tr>
<tr>
<td>$E(r^f)$</td>
<td>.094*</td>
<td></td>
<td>.094</td>
<td>2.92</td>
</tr>
<tr>
<td>$E(r - r^f)/\sigma(r - r^f)$</td>
<td>.43*</td>
<td>.33</td>
<td>.43</td>
<td>.22</td>
</tr>
<tr>
<td>$E(R - R^f)/\sigma(R - R^f)$</td>
<td>.50</td>
<td></td>
<td>.50</td>
<td></td>
</tr>
<tr>
<td>$E(r - r^f)$</td>
<td>6.64</td>
<td>6.52</td>
<td>6.69</td>
<td>3.90</td>
</tr>
<tr>
<td>$\sigma(r - r^f)$</td>
<td>15.2</td>
<td>20.0</td>
<td>15.7</td>
<td>18.0</td>
</tr>
<tr>
<td>$\exp[E(\bar{p} - d)]$</td>
<td>18.3</td>
<td>18.7</td>
<td>24.7</td>
<td>21.1</td>
</tr>
<tr>
<td>$\sigma(\bar{p} - d)$</td>
<td>.27</td>
<td>.29</td>
<td>.26</td>
<td>.27</td>
</tr>
</tbody>
</table>

**Note.**—The model is simulated at a monthly frequency; statistics are calculated from artificial time-averaged data at an annual frequency. All returns are annual percentages. *Statistics that model parameters were chosen to replicate.
Partial sum of autocorrelation coefficients measures mean reversion (more evident in long sample).

The autocorrelation of absolute returns reveals long-horizon conditional heteroskedasticity in the model.
verifies that the price/consumption ratio forecasts long-horizon returns with the right sign: high prices forecast low returns.

cross-correlations between returns and subsequent absolute returns show that a big price decline signals high volatility for several years ahead (leverage effect)
Long Horizon Regressions

- Regressions of log excess stock returns on the log price/dividend ratio in simulated and historical data

High prices forecast low returns for many years in the future, the forecastability of returns increases with the horizon.
Fig. 9.—Historical price/dividend ratio and model predictions based on the history of consumption.
Equity Premium Puzzle

- Under power utility,
  \[ M_{t+1} = \beta (C_{t+1}/C_t)^{-\eta}, \]
  \[ r^f_t = -\ln(\beta) + \eta g - \eta^2 \sigma^2 / 2. \]
  \[ \frac{\mathbb{E}(R^e)}{\sigma(R^e)} \leq e^{\eta^2 \sigma^2 / 2} - 1 \approx \eta \sigma, \]

- To explain a (gross return) Sharpe ratio of 0.50 with \( \sigma = 1.22 \) percent, the power utility model needs a risk aversion coefficient \( \eta > 41 \).

- Risk-free rate puzzle: If \( \eta = 41 \) and \( g = 1.89 \), imposing \( \beta < 1 \) would predict an effective risk-free rate of 90% per year. Also, it implies that the risk-free rate should be sensitive to mean consumption growth rate (\( g \)).

- Here, avoid the risk-free rate puzzle.

- The power parameter \( \gamma = 2 \), much lower than utility curvature \( \gamma / S \), controls the relationship between average consumption growth and the risk-free interest rate. The model does not predict sensitive relationship between risk-free rate and mean consumption growth.

- The model also uses non-time separable preference as opposed to non-state separable preference (Epstein-Zin).
Potential shortcomings

- Does not allow for any heterogeneity across consumers
- Assumes implausibly high risk aversion

\[ \eta_t \equiv - \frac{C_t u_{cc}(C_t, X_t)}{u_c(C_t, X_t)} = \frac{\gamma}{S_t} = 35 \text{ at steady state} \]

- Relies on an external-habit rather than the more common internal-habit specification