Risks for the Long Run and the Real Exchange Rate
by Riccardo Colacito and Mariano Croce (2010)

Paper Summary by
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The International Equity Premium Puzzle

- **PUZZLE**: Given the observed volatility of the US dollar, *power utility* implies a higher correlation between SDFs (consumption growth) across countries than observed.

- **SOLUTION**: A model incorporating Epstein-Zin preferences and long-run consumption growth dynamics resolves this puzzle.

- **KEY ASSUMPTIONS**: Agents care about the temporal distribution of risk *and* countries share very similar long-run growth prospects.
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To verify that countries long-run growth prospects are similar the authors:

- Measure long-run movements of consumption trends via projections on the set of predictive variables (Bansal, Kiku, and Yaron 2006)

- Find that the predictable components of consumption growth rates are highly persistent and that their correlation increases over time, just as the volatility of the exchange rate growth decreases
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Outline

1. The International Equity Premium Puzzle
2. Using Long Run Risk to Resolve the Puzzle
3. Estimating International Long-run Risks
Deriving the Puzzle

Domestic ($M_t$) and foreign ($M^*_t$) discount factors are related via a simple change of units:

\[ M^*_t = M_t \frac{E_t}{E_{t-1}} \]

where $E_t$ is the price of 1 unit of the foreign currency in terms of the domestic currency, e.g. the cost of 1 pound in terms of dollars.

**For the remainder of the presentation:

Domestic = US         Foreign = UK
NOTE: $M_t$ is the dollar-denominated pricing kernel, i.e. it prices dollar-denominated cash flows

Thus, if $R_t$ is the return on a dollar-denominated asset, then the following equation holds

$$1 = \mathbb{E}_{t-1}(M_t R_t)$$

Similarly $M^*_t$ prices pound-denominated assets, and so if $R^*_t$ is the return on a pound-denominated asset the following holds

$$1 = \mathbb{E}_{t-1}(M^*_t R^*_t)$$
Deriving the Puzzle

Alternatively, we could convert pound returns into dollars via $R_t = (E_t/E_{t-1})R_t^*$ and price them using $M_t$ yielding

$$1 = \mathbb{E}_{t-1} (M_t (E_t/E_{t-1})R_t^*)$$

Combining equations

$$\mathbb{E}_{t-1} (M_t^* R_t^*) = \mathbb{E}_{t-1} (M_t (E_t/E_{t-1})R_t^*)$$

which assuming complete markets implies

$$M_t^* = M_t \frac{E_t}{E_{t-1}}$$
Taking the log of both sides of $M^*_t = M_t \frac{E_t}{E_{t-1}}$ yields the following key equation:

$$\Delta e_t = m^*_t - m_t$$

where $m_t = \log M_t$, $m^*_t = \log M^*_t$, and $\Delta e_t = \log E_t - \log E_{t-1}$
Risks for the Long Run and the Real Exchange Rate

The International Equity Premium Puzzle

The Puzzle

\[ \Delta e_t = m^*_t - m_t \text{ implies} \]

\[ \sigma^2_{\Delta e_t} = \underbrace{\sigma^2_{m^*_t}}_{1.5\%} + \underbrace{\sigma^2_{m_t}}_{20\%} - 2 \underbrace{\rho_{m,m^*}}_{0.96} \underbrace{\sigma_{m^*_t} \sigma_{m_t}}_{20\%} \]

If investors have power utility then \( m_t^i = -\gamma \Delta c_t^i \).

\[ \sigma^2_{\Delta e_t} = \underbrace{\gamma^2 \sigma^2_{\Delta c_t^*}}_{28\%} + \underbrace{\gamma^2 \sigma^2_{\Delta c_t}}_{20\%} - 2 \underbrace{\rho_{\Delta c,\Delta c^*}}_{0.3} \underbrace{\gamma \sigma_{\Delta c_t^*} \sigma_{\Delta c_t}}_{20\%} \]

SOLUTION: Allow for long-run consumption growth components to be highly correlated \( \Rightarrow \) High \( \rho_{m,m^*} \) and low \( \sigma^2_{\Delta e_t} \), but ALSO low \( \rho_{\Delta c,\Delta c^*} \).
The Puzzle

$$\Delta e_t = m_t^* - m_t$$ implies

$$\sigma_{\Delta e_t}^2 = \sigma_{m_t^*}^2 + \sigma_{m_t}^2 - 2 \rho_{m,m^*} \sigma_{m_t^*} \sigma_{m_t}$$

1.5% 20% 20% 0.96 20%

If investors have **power utility** then $m_t^i = -\gamma \Delta c_t^i$.

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28% 20% 20% 0.3 20%

**SOLUTION:** Allow for long-run consumption growth components to be highly correlated $\Rightarrow$ High $\rho_{m,m^*}$ and low $\sigma_{\Delta e_t}^2$, but ALSO low $\rho_{\Delta c,\Delta c^*}$. 
The Puzzle

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\[
\frac{\sigma^2_{\Delta e_t}}{1.5\%} = \frac{\sigma^2_{m_t^*}}{20\%} + \frac{\sigma^2_{m_t}}{20\%} - 2 \rho_{m_t,m_t^*} \frac{\sigma_{m_t^*} \sigma_{m_t}}{0.96 \% 20\%}
\]

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\[
\frac{\sigma^2_{\Delta e_t}}{28\%} = \frac{\gamma^2 \sigma^2_{\Delta c_t^*}}{20\%} + \frac{\gamma^2 \sigma^2_{\Delta c_t}}{20\%} - 2 \rho_{\Delta c_t,\Delta c_t^*} \frac{\gamma \sigma_{\Delta c_t^*} \sigma_{\Delta c_t}}{0.3 \% 20\%}
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The Model: An Overview

- Two representative agent countries each producing a unique good
- Simplifying assumption: Countries only consume their own good (i.e. complete home bias)
- Both countries have Epstein-Zin preferences ($\gamma$ CRA, $\psi$ IES, $\delta$ discount rate)
- Economies are symmetric: same preference and transition law parameters
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- Economies are symmetric: same preference and transition law parameters
Epstein-Zin preferences imply the following log-pricing kernel:

\[ m_{t+1} = \frac{1 - \gamma}{1 - 1/\psi} \log \delta - \frac{1 - \gamma}{\psi - 1} \Delta c_t + \frac{1/\psi - \gamma}{1 - 1/\psi} r_{c,t+1} \]

were \( r_{c,t+1} \) log-return on the asset that pays consumption.

Consumption and dividend growth rate dynamics are specified as follows:

\[ \Delta c_t = \mu_c + x_{t-1} + \varepsilon_{c,t} \]
\[ \Delta d_t = \mu_d + \lambda x_{t-1} + \varepsilon_{d,t} \]
\[ x_t = \rho x x_{t-1} + \varepsilon_{x,t} \]
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The Model

The vector of shocks are distributed

$$\begin{bmatrix} \varepsilon_{c,t} & \varepsilon_{d,t} & \varepsilon_{x,t} & \varepsilon_{c,t}^* & \varepsilon_{d,t}^* & \varepsilon_{x,t}^* \end{bmatrix} \sim N(0, \Sigma) \text{ i.i.d.}$$

with

$$\Sigma = \begin{bmatrix}
\sigma & 0 & 0 & \rho_c^{hf} & 0 & 0 \\
0 & \varphi_d \sigma & 0 & 0 & \rho_d^{hf} & 0 \\
0 & 0 & \varphi_e \sigma & 0 & 0 & \rho_x^{hf} \\
\rho_c^{hf} & 0 & 0 & \sigma & 0 & 0 \\
0 & \rho_d^{hf} & 0 & 0 & \varphi_d \sigma & 0 \\
0 & 0 & \rho_x^{hf} & 0 & 0 & \varphi_e \sigma \\
\end{bmatrix}$$
Risks for the Long Run and the Real Exchange Rate

Using Long Run Risk to Resolve the Puzzle

The Model: First-order Linear Approximation

\[ m_{t+1} = \log \delta - \frac{1}{\psi} x_t + \delta \frac{1 - \gamma \psi}{\psi (1 - \rho_x \kappa_c)} \varepsilon_{x,t+1} - \gamma \varepsilon_{c,t+1} \]

\[ r_{d,t+1} = \bar{r}_d + \frac{1}{\psi} x_t + \kappa_d \frac{\lambda - \frac{1}{\psi}}{1 - \rho_x \kappa_d} \varepsilon_{x,t+1} + \varepsilon_{d,t+1} \]

\[ r_{f,t+1} = \bar{r}_f + \frac{1}{\psi} x_t \]

\[ v_{d,t} = \bar{v}_d + \frac{\lambda - \frac{1}{\psi}}{1 - \rho_x \kappa_d} x_t \quad \text{price-dividend ratio} \]

\[ v_{c,t} = \bar{v}_c + \frac{\lambda - \frac{1}{\psi}}{1 - \rho_x \kappa_c} x_t \quad \text{price-consumption ratio} \]

\[ \Delta e_{t+1} = m^*_t - m_t \]
The Model: Exchange Rate Dynamics

Using the first and last equation

\[ m_{t+1} = \log \delta - \frac{1}{\psi} x_t + \delta \frac{1 - \gamma \psi}{\psi (1 - \rho \kappa_c)} \varepsilon_{x,t+1} - \gamma \varepsilon_{c,t+1} \]

\[ \Delta e_{t+1} = m_{t+1}^* - m_{t+1} \]

we obtain the following equation describing exchange rate dynamics

\[ \Delta e_{t+1} - \mathbb{E}_t [\Delta e_{t+1}] = \delta \frac{1}{\psi} - \gamma \left( \varepsilon_{x,t+1}^* - \varepsilon_{x,t+1} \right) - \gamma \left( \varepsilon_{c,t+1}^* - \varepsilon_{c,t+1} \right) \]
The Model: Exchange Rate Dynamics

\[ \Delta e_{t+1} - \mathbb{E}_t [\Delta e_{t+1}] = \delta \frac{1 - \gamma}{(1 - \rho_x \kappa_c)} (\varepsilon^*_{x,t+1} - \varepsilon_{x,t+1}) - \gamma (\varepsilon^*_{c,t+1} - \varepsilon_{c,t+1}) \]

1. Must have \( \gamma \neq \frac{1}{\psi} \) for long-run growth prospects to affect exchange rate

2. If \( \gamma > \frac{1}{\psi} \) and \( \varepsilon^*_{x,t+1} - \varepsilon_{x,t+1} > 0 \), then \( \Delta e_{t+1} \downarrow \)

3. The more persistent the long-run components are (high \( \rho_x \)), the stronger the impact of long-run news

4. The degree of cross-country correlation between the shocks matters, e.g. if shocks are perfectly correlated then they won’t alter the exchange rate
The Model: Exchange Rate Dynamics

\[ \Delta e_{t+1} - \mathbb{E}_t [\Delta e_{t+1}] = \delta \frac{1}{\psi} - \gamma \left( \frac{\varepsilon_{x,t+1}^* - \varepsilon_{x,t+1}}{(1 - \rho_x \kappa_c)} (\varepsilon_{x,t+1}^* - \varepsilon_{x,t+1}) - \gamma (\varepsilon_{c,t+1}^* - \varepsilon_{c,t+1}) \right) \]

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\[ \Delta e_{t+1} - \mathbb{E}_t [\Delta e_{t+1}] = \delta \frac{1 - \gamma}{(1 - \rho_x \kappa_c)} (\epsilon_x^{*,t+1} - \epsilon_x^{*,t+1}) - \gamma (\epsilon_c^{*,t+1} - \epsilon_c^{*,t+1}) \]

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The Model: Exchange Rate Dynamics

\[ \Delta e_{t+1} - \mathbb{E}_t [\Delta e_{t+1}] = \delta \frac{1}{\psi} \left( \frac{1}{\psi} - \gamma \right) (\varepsilon^*_x, t+1 - \varepsilon_x, t+1) - \gamma (\varepsilon^*_c, t+1 - \varepsilon_c, t+1) \]

1. Must have \( \gamma \neq 1/\psi \) for long-run growth prospects to affect exchange rate

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Model Calibration

Model is calibrated to match US and UK data averages

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Psi$</td>
<td>Intertemporal elasticity of substitution</td>
<td>2</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Risk aversion</td>
<td>4.25</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Subjective discount factor</td>
<td>0.998</td>
</tr>
<tr>
<td>$\mu_c$</td>
<td>Average consumption growth</td>
<td>$15 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Autoregressive coefficient of the long-run component $x_t$</td>
<td>0.987</td>
</tr>
<tr>
<td>$\varphi_e$</td>
<td>Ratio of long-run shock and short-run shock volatilities</td>
<td>0.048</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Standard deviation of the short-run shock to consumption</td>
<td>$68 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\mu_d$</td>
<td>Average dividend growth</td>
<td>0.0007</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Leverage</td>
<td>3.0</td>
</tr>
<tr>
<td>$\varphi_d$</td>
<td>Volatility ratio of short-run shocks to dividend and consumption growth</td>
<td>5.0</td>
</tr>
<tr>
<td>$\rho^{hf}$</td>
<td>Cross-country correlation of the long-run shock</td>
<td>1.0</td>
</tr>
<tr>
<td>$\rho^{hf}$</td>
<td>Cross-country correlation of the short-run shock to consumption</td>
<td>0.3</td>
</tr>
<tr>
<td>$\rho^{hf}$</td>
<td>Cross-country correlation of the short-run shock to dividends</td>
<td>-0.1</td>
</tr>
</tbody>
</table>
Model Calibration: Varying EIS

Dark-line corresponds to the baseline calibration.

Don’t need EIS > 1. High correlation between SDF’s is driven by high persistence and high correlation of long-run components (i.e. high \( \rho_x \) and \( \rho_{hf} \)).
Model Calibration: Varying CRRA

Horizontal lines indicate feasible range for exchange rate growth volatility
**Model Calibration**

<table>
<thead>
<tr>
<th>Key Moments of International Markets - Symmetric Calibration</th>
<th>US</th>
<th>UK</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho (m^h, m^f)$</td>
<td>Correlation of pricing kernels</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma (\Delta e)$</td>
<td>Volatility of FX growth</td>
<td>11.211</td>
<td>11.832</td>
</tr>
<tr>
<td>$E(\Delta e)$</td>
<td>Average FX growth</td>
<td>1.330</td>
<td>0.000</td>
</tr>
<tr>
<td>$\rho (\Delta e_{t+1}, \Delta e_t)$</td>
<td>Autocorrelation of FX growth</td>
<td>0.002</td>
<td>0.003</td>
</tr>
<tr>
<td>$\rho (\Delta e_{t+1}, v_{d,t} - v_{d,t}^*)$</td>
<td>Correlation of FX growth and price-dividend ratios differences</td>
<td>0.070</td>
<td>-0.003</td>
</tr>
<tr>
<td>$\rho (\Delta e, \Delta c - \Delta c^*)$</td>
<td>Correlation of FX growth and consumption growth differentials</td>
<td>0.15</td>
<td>0.80</td>
</tr>
<tr>
<td>$\rho (v_d, v_{d,t-1})$</td>
<td>Autocorrelation of price-dividend ratio</td>
<td>0.624</td>
<td>0.716</td>
</tr>
<tr>
<td>$\sigma (v_d)$</td>
<td>Volatility of price-dividend ratio</td>
<td>31.207</td>
<td>32.210</td>
</tr>
<tr>
<td>$E(v_d)$</td>
<td>Average price-dividend ratio</td>
<td>3.331</td>
<td>2.890</td>
</tr>
<tr>
<td>$E(r_d - r_f)$</td>
<td>Average excess return</td>
<td>5.504</td>
<td>6.501</td>
</tr>
<tr>
<td>$\sigma (r_d - r_f)$</td>
<td>Volatility of excess return</td>
<td>17.130</td>
<td>22.830</td>
</tr>
<tr>
<td>$E(r_f)$</td>
<td>Average risk free rate</td>
<td>1.470</td>
<td>1.620</td>
</tr>
<tr>
<td>$\sigma (r_f)$</td>
<td>Volatility of risk free rate</td>
<td>1.530</td>
<td>2.920</td>
</tr>
<tr>
<td>$\rho (r_d - r_f, r_d^* - r_f^*)$</td>
<td>Correlation of excess returns</td>
<td>0.670</td>
<td>0.603</td>
</tr>
<tr>
<td>$\rho (v_d, v_d^*)$</td>
<td>Correlation of price-dividend ratios</td>
<td>0.770</td>
<td>0.925</td>
</tr>
<tr>
<td>$\rho (\Delta d, \Delta d^*)$</td>
<td>Correlation of dividend growth</td>
<td>-0.03</td>
<td>-0.07</td>
</tr>
<tr>
<td>$\rho (r_f, r_f^*)$</td>
<td>Correlation of risk free rates</td>
<td>0.653</td>
<td>1.000</td>
</tr>
</tbody>
</table>
Conclusion: We can’t reject that US and UK predictive components are highly persistent and perfectly correlated.

Real Conclusion: The likelihood function is very uninformative.
Risks for the Long Run and the Real Exchange Rate

Estimating International Long-run Risks

Monte-Carlo Experiment

Suppose you have 2 time-series for consumption which are both \( T \) years in length.

**QUESTION:** What is the minimum sample size \( T \) needed to sharply identify the long-run components and reject the random walk model?

**ANSWER:** If \( \rho(x, x^*) = 1 \), then you need \( T = 80 \). However, if \( \rho(x, x^*) = .95 \), then you \( T = 200 \).

**CONCLUSION:** Consumption data alone are not enough to identify the low frequency dynamics of the long-run components of consumption growth.
Suppose you have 2 time-series for consumption which are both $T$ years in length.

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Predictive Regressions Approach

Bansal et al. (2006) document that predictive variables (e.g. lagged price-dividend, risk free rates, etc.) contain a direct measure of long-run risk at each date and state.

To see this, recall the price dividend ratio formula from the linearized model:

\[ v_{d,t} = \bar{v}_d + \frac{\lambda - \frac{1}{\psi}}{1 - \rho_x \kappa_d} x_t \quad \Leftrightarrow \quad x_t = k_0 + k_1 v_{d,t} \]

Thus, estimates of time-series \( \hat{x}_t^i \) and \( i \in \{US, UK\} \) can be obtained from the fitted values of

\[ \Delta c_t^i \text{ on } pd_{t-1}^i, r_{f,t-1}^i, \Delta c_{t-1}^i, cy_{t-1}^i, \text{ default}_{t-1}^i \]
Plot of $\hat{x}_t^{US}$ (red) and $\hat{x}_t^{UK}$ (black)

Notice the changing correlation over time!
Negative Relationship between $\sigma(\Delta e)$ and $\text{corr}(\hat{x}^{US}, \hat{x}^{UK})$
Euler Equations’ GMM

<table>
<thead>
<tr>
<th></th>
<th>Conditional Estimation</th>
<th>Joint Estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P/D</td>
<td>P/D and R_f</td>
</tr>
<tr>
<td>ψ</td>
<td>5.526</td>
<td>3.881</td>
</tr>
<tr>
<td></td>
<td>(-38.864)</td>
<td>(-31.361)</td>
</tr>
<tr>
<td>γ</td>
<td>13.537</td>
<td>13.915</td>
</tr>
<tr>
<td></td>
<td>(-1.053)</td>
<td>(-2.182)</td>
</tr>
<tr>
<td>ρx</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Wald-stat</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>p-value</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>γ − 1/ψ</td>
<td>5.872</td>
<td>16.644</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.001)</td>
</tr>
</tbody>
</table>

First three columns are counterparts to exercises in Bansal et al. (2006). However, γ estimates are half as large as Bansal et al. (2006), and ψ > 1! Exchange-rate info matters.
Want to directly compare the time series of returns and exchange rates implied by the model i.e. want to estimate

\[
E \left[ (\tilde{r}_m^{i,t+1} - \tilde{r}_f^{i,t+1}) (r_m^{i,t+1} - r_f^{i,t+1}) \right] = 0 \quad i \in \{\text{US, UK}\}
\]

\[
E \left[ (\tilde{r}_f^{i,t+1} - \tilde{r}_f^{i,t+1}) \right] = 0 \quad i \in \{\text{US, UK}\}
\]

But to estimate these equations we need to construct the time series for \( \Delta d_t \) to calculate \( \tilde{r}_m^{i,t+1} \)
Verifying Common Long-run Component in Consumption and Dividends

Thus to verify the imposed structure on consumption and dividends compare the joint estimation of

\[
\Delta c_t^i = \beta_0^i \Delta c_{t-1}^i + \beta_1^i p d_{t-1}^i + \beta_2^i \Delta c_{t-1}^i + \beta_3^i \text{default}_{t-1}^i + \beta_4^i r_{f,t-1}^i + \varepsilon_{c,t}^i
\]
\[
\Delta d_t^i = \lambda^i (\beta_0^i \Delta c_{t-1}^i + \beta_1^i p d_{t-1}^i + \beta_2^i \Delta c_{t-1}^i + \beta_3^i \text{default}_{t-1}^i + \beta_4^i r_{f,t-1}^i) + \varepsilon_{d,t}^i
\]

to the joint estimation of

\[
\Delta c_t^i = \phi_0^i \Delta c_{t-1}^i + \phi_1^i p d_{t-1}^i + \phi_2^i \Delta c_{t-1}^i + \phi_3^i \text{default}_{t-1}^i + \phi_4^i r_{f,t-1}^i + \varepsilon_{c,t}^i
\]
\[
\Delta d_t^i = \phi_0^i \Delta c_{t-1}^i + \phi_1^i p d_{t-1}^i + \phi_2^i \Delta c_{t-1}^i + \phi_3^i \text{default}_{t-1}^i + \phi_4^i r_{f,t-1}^i \varepsilon_{d,t}^i
\]

Null test that \( \phi_j^i = \lambda^i \beta_j^i \) cannot be rejected at the 5% level \( \Rightarrow \) common long-run component in consumption and dividends
Verifying Common Long-run Component in Consumption and Dividends

Thus to verify the imposed structure on consumption and dividends compare the joint estimation of

$$
\Delta c_t^i = \beta_0^i \Delta c_{t-1}^i + \beta_1^i p_{d_{t-1}}^i + \beta_2^i \Delta c_{y_t}^i + \beta_3^i \text{default}_t^i + \beta_4^i r_{f,t}^i + \varepsilon_{c,t}^i
$$
$$
\Delta d_t^i = \lambda^i (\beta_0^i \Delta c_{t-1}^i + \beta_1^i p_{d_{t-1}}^i + \beta_2^i \Delta c_{y_t}^i + \beta_3^i \text{default}_t^i + \beta_4^i r_{f,t}^i) + \varepsilon_{d,t}^i
$$

to the joint estimation of

$$
\Delta c_t^i = \phi_0^i \Delta c_{t-1}^i + \phi_1^i p_{d_{t-1}}^i + \phi_2^i \Delta c_{y_t}^i + \phi_3^i \text{default}_t^i + \phi_4^i r_{f,t}^i + \varepsilon_{c,t}^i
$$
$$
\Delta d_t^i = \phi_0^i \Delta c_{t-1}^i + \phi_1^i p_{d_{t-1}}^i + \phi_2^i \Delta c_{y_t}^i + \phi_3^i \text{default}_t^i + \phi_4^i r_{f,t}^i \varepsilon_{d,t}^i
$$

Null test that $\phi_j^i = \lambda^i \beta_j^i$ cannot be rejected at the 5% level $\Rightarrow$ common long-run component in consumption and dividends
### GMM Estimation with Dividends

<table>
<thead>
<tr>
<th></th>
<th>Conditional Estimation</th>
<th>Joint Estimation</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$P/D$</td>
<td>$P/D$ and $R_f$</td>
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<tr>
<td>$\psi$</td>
<td>2.139</td>
<td>1.538</td>
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<td></td>
<td>(0.331)</td>
<td>(0.043)</td>
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<tr>
<td>$\gamma$</td>
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<tr>
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<td>$\gamma - 1/\psi$</td>
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<td>(0.438)</td>
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<tr>
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<tr>
<td></td>
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<td>(0.048)</td>
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