International Asset Pricing and Risk Sharing with Recursive Preferences

Riccardo Colacito

Prepared for Tom Sargent’s PhD class (Part 1)
Roadmap

Today

- International asset pricing (exchange rates, co-movements of int’l stock market returns, ...)
- Agents have recursive preferences
- No equilibrium trade
Roadmap

Today

▶ International asset pricing (exchange rates, co-movements of int’l stock market returns, ...)
▶ Agents have recursive preferences
▶ No equilibrium trade

Tomorrow

▶ Agents consume bundles of domestic and foreign goods
▶ Trade arises as an equilibrium outcome
▶ Efficient risk-sharing with recursive preferences
Set the stage

- Study the link between international stochastic discount factors and the depreciation of the US dollar:

$$\frac{M_{t+1}^{uk}}{M_{t+1}^{us}} = \frac{e_{t+1}}{e_t}$$

where

$$E_t \left[ M_{t+1}^i R_{t+1}^i \right] = 1, \quad \forall i \in \{us, uk\}$$
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- How to evaluate \( M_{t+1}^i \)?

  1. From prices: \( \sigma \left( M_{t+1}^i \right) \geq \frac{E[R_{t+1}^i - R_{t+1}^f]}{\sigma(R_{t+1}^i - R_{t+1}^f)} \)
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   ▶ e.g. if \( E[R^i - R^f] \approx 7\%, \sigma (R^i - R^f) \approx 17\% \) then \( \sigma (M^i) \approx 40\% \)
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2. From quantities: \( M_{t+1}^i = \frac{\partial U^i / \partial C_{t+1}^i}{\partial U^i / \partial C_t^i} \)
   - e.g. with CRRA preferences \( M_{t+1}^i = \delta \left( \frac{C_{t+1}^i}{C_t^i} \right)^{-\gamma} \)
The International equity premium puzzle

By no arbitrage

\[ E_t \left[ M_{t+1}^f R_{t+1}^f \right] = 1 = E_t \left[ M_{t+1}^h R_{t+1}^h \right] \]
The International equity premium puzzle

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The International equity premium puzzle

- By no arbitrage

$$\log M_{t+1}^{f} - \log M_{t+1}^{h} = \log \frac{e_{t+1}}{e_{t}}$$
The International equity premium puzzle

By no arbitrage

$$\log M_{t+1}^f - \log M_{t+1}^h = \log \frac{e_{t+1}}{e_t}$$

Correlation of stochastic discount factors

$$\sigma_{m^f}^2 + \sigma_{m^h}^2 - 2\rho_{m^f,m^h}\sigma_{m^f}\sigma_{m^h} = \sigma_\pi^2$$
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\[
\begin{array}{c}
20\% \\
20\% \\
0.96 \\
20\%
\end{array}
\]

\[
\begin{array}{c}
1.5\%
\end{array}
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- Assuming identical CRRA preferences:

\[ \log M_{t+1}^i = -\gamma \Delta c_{t+1}^i \]

\[ \gamma^2 \sigma_{\Delta c^f}^2 + \gamma^2 \sigma_{\Delta c^h}^2 - 2 \rho_{\Delta c^f,\Delta c^h} \gamma \sigma_{\Delta c^f} \sigma_{\Delta c^h} = \sigma_{\pi}^2 \]
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By no arbitrage

\[ \log M^f_{t+1} - \log M^h_{t+1} = \log \frac{e_{t+1}}{e_t} \]

Correlation of stochastic discount factors

\[ \sigma^2_{m^f} + \sigma^2_{m^h} - 2 \rho_{m^f,m^h} \sigma_{m^f} \sigma_{m^h} = \sigma^2_\pi \]

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► By no arbitrage

\[ \log M_{t+1}^f - \log M_{t+1}^h = \log \frac{e_{t+1}}{e_t} \]

► Correlation of stochastic discount factors

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\begin{align*}
\sigma_{m,f}^2 + \sigma_{m,h}^2 - 2 \rho_{m,f,m,h} \sigma_{m,f} \sigma_{m,h} &= \sigma_{\pi}^2 \\
20\% &\quad 20\% &\quad 0.96 &\quad 20\% &\quad 1.5\%
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20\% &\quad 20\% &\quad 0.3 &\quad 20\% &\quad 28\%
\end{align*}
\]
The puzzle in a cross section of countries

- HJ bound and depreciation rate: high correlation of SDF and low variance of depreciation rate.

![Graph showing correlations and variances of different countries](image-url)
The puzzle in a cross section of countries

- Consumption data and CRRA preferences: low correlation of SDF and high variance of depreciation rate.

![Diagram](chart.png)
The puzzle in a cross section of countries

This paper: high correlation of SDF, low correlation of $\Delta c$ and low variance of depreciation rate.
The plan

1. Asset pricing within each country
   - The long-run risk model by Bansal and Yaron

2. Asset pricing across countries
   - The international long-run risks model by Colacito and Croce
Comparison of Utilities

▶ Expected Utility

\[ U_t = (1 - \delta) \log C_t + \delta E_t U_{t+1} \]

▶ Risk Sensitive preferences

\[ U_t = (1 - \delta) \log C_t + \delta \theta \log E_t \exp \left\{ \frac{U_{t+1}}{\theta} \right\} \]
Comparison of Utilities

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\[ \approx (1 - \delta) \log C_t + \delta E_t U_{t+1} + \frac{\delta}{2\theta} V_t U_{t+1} \]
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► What about SDF’s?
Comparison of SDF’s

- SDF’s can be calculated as

\[ m_{t+1} = \log \frac{\partial U_t / \partial C_{t+1}}{\partial U_t / \partial C_t} \]
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2. Risk Sensitive Preferences

\[ m_{t+1} = \delta - \Delta c_{t+1} + \frac{U_{t+1}}{\theta} - \log E_t \exp \left\{ \frac{U_{t+1}}{\theta} \right\} \]
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- Hansen-Jagannathan: SDF’s should be highly volatile
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  - Consumption growth is not very volatile in the data.
Comparison of SDF’s

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Hansen-Jagannathan: SDF’s should be highly volatile

- Consumption growth is not very volatile in the data.
- What about utility?
Two states examples

Log Consumption

\[ c_{t+1} = 2 \]

\[ c_{t+1} = 1 \]
Two states examples

\[ c_{t+1} = 2 + 0.1 \]

\[ c_{t+1} = 1 \]

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Two states examples

\begin{align*}
ct_{+1} &= 2 + 0.1 \\
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Utility

\[ E_0 [U_1] = 1.5, V_0 [U_1] = (0.5)^2 \]
\[ E_0 [U_1] = 1.5, V_0 [U_1] = (0.6)^2 \]
Two states examples

\[ c_{t+1} = 2 + \left( \sum_{j=0}^{t} \rho^j \right) x \]

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Exercise

- Let log-consumption growth evolve according to

\[ \Delta c_{t+1} = c_0 + \left( \sum_{j=0}^{t} \rho^j \right) x_0 \]

where \( c_0 \sim N(0, \sigma_c^2) \) and \( x_0 \sim N(0, \sigma_x^2) \)

- Let preferences be described by

\[ U_t = (1 - \delta) \log C_t + \delta \theta \log E_t \exp \left\{ \frac{U_{t+1}}{\theta} \right\} \]

- Show that the premium of a claim to consumption in excess of the risk-free rate is increasing in

1. \( \sigma_x^2 \)
2. \( \rho \)
Exercise: hints

- By definition the gross return of a claim to consumption is

\[
R_{t+1}^c = \frac{P_{t+1}^c + C_{t+1}}{P_t^c} = \frac{1 + v_{t+1}^c}{v_t^c} \exp \{\Delta c_{t+1}\}
\]

where \(v_t = P_t/C_t\) is the price-consumption ratio.

- Define the log-stochastic discount factor as

\[
m_{t+1} = \log \frac{\partial U_t/\partial C_t}{\partial U_t/\partial C_{t+1}} = \delta - \Delta c_{t+1} + \frac{U_{t+1}}{\theta} - \log E_t \exp \left\{ \frac{U_{t+1}}{\theta} \right\}
\]

- Solve for \(v_t\) using \(E_t \left[ \exp\{m_{t+1}\} R_{t+1}^c \right] = 1\). (Hint: \(v_t\) is constant).

- Solve for the risk-free rate, \(R_{t+1}^f\), using \(E_t \left[ \exp\{m_{t+1}\} R_{t+1}^f \right] = 1\). (Hint: \(R_{t+1}^f\) is known at date \(t\)).

- Compute the premium.
Summary

- The equity premium puzzle: consumption is too smooth to make the stochastic discount factor “volatile enough” in the HJ sense.

- The Bansal and Yaron recipe:
  1. small, but long-lasting shocks to consumption ($\rho$ is large)
  2. that investors care about ($1/\theta \neq 0$)
     can increase the volatility of the stochastic discount factor.

- How does this extend to international asset pricing?
International Asset Pricing

Facts:

1. Equity Sharpe ratios are large in the cross-section of major industrialized countries
   ▶ International SDF’s must be very volatile!

2. Correlation of equity markets’ returns is large ($\approx 0.6$)
   ▶ Puzzling because “fundamentals” have low correlations (i.e. cash flows are poorly correlated across countries)

3. The volatility of real exchange rates’ movements is in the $10 - 15\%$.

The fix for 2+3:
▶ the int’l correlation of SDF’s must be high!
Rules of the game and outline

1. We want:
   ▶ to reconcile the correlation of SDF from
     (a) prices
     (b) quantities
Rules of the game and outline

1. We want:
   - to reconcile the correlation of SDF from
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   - low volatility of depreciation rate
   - low volatility of consumption growth
   - low correlation of consumption growths
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2. Can we match other key features of financial markets?
Rules of the game and outline

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   - low volatility of depreciation rate
   - low volatility of consumption growth
   - low correlation of consumption growths

2. Can we match other key features of financial markets?

3. Can we estimate this model?
Setup of the economy

- Endowment economy.
- Two country specific goods.
- Complete home bias in consumption.
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- Endowment economy.
- Two country specific goods.
- Complete home bias in consumption.
- Epstein, Zin and Weil preferences:

\[
U_t^i = \left\{ (1 - \delta)(C_t^i)^{\frac{1-\gamma}{\theta}} + \delta \left[ E_t(U_{t+1}^{i})^{1-\gamma} \right]^{\theta} \right\}^{\frac{1}{1-\gamma}}, \forall i \in \{h, f\}
\]

where

\[
\theta = \frac{1 - \gamma}{1 - 1/\psi}
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- What do stochastic discount factors look like?
Stochastic discount factors

- Assume $\psi = 1$:

$$U_t^i = (1 - \delta) \log C_t^i + \frac{\delta}{1 - \gamma} \log E_t \left[ \exp (1 - \gamma) U_{t+1}^i \right]$$

- The stochastic discount factors are

$$\log M_{t+1}^i = \log \frac{\partial U_t^i / \partial C_{t+1}^i}{\partial U_t^i / \partial C_t^i}$$

$$= \log \delta + \log \frac{C_t^i}{C_{t+1}^i} + \log \frac{\exp \{ (1 - \gamma) U_{t+1}^i \}}{E_t \left[ \exp \{ (1 - \gamma) U_{t+1}^i \} \right]}$$

- Brandt, Cochrane and Santa-Clara use:

$$\log M_{t+1}^i = \log \delta + \log \frac{C_t^i}{C_{t+1}^i}$$
Remainder of the economy

- **Home country**

  \[ \Delta c^h_t = \mu_c + x^h_{t-1} + \sigma \varepsilon^h_{c,t} \]

  \[ x^h_t = \rho x^h_{t-1} + \sigma \varphi_e \varepsilon^h_{x,t} \]

- **Foreign country**

  \[ \Delta c^f_t = \mu_c + x^f_{t-1} + \sigma \varepsilon^f_{c,t} \]

  \[ x^f_t = \rho x^f_{t-1} + \sigma \varphi_e \varepsilon^f_{x,t} \]

- **Shocks are i.i.d. within each country**

- **Shocks are correlated across countries**
  
  \[ \rho_c = \text{corr}(\varepsilon^h_{c,t}, \varepsilon^f_{c,t}) \]

  \[ \rho_x = \text{corr}(\varepsilon^h_{x,t}, \varepsilon^f_{x,t}) \]
 Calibration

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<tr>
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$$m_{t+1}^i = \theta \log \delta - \frac{\theta}{\psi} \Delta c_t^i + (\theta - 1) \log R_{c,t+1}^i$$

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$$x_t^i = \rho x_{t-1}^i + \sigma \varphi_e \varepsilon_{x,t}^i$$

Preferences:

- Low risk aversion ($\gamma$)
- IES from Bansal, Gallant and Tauchen (2004)
- Monthly model: high discounting
## Calibration

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x_t^i = \rho x_{t-1}^i + \sigma \phi e \varepsilon_{x,t}^i
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**Consumption process:**

- Average consumption growth $\approx 2\%$
- Standard deviation of consumption growth $\approx 2.5\%$
- Variance explained by long run risk $\approx 7 - 8\%$
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Cross correlations of shocks:

- Correlation of consumption growths $\approx 0.3$
Three ingredients

- We can solve the puzzle by appropriately combining three ingredients:
Three ingredients

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  1. Use Epstein and Zin preferences:
     - BCSC (2005): \( m^i_{t+1} = E \left[ g(\Delta c^i_{t+1}) | I_{t+1} \right] = -\gamma \Delta c^i_{t+1} \)
     - This paper: \( m^i_{t+1} = E \left[ \tilde{g}(\Delta c^i_{t+1}, \Delta c^i_{t+2}, \Delta c^i_{t+3}, \ldots) | I_{t+1} \right] \)
Three ingredients

▶ We can solve the puzzle by appropriately combining three ingredients:

1. Use Epstein and Zin preferences:
   - BCSC (2005): \( m^i_{t+1} = E \left[ g(\Delta c^i_{t+1})|I_{t+1} \right] = -\gamma \Delta c^i_{t+1} \)
   - This paper: \( m^i_{t+1} = E \left[ \tilde{g}(\Delta c^i_{t+1}, \Delta c^i_{t+2}, \Delta c^i_{t+3}, ...)|I_{t+1} \right] \)

Alter the conditional distribution of \((\Delta c^h, \Delta c^f):\)

\[
\Delta c^i_{t+1} = \mu_c + x^i_t + \sigma \varepsilon^i_{t,t+1}
\]
\[
x^i_{t+1} = \rho^i x^i_t + \sigma \varphi \varepsilon^i_{x,t+1}
\]

by assuming

2. High persistence \(\rho^i\)
3. High cross country correlation \(corr \left( \varepsilon^h_{x,t+1}, \varepsilon^f_{x,t+1} \right)\)
Stochastic discount factors

\[ m_t^i = \theta \log \delta - \frac{1}{\psi} x_t^i - \gamma \sigma \varepsilon_{c,t+1}^i + \frac{\delta(1-\gamma\psi)}{\psi(1-\rho\delta)} \sigma \varphi \varepsilon_{x,t+1}^i \]
Stochastic discount factors

\[ m_{t+1}^i = \theta \log \delta - \frac{1}{\psi} x_t^i - \gamma \sigma e_{c,t+1}^i + \frac{\delta (1-\gamma \psi)}{\psi (1-\rho \delta)} \sigma \varphi e_{x,t+1}^i \]
Stochastic discount factors

\[ m_{t+1}^i = \theta \log \delta - \frac{1}{\psi} x_t^i - \gamma \sigma \varepsilon_{c,t+1}^i + \frac{\delta (1-\gamma \psi)}{\psi (1-\rho \delta)} \sigma \varphi e_{x,t+1}^i \]

Correlation of stochastic discount factors

Intertemporal Elasticity of substitution (\(\psi\))
Stochastic discount factors

\[ m^i_{t+1} = \theta \log \delta - \frac{1}{\psi} x^i_t - \gamma \sigma \varepsilon^i_{c,t+1} + \frac{\delta(1-\gamma \psi)}{\psi(1-\rho \delta)} \sigma \varphi \xi^i_{x,t+1} \]
Stochastic discount factors

\[ m_t^i = \theta \log \delta - \frac{1}{\psi} x_t^i - \gamma \sigma \varepsilon_{c,t+1} + \frac{\delta (1-\gamma \psi)}{\psi (1-\rho \delta)} \sigma \varphi \varepsilon_{x,t+1} \]
Stochastic discount factors

\[ m^i_{t+1} = \theta \log \delta - \frac{1}{\psi} x^i_t - \gamma \sigma \epsilon^i_{c,t+1} + \frac{\delta (1-\gamma \psi)}{\psi (1-\rho \delta)} \sigma \varphi \epsilon^i_{x,t+1} \]
Exchange rate depreciation

\[
Var \left( \frac{e_{t+1}}{e_t} \right) = \frac{2(1-\rho_x)}{\psi^2} \left\{ \frac{\frac{1}{1-\rho^2} + \left[ \frac{\delta(1-\gamma \psi)}{(1-\rho \delta)} \right]^2}{\varphi_e \sigma^2} \right\} + 2\gamma^2 (1 - \rho_c) \sigma^2
\]
$$\text{Var} \left( \frac{e_{t+1}}{e_t} \right) = \frac{2(1-\rho_x)}{\psi^2} \left\{ \frac{1}{1-\rho^2} + \left[ \frac{\delta(1-\gamma \psi)}{(1-\rho \delta)} \right]^2 \right\} \varphi_e^2 \sigma^2 + 2 \gamma^2 (1 - \rho_c) \sigma^2$$
Exchange rate depreciation

\[
Var \left( \frac{e_{t+1}}{e_t} \right) = \frac{2(1-\rho_x)}{\psi^2} \left\{ \frac{1}{1-\rho^2} + \left[ \frac{\delta(1-\gamma)}{\psi(1-\rho)} \right]^2 \right\} \phi \sigma^2 + 2\gamma^2 (1 - \rho_c) \sigma^2
\]
Exchange rate depreciation

\[ \text{Var} \left( \frac{e_{t+1}}{e_t} \right) = \frac{2(1-\rho_x)}{\psi^2} \left\{ \frac{1}{1-\rho^2} + \left[ \frac{\delta(1-\gamma\psi)}{(1-\rho\delta)} \right]^2 \right\} \varphi_e^2 \sigma^2 + 2\gamma^2 (1 - \rho_c) \sigma^2 \]
Every assumption counts

Ingredients needed to solve the puzzle:
1. Disentangle elasticity of substitution from risk aversion
2. Highly persistent predictable component
3. Highly correlated predictable components
Every assumption counts

Ingredients needed to solve the puzzle:
1. Disentangle elasticity of substitution from risk aversion
2. Highly persistent predictable component
3. Highly correlated predictable components

Can we match key moments of international financial markets?
Introducing dividends

The system becomes

\[
\Delta c_t^i = \mu_c + x_{t-1}^i + \sigma \varepsilon_{c,t}^i \\
\Delta d_t^i = \mu_d + \lambda x_{t-1}^i + \sigma \varphi_d \varepsilon_{d,t}^i \\
x_t^i = \rho x_{t-1}^i + \sigma \varphi_e \varepsilon_{x,t}^i
\]

\[
\forall i \in \{h, f\}
\]

Shocks are \textit{i.i.d.} within each country

Shocks are correlated across countries
Introducing dividends

The system becomes

\[
\Delta c_t^i = \mu_c + x_{t-1}^i + \sigma \varepsilon_{c,t}^i
\]
\[
\Delta d_t^i = 0.0007 + 3 \cdot x_{t-1}^i + \sigma \cdot 5 \cdot \varepsilon_{d,t}^i
\]
\[
x_t^i = \rho x_{t-1}^i + \sigma \varphi_e \varepsilon_{x,t}^i
\]

\(\forall i \in \{h, f\}\)

Shocks are i.i.d. within each country

Shocks are correlated across countries

Calibrate coefficients of dividend growth to match:

- Average dividend growth \(\approx 1\%\)
- Standard deviation of dividend growth \(\approx 12\%\)
- Leverage is 5
- Small correlation of dividend growths: \(corr(\varepsilon_{d,t}^h, \varepsilon_{d,t}^f) \approx 0\)
## Introducing dividends: results

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>UK</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho \left( m^h, m^f \right)$</td>
<td>-</td>
<td>-</td>
<td>0.93</td>
</tr>
<tr>
<td>$\sigma \left( \frac{e_{t+1}}{e_t} \right)$</td>
<td>11.21</td>
<td>11.83</td>
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<tr>
<td>$E \left( r_d - r_f \right)$</td>
<td>7.02</td>
<td>9.17</td>
<td>7.01</td>
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<tr>
<td>$\sigma \left( r_d - r_f \right)$</td>
<td>17.13</td>
<td>22.83</td>
<td>19.60</td>
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<tr>
<td>$\rho \left( r^h_d - r^h_f, r^f_d - r^f_f \right)$</td>
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<tr>
<td>$E \left( r_f \right)$</td>
<td>1.47</td>
<td>1.62</td>
<td>1.33</td>
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<tr>
<td>$\sigma \left( r_f \right)$</td>
<td>1.53</td>
<td>2.92</td>
<td>1.19</td>
</tr>
<tr>
<td>$\rho \left( r^h_f, r^f_f \right)$</td>
<td>0.65</td>
<td>1.00</td>
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</tr>
</tbody>
</table>
Estimating long run risks

Can we estimate this model?

\[
\begin{bmatrix}
\Delta c^h_t \\
\Delta c^f_t
\end{bmatrix}
= 
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
x^h_{t-1} \\
x^f_{t-1}
\end{bmatrix}
+ 
\begin{bmatrix}
\sigma & 0 \\
0 & \sigma
\end{bmatrix}
\begin{bmatrix}
\varepsilon^h_{c,t} \\
\varepsilon^f_{c,t}
\end{bmatrix}
\]

\[
\begin{bmatrix}
x^h_t \\
x^f_t
\end{bmatrix}
= 
\begin{bmatrix}
\rho^h & 0 \\
0 & \rho^f
\end{bmatrix}
\begin{bmatrix}
x^h_{t-1} \\
x^f_{t-1}
\end{bmatrix}
+ 
\begin{bmatrix}
1 & \rho x \\
\rho x & 1
\end{bmatrix}
\frac{1}{2}
\begin{bmatrix}
\sigma \varphi_e & 0 \\
0 & \sigma \varphi_e
\end{bmatrix}
\begin{bmatrix}
\varepsilon^h_{x,t} \\
\varepsilon^f_{x,t}
\end{bmatrix}
\]
Estimating long run risks

Can we estimate this model?

\[
\begin{bmatrix}
\Delta c^h_t \\
\Delta c^f_t
\end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x^h_{t-1} \\
x^f_{t-1} \end{bmatrix} + \begin{bmatrix} \sigma & 0 \\ 0 & \sigma \end{bmatrix} \begin{bmatrix} \varepsilon^h_{c,t} \\
\varepsilon^f_{c,t} \end{bmatrix}
\]

\[
\begin{bmatrix}
x^h_t \\
x^f_t
\end{bmatrix} = \begin{bmatrix} \rho^h & 0 \\ 0 & \rho^f \end{bmatrix} \begin{bmatrix} x^h_{t-1} \\
x^f_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & \rho x \\ \rho x & 1 \end{bmatrix} \frac{1}{2} \begin{bmatrix} \sigma \varphi_e & 0 \\ 0 & \sigma \varphi_e \end{bmatrix} \begin{bmatrix} \varepsilon^h_{x,t} \\
\varepsilon^f_{x,t} \end{bmatrix}
\]

Roadmap:

1. Use consumption data only
   - Use Kalman filter to get a recursive representation of the likelihood function
   - Multi-country provide inconclusive evidence
Estimating long run risks

Can we estimate this model?

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\]

\[
\begin{bmatrix}
x^h_t \\
x^f_t
\end{bmatrix}
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\end{bmatrix}
\begin{bmatrix}
x^h_{t-1} \\
x^f_{t-1}
\end{bmatrix}
+ \begin{bmatrix}
1 & \rho_x \\
\rho_x & 1
\end{bmatrix}
\frac{1}{2}
\begin{bmatrix}
\sigma \varphi_e & 0 \\
0 & \sigma \varphi_e
\end{bmatrix}
\begin{bmatrix}
\varepsilon^h_{x,t} \\
\varepsilon^f_{x,t}
\end{bmatrix}
\]

Roadmap:

1. Use consumption data only
   - Use Kalman filter to get a recursive representation of the likelihood function
   - Multi-country provide inconclusive evidence

2. Use consumption and price data
   - Predictive regressions
   - Identify departure from \textit{i.i.d.}
Likelihood ratio tests

\[ \frac{\text{corr}(x_{\text{US}}, x_{\text{UK}})}{\text{corr}(x_{\text{US}}, x_{\text{UK}})} = 0.10 \]

\[ \frac{\text{corr}(x_{\text{US}}, x_{\text{UK}})}{\text{corr}(x_{\text{US}}, x_{\text{UK}})} = 0.05 \]

\[ \frac{\text{corr}(x_{\text{US}}, x_{\text{UK}})}{\text{corr}(x_{\text{US}}, x_{\text{UK}})} = 0.025 \]

\[ \frac{\text{corr}(x_{\text{US}}, x_{\text{UK}})}{\text{corr}(x_{\text{US}}, x_{\text{UK}})} = 0.01 \]
### A simulation exercise

<table>
<thead>
<tr>
<th>$\rho (x, x^*)$</th>
<th># of $\Delta c$'s</th>
<th># of $x$'s</th>
<th>50</th>
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| 0.9             | 2                 | 2           | 2     | 2     | 2     | 2     |
| 5               | 5                 | 5           | 5     | 5     | 5     | 5     |

**Notes** - Each column reports the 95% confidence interval for the estimated $\varphi_e$ parameter for simulated samples of increasing size. The true value of $\varphi_e$ is 0.34.
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# A simulation exercise

## Sample Size

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<td>(0.000, 1.234)</td>
<td>(0.000, 0.746)</td>
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<td>(0.066, 0.667)</td>
<td>(0.093, 0.637)</td>
<td>(0.178, 0.527)</td>
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<td>(0.182, 0.500)</td>
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<td>5</td>
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<td>(0.089, 0.564)</td>
<td>(0.131, 0.531)</td>
<td>(0.217, 0.457)</td>
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</table>

Notes - Each column reports the 95% confidence interval for the estimated $\varphi_e$ parameter for simulated samples of increasing size. The true value of $\varphi_e$ is 0.34.
Predictive regressions

- Regress consumption growth on lagged values of
  - price-dividend ratio
  - risk-free rate
  - consumption-output ratio
  - consumption growth

- Fitted consumption growth is predictive component ($x$)

- Use annual data from 1929 to 2006
- Repeat analysis for US and UK separately
## Predictive regressions: results

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>UK</th>
<th>US</th>
<th>UK</th>
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<tr>
<td>$R^2$</td>
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</tr>
<tr>
<td>$\rho_x$</td>
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</tr>
<tr>
<td>corr $(x_{US}, x_{UK})$</td>
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Pd and risk-free

All predictive variables

Pd only
## Predictive regressions: results

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<td>3.476</td>
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</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.036)</td>
<td></td>
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<td>6.852</td>
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<td>(0.000)</td>
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<tr>
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<td>7.299</td>
<td>6.013</td>
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<td></td>
<td>(0.008)</td>
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Predictive regressions: results

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# Predictive regressions: results

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<th>( F - \text{stat} )</th>
<th>( R^2 )</th>
<th>( \rho_x )</th>
<th>( \text{corr} \left( x^{US}, x^{UK} \right) )</th>
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<td>(0.008)</td>
<td>(0.016)</td>
<td>(0.065)</td>
<td>(0.099)</td>
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</table>
FX volatility and correlation of long-run risks

\[ \rho(x^{\text{us}}, x^{\text{uk}}) = 0.22, \quad \sigma_\pi = 17.55 \]

\[ \rho(x^{\text{us}}, x^{\text{uk}}) = 0.68, \quad \sigma_\pi = 7.67 \]

\[ \rho(x^{\text{us}}, x^{\text{uk}}) = 0.73, \quad \sigma_\pi = 13.81 \]

\[ \rho(x^{\text{us}}, x^{\text{uk}}) = 0.87, \quad \sigma_\pi = 11.02 \]
FX volatility and correlation of long-run risks: post 1970

\[ R^2 = 0.9238 \]
### FX volatility and correlation of long-run risks: table

<table>
<thead>
<tr>
<th></th>
<th>( R^2 )</th>
<th>( \beta )</th>
<th>( \frac{\text{Var}(x^{US} - x^{UK})}{\text{Var}(\Delta e)} )</th>
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</thead>
<tbody>
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<tr>
<td>Pd only</td>
<td>0.792</td>
<td>-12.34</td>
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<td></td>
<td>[0.014]</td>
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</table>
Estimating preference parameters

▶ Use Euler equation restrictions (each country has two domestic and two foreign assets)

▶ and first two moments of FX change

▶ to estimate preference parameters

▶ Two exercises:
  1. use OLS estimates of previous tables
  2. jointly estimate OLS parameters and preference parameters
## GMM estimation

<table>
<thead>
<tr>
<th></th>
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<th>Joint Estimation</th>
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<td>$P/D, R_f$</td>
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<td>1.371</td>
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<td></td>
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<td>[0.391, 1.792]</td>
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<td>$\gamma$</td>
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Notes: Each column reports the parameters estimated using GMM on the 1971-2006 sample.
Concluding remarks

- Key ingredients
  - Separate elasticity of substitution from risk aversion
  - Highly persistent predictable component
  - Highly correlated predictable components

- It is possible to explain
  - low volatility of the depreciation of the US dollar
  - high equity premium
  - high persistence of the risk free rate
  - high correlation of int’l financial markets
  - correlation of bonds
  - low correlation of consumption growths
  - low persistence of consumption growths