There is a risk-return trade-off after all

Eric Ghysels, Pedro Santa-Clara, Rossen Valkanov (JFE 2005)
- Presented by Joon Ho Hur -
Objectives

- Intertemporal relation between the conditional mean and the conditional variance of the aggregate stock market returns

- Introduction of MIDAS (Mixed Data Sampling) regression

- Comparison the MIDAS results with tests of ICAPM based on alternative conditional variance models (GARCH, Rolling Window)
Risk-Return Trade-off

- Conditional expected excess return on the stock market varies positively with the market’s conditional variance

\[ E_t[R_{t+1}] = \mu + \gamma \text{Var}_t[R_{t+1}], \quad \gamma > 0 \]

- Merton (1973)

- Previous estimates of the relation between risk and return often have been insignificant and sometimes even negative
  
  - Baillie and DeGennaro (1990), French et al. (1987), Campbell and Hentschel (1992), Campbell (1987), Nelson (1991), Glosten et al. (1993), and many others
Midas (Mixed Data Sampling)

- Construct regressions combining data with different sampling frequencies

\[
Y_t = \alpha_0 + \alpha_1 \sum_{i=0}^{i_{\text{max}}} B(i, \theta) x^{(m)}_{t-i/m} + \varepsilon_t
\]

\[
= \alpha_0 + \alpha_1 B(L^{1/m}, \theta) x^{(m)}_t + \varepsilon_t
\]

Where \( B(L^{1/m}, \theta) = B(0, \theta) + B(1, \theta)L^{1/m} + \ldots + B(i_{\text{max}}, \theta)L^{i_{\text{max}}/m} \)

- \( Y_t \) is sampled at some fixed frequency (annual, quartely, monthly or daily), \( X^{(m)}_t \) is sampled \( m \) times faster than \( Y_t \)

- \( B(i, \theta) \) is a polynomial weighting function depending on both the elapsed time \( i \) and the parameter vector \( \theta \)
Parameterization of B(i, \( \theta \))

- **Exponential Almon lag polynomial**

\[
B(i, \theta) = \frac{e^{\theta_1 i + \theta_2 i^2 + \ldots + \theta_p i^p}}{\sum_{i=1}^{i_{\text{max}}} e^{\theta_1 i + \theta_2 i^2 + \ldots + \theta_p i^p}}
\]

which is related to “Alman lags” (Alman, 1965)

- In general, the weights can decline more or less rapidly or take on the desired form according to the value of the parameter \( \theta_i \), for \( i=1, 2, \ldots, p \)
  - If \( \theta_1 = \theta_2 = \ldots = \theta_i = 0 \), then the weight of each lag is the same

- There are other parameterization ways such as “Beta lag polynomial” (Ghysels et al., 2006)
MIDAS tests of the risk-return trade-off

- Estimate $\theta_i$, $\mu$, and $\gamma$ via QMLE using MIDAS regression

$$ R_{t+1} \sim N(\mu + \gamma V_t^{MIDASS}, V_t^{MIDASS}) $$

where

$$ V_t^{MIDASS} = \text{Var}_t[R_{t+1}] = 22 \sum_{d=0}^{\infty} w(d, \kappa_1, \kappa_2) r_{t-d}^2 $$

and

$$ w(d, \kappa_1, \kappa_2) = \frac{\exp\{ \kappa_1 d + \kappa_2 d^2 \}}{\sum_{i=0}^{\infty} \exp\{ \kappa_1 i + \kappa_2 i^2 \}} $$

- $R_{t+1}$: Monthly returns from date $t$ to date $t+1$
- $r_{t-d}$: Daily return $d$ days before date $t$
Data

- Excess returns on the stock market from January 1928 to December 2000
  - Two subsamples of approximately equal length, 1928-1963 and 1964-2000

- Center for Research in Security Prices (CRSP) value-weighted portfolio as a proxy for the stock market

- The yield of the three-month Treasury bill as the risk-free interest rate
MIDAS regression results

<table>
<thead>
<tr>
<th>Sample</th>
<th>$\mu$ ($\times 10^3$)</th>
<th>$\gamma$ ($\times 10^3$)</th>
<th>$\kappa_1$ ($\times 10^3$)</th>
<th>$\kappa_2$ ($\times 10^5$)</th>
<th>$R^2_R$</th>
<th>$R^2_{\sigma^2}$</th>
<th>LLF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1928:01–2000:12</td>
<td>6.430</td>
<td>2.606</td>
<td>-5.141</td>
<td>-10.580</td>
<td>0.019</td>
<td>0.407</td>
<td>1421.989</td>
</tr>
<tr>
<td>1928:01–1963:12</td>
<td>11.676</td>
<td>1.547</td>
<td>-0.909</td>
<td>-10.807</td>
<td>0.011</td>
<td>0.444</td>
<td>681.237</td>
</tr>
<tr>
<td>1964:01–2000:12</td>
<td>3.793</td>
<td>3.748</td>
<td>-6.336</td>
<td>-18.586</td>
<td>0.050</td>
<td>0.082</td>
<td>807.193</td>
</tr>
<tr>
<td></td>
<td>[5.673]</td>
<td>[8.612]</td>
<td>[-7.862]</td>
<td>[-7.710]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- $\gamma$ is 2.606 in the full sample, with a highly significant t-statistics
- $\mu$ is significant → could capture other predictable variables
- $\kappa_1, \kappa_2$ are statistically significant
MIDAS regression results

- Daily, weekly, monthly, bimonthly, and quarterly horizons on the MIDAS

<table>
<thead>
<tr>
<th>Horizon</th>
<th>μ (×10^3)</th>
<th>γ</th>
<th>$R^2_R$</th>
<th>$R^2_{e2}$</th>
<th>LLF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample: 1928:01–2000:12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Daily</td>
<td>0.275</td>
<td>2.684</td>
<td>0.004</td>
<td>0.059</td>
<td>57098.422</td>
</tr>
<tr>
<td></td>
<td>[13.422]</td>
<td>[1.154]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weekly</td>
<td>1.320</td>
<td>2.880</td>
<td>0.009</td>
<td>0.119</td>
<td>8441.573</td>
</tr>
<tr>
<td></td>
<td>[13.156]</td>
<td>[3.127]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Monthly</td>
<td>6.430</td>
<td>2.606</td>
<td>0.019</td>
<td>0.407</td>
<td>1421.989</td>
</tr>
<tr>
<td></td>
<td>[11.709]</td>
<td>[6.710]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BiMonthly</td>
<td>14.218</td>
<td>1.964</td>
<td>0.018</td>
<td>0.309</td>
<td>583.383</td>
</tr>
<tr>
<td></td>
<td>[12.007]</td>
<td>[4.158]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quarterly</td>
<td>24.992</td>
<td>2.199</td>
<td>0.016</td>
<td>0.329</td>
<td>377.901</td>
</tr>
<tr>
<td></td>
<td>[12.029]</td>
<td>[4.544]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
MIDAS weights

- The estimated volatility process is persistent
- The one month of daily returns is not sufficient
Rolling window tests: use within month daily squared returns to forecast next month’s variance

- French et al. (1987)

\[ V_t^{RW} = 22 \sum_{d=0}^{D} \frac{1}{D} r_{t-d}^2 \]

where \( D \) is the number of days (\( D=22 \), one month)

- The use of daily data increases the precision of the variance estimator
- The realized variance on a given month ought to be a good forecast of next month’s variance

Realized variance: \[ \sigma_{t+1}^2 = \sum_{d=0}^{22} r_{t+1-d}^2 \]
Rolling window tests results

<table>
<thead>
<tr>
<th>Horizon (months)</th>
<th>$\mu$ ($\times 10^3$)</th>
<th>$\gamma$</th>
<th>$R^2_R$</th>
<th>$R^2_{\sigma^2}$</th>
<th>LLF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.523 [4.155]</td>
<td>0.546 [0.441]</td>
<td>0.007</td>
<td>0.370</td>
<td>1292.454</td>
</tr>
<tr>
<td>2</td>
<td>7.958 [4.071]</td>
<td>1.494 [1.532]</td>
<td>0.009</td>
<td>0.379</td>
<td>1325.528</td>
</tr>
<tr>
<td>3</td>
<td>6.823 [3.240]</td>
<td>2.171 [1.945]</td>
<td>0.011</td>
<td>0.382</td>
<td>1308.923</td>
</tr>
<tr>
<td>4</td>
<td>6.830 [3.344]</td>
<td>2.149 [2.212]</td>
<td>0.012</td>
<td>0.384</td>
<td>1346.685</td>
</tr>
<tr>
<td>5</td>
<td>7.972 [3.506]</td>
<td>1.458 [1.325]</td>
<td>0.011</td>
<td>0.383</td>
<td>1335.114</td>
</tr>
</tbody>
</table>

- One month size D $\rightarrow$ insignificant $\gamma$ (0.546, 0.441)
- Four month size D $\rightarrow$ significant $\gamma$ (2.149, 2.212)
  - A constant weight of $1/(22\times4) = 0.0111$
Comparison with others (GARCH-M)

- Estimate GARCH-M via QMLE

\[ R_{t+1} \sim N(\mu + \gamma V^GARCH_t, V^GARCH_t) \]

where

\[ V^GARCH_t = \omega + \alpha \varepsilon^2_t + \beta V^GARCH_{t-1} \]

\[ = \frac{\omega}{1 - \beta} + \alpha \sum_{i=0}^{\infty} \beta^i \varepsilon^2_{t-i} \]

\[ \Rightarrow \text{Approximately a weighted average of past monthly squared returns rather than daily squared returns} \]

- ABS-GARCH

\[ (V^ABSGARCH_t)^{1/2} = \omega + \alpha |\varepsilon_t| + \beta (V^ABSGARCH_{t-1})^{1/2} \]
GARCH-M tests results

- GARCH-M → insignificant $\gamma$ (1.060, 1.292)
- ABSGARCH → insignificant $\gamma$ (1.480, 1.415)
- MIDAS has more power than GARCH because it has a more flexible function form for the weights on past squared returns
Comparison of three tests  
(In-sample forecasted variances vs realized variance)
Comparison of three models
(Scatterplots of forecasted variances vs realized variance)
Comparison of GARCH and MIDAS

- Comparing MIDAS with GARCH estimated with mixed-frequency data (To analyze the gains from flexible form for the weights)
  - To estimate the mixed-frequency GARCH, assume that daily variance follows a GARCH(1,1)
  - Summing the forecasted variance over the following 22 days yields a forecast of next month’s variances
  - The difference between the MIDAS and the mixed-frequency GARCH estimator is the shape of the weight function

- Comparing monthly GARCH with MIDAS estimated from monthly data (To analyze the gains from mixing frequencies)

\[
V_{t}^{MIDASS} = \sum_{m=0}^{\infty} w_{m} R_{t-m}^{2}
\]
Comparison of GARCH and MIDAS

- Flexible form for the weights is important
- Using mixed-frequency data increases the power of the risk-return trade-off test
Comparison of GARCH and MIDAS

- The decay of the daily GARCH weights is much faster than MIDAS model.
  - The persistence of the estimated GARCH variance process is lower than that of MIDAS.
Asymmetries

Using asymmetric MIDAS estimator, examine whether the risk-return trade-off is robust to the inclusion of asymmetric effects in the conditional variance

\[ V_t^{ASYMIDASS} = 22 \left[ \phi \sum_{d=0}^{\infty} w_d (\kappa_1^-, \kappa_2^-) l_{t-d}^- r_{t-d}^2 + (2 - \phi) \sum_{d=0}^{\infty} w_d (\kappa_1^+, \kappa_2^+) l_{t-d}^+ r_{t-d}^2 \right] \]

where \( l_{t-d}^+ \) denotes the indicator function for \( \{ r_{t-i} \geq 0 \} \)
\( l_{t-d}^- \) denotes the indicator function for \( \{ r_{t-i} < 0 \} \)
and \( \phi \) is in the interval \((0, 2)\)
Asymmetric MIDAS test results

- The $\gamma$ is 2.482 and highly significant in the entire sample.
- Hence, asymmetries in the conditional variance are consistent with a positive coefficient $\gamma$ in the ICAPM relation.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$\mu$ ($\times 10^3$)</th>
<th>$\gamma$</th>
<th>$\kappa_1^-$ ($\times 10^2$)</th>
<th>$\kappa_2^-$ ($\times 10^3$)</th>
<th>$\kappa_1^+$ ($\times 10^2$)</th>
<th>$\kappa_2^+$ ($\times 10^5$)</th>
<th>$\phi$</th>
<th>$R^2_R$</th>
<th>$R^2_{\sigma^2}$</th>
<th>LLF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1928:01–2000:12</td>
<td>7.912</td>
<td>2.482</td>
<td>18.838</td>
<td>-12.694</td>
<td>0.188</td>
<td>-2.230</td>
<td>0.572</td>
<td>0.041</td>
<td>0.429</td>
<td>1482.667</td>
</tr>
<tr>
<td>1928:01–1963:12</td>
<td>10.114</td>
<td>2.168</td>
<td>13.866</td>
<td>-10.924</td>
<td>0.176</td>
<td>-3.241</td>
<td>0.537</td>
<td>0.023</td>
<td>0.461</td>
<td>698.835</td>
</tr>
<tr>
<td>1964:01–2000:12</td>
<td>5.521</td>
<td>2.603</td>
<td>27.616</td>
<td>-15.767</td>
<td>-0.392</td>
<td>-0.050</td>
<td>0.697</td>
<td>0.092</td>
<td>0.088</td>
<td>819.237</td>
</tr>
</tbody>
</table>
Negative shocks have a strong impact on the conditional variance, but that impact is transitory.
Comparison with asymmetric GARCH-M

- Following Hentschel (1995), a general class of GARCH models can be written as

\[
\frac{V_t^\lambda - 1}{\lambda} = \omega + \alpha V_{t-1}^\lambda (|u_t + b| + c(u_t + b))^v + \beta \frac{V_{t-1}^\lambda - 1}{\lambda}
\]

- GARCH(\(\lambda=1, v=2, b=c=0\))
- ASYGARCH (\(\lambda=1, v=2, b=0, c: \text{unrestricted}\))
- EGARCH (\(\lambda \to 0, v=1, b=0, c: \text{unrestricted}\))
  - Negative \(c\) => negative returns have a stronger impact on the conditional variance
- QGARCH (\(\lambda=1, v=2, c=0, b: \text{unrestricted}\))
  - Negative \(b\) => variance reacts more to negative returns
Comparison with asymmetric GARCH-M

Asymmetries in the ASYGARCH and EGARCH produce a negative, statistically insignificant, estimate of the risk-return trade-off parameter \( \gamma \)

- Results are consistent with Glosten et al. (1993) puzzling

The estimates of the persistence parameter \( \beta \) in the asymmetric GARCH models are much lower than in the symmetric GARCH models
Contributions

- In support of the ICAPM, they find a positive and significant relation between risk and return
  - This relation does not change when the conditional variance is allowed to react asymmetrically to positive and negative returns
- Midas estimator is a better forecaster of the stock market variance than rolling window or GARCH estimators
- Positive shocks have a bigger effect overall on the conditional mean of returns and are much more persistent
  - Negative shocks have a large initial, but temporary, effect on the variance of returns
Further Enquiries

- Out-of-sample forecasted variances against realized variance
- Performance over other markets besides U.S. such as U.K., Germany, Japan, and emerging markets.