Risks for the Long Run and the Real Exchange Rate

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We propose an equilibrium model that can explain a wide range of international finance puzzles, including the high correlation of international stock markets, despite the lack of correlation of fundamentals. We conduct an empirical analysis of our model, which combines cross-country-correlated long-run risk with Epstein and Zin preferences, using U.S. and U.K. data, and show that it successfully reconciles international prices and quantities, thereby solving the international equity premium puzzle. These results provide evidence suggesting a link between common long-run growth perspectives and exchange rate movements.

I. Introduction

Since Backus and Smith (1993), a rich strand of the literature has documented the tension between international prices and quantities. As noted by Backus, Foresi, and Telmer (2001) and Lustig and Verdelhan (2007), among others, if markets are complete, the growth of the log

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exchange rate would equal the difference of the log stochastic discount factors for foreign currency– and dollar-denominated assets:

$$\Delta e_{t+1} = m^{f}_{t+1} - m_{t+1}. \tag{1}$$

If investors have power utilities, their log stochastic discount factors will be proportional to log consumption growth rates. Within this framework, it is well documented that (i) risk aversion has to be large to reconcile the low volatility of consumption growth rates with highly volatile stochastic discount factors and (ii) consumption is poorly correlated across countries at annual or higher frequencies. Indeed, if we were to set the coefficient of risk aversion to a number as high as 30, the fluctuation of exchange rates would be 15–30 times greater than what we observed in the data. On the basis of this insight, Brandt, Cochrane, and Santa-Clara (2006) conclude that given the degree of volatility of the U.S. dollar that we observed in the data, there should be a larger correlation of stochastic discount factors across countries.

We like to view this as an international equity premium puzzle. In a one-country model, consumption growth does not vary enough to explain the excess return over the risk-free rate. In a two-country model, consumption growth does not covary enough to track movements in the exchange rate and returns. This dichotomy of prices and quantities strikes us as an important unresolved puzzle in international finance.

The real exchange rate reflects the relative value of consumption in two countries. With power utility, only short-run consumption differences affect the current currency value. In this paper, we propose an equilibrium model in which exchange rates adjust to reflect differences in both current and future relative consumption across countries. We show that the marginal valuation of a unit of consumption in the two countries can be similar even when their current consumption growth rates are different. This requires both that agents care about the temporal distribution of risk and that countries share very similar long-run growth prospects.

We assume that consumers have Epstein and Zin (1989) recursive...
preferences. In this economy, agents are extremely sensitive to uncertainty about the long-run perspectives of the economy; news about future consumption growth affects their marginal utility of consumption today. If the long-run components of the endowments are highly correlated across countries, then stochastic discount factors will be too. Ultimately, exchange rates need to fluctuate much less to reflect differences in the marginal valuation of consumption in the two countries.

Following Bansal and Yaron (2004), we refer to these components as the long-run risks. These risks enter as small, slowly moving, predictable components of consumption growth rates that are consistent with the empirical evidence showing consumption to be close to a random walk. The addition of a long-run component changes the joint conditional distribution of consumption growth rates quite a bit. In particular, although over a short horizon this component explains only a very small share of consumption volatility, over a longer horizon the long-run risks are responsible for most of the fluctuations and the comovements of domestic and foreign consumption. We can view these components as capturing long-run movements in national technological frontiers, and we assume that technological frontiers are highly correlated across countries in the long run. This is consistent with the analysis of Eaton and Kortum (1999), whose estimates of international technological diffusion suggest substantial, but not perfect, sharing of ideas.

Our consumption-based equilibrium model simultaneously reproduces several other findings regarding international quantities and prices: (i) consumption growth rates prove to be as volatile, as autocorrelated, and as cross-country correlated as they are in the data; (ii) the equity premium puzzle is solved within each country; (iii) international asset returns are highly correlated, despite the virtual lack of correlation of fundamentals; and (iv) the contemporaneous correlation between log exchange rate growth and international returns is as low as in the data.

Our theoretical analysis suggests the following empirical question: is it possible to provide a direct estimate of the low-frequency components in consumption and make the case that long-run growth rate prospects look similar across countries? We address this question by adopting the view that the long-run movements of the consumption trend can be identified by means of predictive variables. Similarly to Bansal, Kiku, and Yaron (2006), we project consumption growth on lagged values of the price-dividend ratio, consumption-output ratio, and consumption growth itself to measure long-run risks. Applying this methodology separately to data from the United States and the United Kingdom, we conclude that the predictable components of consumption growth rates are highly persistent and that their correlation increases over time, just as the volatility of exchange rate growth decreases. The latter result
supports our conjecture regarding the link between common long-run growth perspectives and exchange rate movements.

Using this measure of long-run risks, we can also test the moment restrictions resulting from our model and estimate preference parameters. Results confirm that the intertemporal elasticity of substitution should be greater than one and that a coefficient of risk aversion smaller than 10 can account for both the domestic and international versions of the equity premium puzzle. We focus on the post–Bretton Woods sample and show that our model is not rejected by the data.

One of the novelties of our empirical work is that we identify long-run consumption risk using information from domestic financial markets and better assess the impact that such news has on currency markets. This enables our model to bridge part of the gap between international prices and quantities.

This paper is organized as follows. In Section II, we propose a simple model that we can both solve and calibrate. This provides a useful instrument to show the internal transmission mechanism of the economy. We document the ability of the model to account for the degree of volatility of exchange rate movements and a large number of other international moments. The focus of Section III is on the estimation of the model. We first show that the likelihood function obtained from consumption data only is quite uninformative about the existence and the nature of the predictive components of consumption growth in the cross section of countries. We then proceed with our predictive regressions approach and test a number of predictions of the model. Section IV concludes the paper with a discussion of the potential extensions of the model and a summary of the main findings.

II. Reconciling International Prices and Quantities

A. Exchange Rate Movements

Basics.—If markets are complete, the movements of the real exchange rate are pinned down by the simple no-arbitrage condition in equation (1). According to this equation, the more correlated the pricing kernels are, the smoother exchange rate movements should be.

Is correlation the same as risk sharing?—Not necessarily, as this crucially depends on the model’s assumptions. For example, consider an economy populated by two consumers with standard time-additive preferences defined over an aggregate of two tradable goods (e.g., one domestic and one foreign good). Risk sharing implies that the intertemporal marginal rates of substitution (IMRS) with respect to each good are equalized and therefore perfectly correlated across agents. If agents’ preferences are biased toward the consumption of opposite goods (e.g., there is con-
consumption home bias), however, their IMRS with respect to the consumption bundle will be less than perfectly correlated, even if in equilibrium all risk-sharing opportunities have been exhausted. For this reason, we analyze the volatility of the exchange rate fluctuations in terms of the correlation of the log pricing kernels, without drawing any conclusion about international risk sharing.

Exchange rates as relative consumption growth.—The real exchange rate is the relative price of two consumption bundles. As such, it should adjust to reflect differences in both current and future relative consumption across countries. In the special case of time-additive constant relative risk aversion (CRRA) preferences, only current consumption matters, since equation (1) reduces to

$$\Delta e_t \propto (\Delta c_t - \Delta c^*_t).$$

In this paper, we consider the case in which agents care about the entire sequence of future consumption streams. As a result, both current and future consumption growth differences matter for exchange rates today. In the next subsection we introduce the model and highlight the two sources of international consumption risk.

B. The Model

Setup of the economy.—We analyze an economy with two countries that we denote as home and foreign. For convenience, we characterize preferences, endowments, and prices only for the home country. Identical expressions indexed by an asterisk apply to the foreign country. The home representative consumer has the following Epstein and Zin (1989) preferences:

$$U_t = \frac{1}{\psi} \log \left( \frac{C_t^{1-\gamma}}{(1-\psi)^{1/(1-\psi)}} \right),$$

where $\gamma$ is the coefficient of risk aversion and $\psi$ is the intertemporal elasticity of substitution. There are two country-specific goods in the economy. To simplify the setup, we impose preferences such that there is complete home bias, meaning that the representative consumer in each country is willing to consume only the good with which she is endowed. Markets are complete. An equilibrium of this economy exists in which each country behaves as in autarky for both consumption and asset holdings.

As shown by Epstein and Zin (1991), the logarithm of the pricing kernel, $m_{t+1}$, is a stochastic process that depends on both log consumption growth, $\Delta c_t$, and the log return on the asset that pays consumption, $r_{c,t+1}$:

$$m'_{t+1} = \frac{1-\gamma}{1-1/\psi} \log \delta - \frac{1-\gamma}{\psi-1} \Delta c_{t+1} + \frac{1/\psi - \gamma}{1-1/\psi} r_{c,t+1}. \quad (2)$$
We complete the system by specifying exogenous laws of motion for consumption and dividend growth rates as follows:

\[
\begin{align*}
\Delta c_t &= \mu_c + x_{t-1} + \varepsilon_{c,t}, \\
\Delta d_t &= \mu_d + \lambda x_{t-1} + \varepsilon_{d,t}, \\
\end{align*}
\]

(3)

\[x_t = \rho_t x_{t-1} + \varepsilon_{x,t},\]

where \(\xi_t \sim \text{i.i.d. } N(0, \Sigma)\),

\[
\begin{align*}
\Delta c_t &= \mu_c + x_{t-1} + \varepsilon_{c,t}, \\
\Delta d_t &= \mu_d + \lambda x_{t-1} + \varepsilon_{d,t}, \\
\end{align*}
\]

(3)

\[x_t = \rho_t x_{t-1} + \varepsilon_{x,t},\]

where \(\xi_t \sim \text{i.i.d. } N(0, \Sigma)\),

In this section, the two economies are assumed to be symmetric, having the same preference and transition laws parameters. Furthermore, we assume that the shocks \(\varepsilon_{c,t}, \varepsilon_{d,t}\), and \(\varepsilon_{x,t}\) are mean zero independent, identically, and normally distributed (i.i.d.) within each country, with volatilities \(\sigma_c, \sigma_d,\) and \(\sigma_x\), respectively. The shocks are allowed to be cross-country correlated. We shall denote \(\rho_{c,c}^j\) as the correlation between \(\varepsilon_{c,t}\) and \(\varepsilon_{c,t}^*\), \(\rho_{d,d}^j\) as the correlation between \(\varepsilon_{d,t}\) and \(\varepsilon_{d,t}^*\), and \(\rho_{x,x}^j\) as the correlation between \(\varepsilon_{x,t}\) and \(\varepsilon_{x,t}^*\). All other correlations are set to zero.

The risk-free rate, \(r_f\), the log price-dividend ratio, \(v_d\), and the log price-consumption ratio, \(v_c\), are computed as follows:

\[
\frac{1}{1 + r_{ft}} = E[\exp (m_{t+1})],
\]

\[
\exp (v_{dt}) = E\left[ \sum_{j=1}^{\infty} \exp (m_{t+j}|d) \frac{D_{t+j}}{D_t} \right],
\]

\[
\exp (v_{ct}) = E\left[ \sum_{j=1}^{\infty} \exp (m_{t+j}|c) \frac{C_{t+j}}{C_t} \right],
\]

where \(m_{t+j} = m_{t+1}^c + m_{t+2}^d + \cdots + m_{t+j}^c\) is the \(j\)-step pricing kernel.

The following system is obtained as a first-order linear approximation of the model:

\[
\begin{align*}
m_{t+1} &= \log \delta - \frac{1}{\psi} x_t + \kappa_t \frac{1 - \gamma \psi}{\psi (1 - \rho_s \kappa_t)} \varepsilon_{x,t+1} - \gamma \varepsilon_{c,t+1}, \\
\Delta \varepsilon_{t+1} &= \varepsilon_{t+1}^* - \varepsilon_{t+1}, \\
\v_{d,t} &= \tilde{v}_d + \frac{\lambda - 1/\psi}{1 - \rho_s \kappa_d} x_t, \\
\v_{c,t} &= \tilde{v}_c + \frac{1 - 1/\psi}{1 - \rho_s \kappa_c} x_t, \\
\end{align*}
\]

(4)

\[
\begin{align*}
r_{d,t+1} &= \tilde{r}_d + \frac{1}{\psi} x_t + \kappa_d \frac{\lambda - 1/\psi}{1 - \rho_s \kappa_d} \varepsilon_{x,t+1} + \varepsilon_{d,t+1}, \\
r_{f,t} &= \tilde{r}_f + \frac{1}{\psi} x_t,
\end{align*}
\]
where \( \bar{\bar{r}}_j \) is the average return on asset \( j \), and the approximation constants \( \kappa_r \) and \( \kappa_d \) are endogenously determined as in Campbell and Shiller (1988).

Note that we abstract from modeling time-varying volatility in the dynamics of consumption and risky cash flows. As a consequence, the model produces constant risk premia and thus cannot replicate the empirical failure of the uncovered interest parity. The role of time-varying uncertainty of a common cross-country component is documented in Lustig, Roussanov, and Verdelhan (2008).

In this paper, the dynamics of the exchange rate growth is jointly affected by both short- and long-run news to international consumption:

\[
\Delta e_{t+1} - E_t[\Delta e_{t+1}] = \kappa_r \frac{1 - \gamma \psi}{\psi (1 - \rho_x \kappa_r)} (e_{x,t+1}^\ast - e_{x,t+1}) - \gamma (e_{c,t+1}^\ast - e_{c,t+1}).
\] (5)

Equation (5) highlights the key ingredients of exchange rate fluctuations in our model. First, we must disentangle risk aversion and the intertemporal elasticity of substitution, \( \gamma \neq 1/\psi \), for long-run growth prospects to affect the current exchange rate. Second, the more persistent the long-run components are (high \( \rho_x \)), the stronger the impact of long-run news on the current exchange rate. Equivalently, news with long-lasting effects on future consumption determines large movements in exchange rates. Third, the degree of cross-country correlation of short- and long-run consumption news matters: perfectly correlated shocks do not change the relative value of consumption bundles across countries; that is, they do not alter the exchange rate.

**Calibration.**—The structure of our two parallel economies mimics the economy discussed by Bansal and Yaron (2004). In table 1 we report our baseline calibration. The subjective discount factor \( \delta \) is set to 0.998 to reflect that our model describes a monthly decision problem. We set the coefficient of risk aversion \( \gamma \) equal to 4.25, which is relatively low compared to both the number commonly found in the equity premium puzzle literature and the coefficient proposed by Brandt et al. (2006). The intertemporal elasticity of substitution, \( \psi \), is equal to two, a number consistent with the literature on long-run risks.\(^1\)

As far as the calibration of the parameters of the laws of motion of consumption growth is concerned, our goal is to reproduce the average behavior of consumption growth in the United States and in the United Kingdom. We motivate the choice of these countries in detail in the

\(^1\) The value of the intertemporal elasticity of substitution has been the subject of a long-lasting debate in the literature. Hall (1988) and many follow-up studies estimate this number to be below unity. Guvenen (2006) reproduces capital and consumption fluctuations as long as most of the wealth is held by a small fraction of the population with a high elasticity of intertemporal substitution. Attanasio and Weber (1989) document an intertemporal elasticity of substitution greater than one in the United Kingdom.
TABLE 1
Baseline Calibration

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Psi$</td>
<td>Intertemporal elasticity of substitution</td>
<td>2</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Risk aversion</td>
<td>4.25</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Subjective discount factor</td>
<td>.998</td>
</tr>
<tr>
<td>$\mu_c$</td>
<td>Average consumption growth</td>
<td>$15 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Autoregressive coefficient of the long-run component $x_t$</td>
<td>.987</td>
</tr>
<tr>
<td>$\varphi_e$</td>
<td>Ratio of long-run shock and short-run shock volatilities</td>
<td>.048</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Standard deviation of the short-run shock to consumption</td>
<td>$68 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\mu_d$</td>
<td>Average dividend growth</td>
<td>.0007</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Leverage</td>
<td>3.0</td>
</tr>
<tr>
<td>$\varphi_d$</td>
<td>Volatility ratio of short-run shocks to dividend and consumption growth</td>
<td>5.0</td>
</tr>
<tr>
<td>$\rho^{by}_c$</td>
<td>Cross-country correlation of the long-run shock</td>
<td>1.0</td>
</tr>
<tr>
<td>$\rho^{by}_e$</td>
<td>Cross-country correlation of the short-run shock to consumption</td>
<td>.3</td>
</tr>
<tr>
<td>$\rho^{by}_d$</td>
<td>Cross-country correlation of the short-run shock to dividends</td>
<td>$-.1$</td>
</tr>
</tbody>
</table>

Note.—The countries share the same calibration. The model describes a monthly decision problem. The parameters of the laws of motion of consumption and dividend growth are set to reproduce the average behavior of consumption growth in the United States and in the United Kingdom. The cross-country correlations of the idiosyncratic shocks to consumption and shocks to the trend are chosen so as to obtain a correlation of consumption growths in the order of 0.3. The coefficient $\lambda$ is set in such a way that the ratio $\sigma_{a_d}/\sigma_{a_c} = 4.86$, which is in the range $(4, 8)$ estimated by Ludvigson, Lettau, and Wachter (2008). The cross-country correlation of the short-run shocks to dividends, $\rho^{by}_d$, is set so as to achieve an almost zero correlation of dividend growths between the United States and the United Kingdom.

The dividends processes require the calibration of four additional parameters, which we have selected with the goal of matching the first two moments of dividend growth rates, their cross-country correlation, and their leverage as the average of actual U.S. and U.K. data. The variance of dividend growth explained by its predictable component is very small, on the order of 3 percent.

We set the correlation of long-run risks, $\rho^{by}_x$, to one. This enables us to keep the volatility of the exchange rate fluctuations to about 11–12
percent. We discuss in the next section the empirical evidence supporting a large correlation of the predictive components of consumption growth. The cross-country correlation of the shocks to consumption, $\rho_y$, is chosen so as to obtain a correlation of consumption growth on the order of 0.3.

C. Correlations and Volatilities

Correlation of stochastic discount factors.—In figure 1 we report the international correlation of stochastic discount factors as a function of the coefficient of the intertemporal elasticity of substitution. The plot allows us to better understand the role of the key elements mentioned in the previous section: (i) long-run risk sensitivity, (ii) high persistence, and (iii) high international correlation of the long-run components.

In each of the two panels, the dark line is drawn according to our baseline calibration. The minimum of this graph falls in the vicinity of $\psi \approx 1/\gamma$, the CRRA case. As $\psi$ departs from $1/\gamma$ in either direction, the increasing sensitivity to long-run risk produces a sharp increase in the correlation of stochastic discount factors. Interestingly, we do not need to take a stand on whether the intertemporal elasticity of substitution is greater or smaller than one in order to obtain highly correlated pricing kernels. Any $\psi$ larger than 0.5 would yield a correlation of the log pricing kernels of at least 80 percent, provided that the remaining coefficients are calibrated as in table 1.

We are able to obtain highly correlated pricing kernels while keeping the unconditional international correlation of consumption as low as in the data. This result is primarily driven by the high correlation of the long-run components. The left panel of figure 1 documents that if...
the correlation of the long-run risks is lowered, the correlation of the pricing kernels decreases as well. The right panel of figure 1 documents that for the small shocks to the consumption trend to matter, they must be long lasting. If investors care about the temporal distribution of risk, near-permanent changes in the average growth of their consumption lead to large responses in marginal utilities.

Volatility of exchange rate fluctuations.—In the limiting case of our benchmark calibration (in which long-run risks are perfectly correlated across countries), short-run shocks are solely responsible for exchange rate fluctuations. In such cases, exchange rate movements are intimately related to investors’ degree of risk aversion.

For this reason, in figure 2 we plot the volatility of exchange rate growth against the coefficient of risk aversion. The two horizontal dashed lines represent the region within which the volatility of the exchange rate growth typically falls for Group of 7 countries. In our baseline calibration we set $\gamma = 4.25$, which yields a volatility that is well within the region of interest, as documented by the dark lines in the two panels of figure 2. Increasing the risk aversion of the representative consumers of the two countries to the high levels required by the equity premium puzzle literature would push the volatility to counterfactual levels—similar to those that motivated Brandt et al. (2006) to postulate the existence of a puzzle. In this case, our calibration again proves itself crucial.

Reducing the correlation of the long-run shocks reduces the correlation of stochastic discount factors, as we documented before, and increases the exchange rate fluctuations. Unless agents are made less risk averse, we are left with exchange rates that are too volatile at the calibrated level of risk aversion (fig. 2, left panel). The high exchange
<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>United States</th>
<th>United Kingdom</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho(m^i, m^j)$</td>
<td>Correlation of pricing kernels</td>
<td>. . .</td>
<td>. . .</td>
<td>.93</td>
</tr>
<tr>
<td>$\sigma(\Delta \epsilon)$</td>
<td>Volatility of FX growth</td>
<td>11.2</td>
<td>11.8</td>
<td></td>
</tr>
<tr>
<td>$E(\Delta \epsilon)$</td>
<td>Average FX growth</td>
<td>1.3</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$\rho(\Delta \epsilon_j, \Delta \epsilon_i)$</td>
<td>Autocorrelation of FX growth</td>
<td>.0</td>
<td>.0</td>
<td></td>
</tr>
<tr>
<td>$\rho(\Delta \epsilon_j, v^i_d - v^j_d)$</td>
<td>Correlation of FX growth and price-dividend ratios differences</td>
<td>.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho(\Delta \epsilon, \Delta \epsilon - \Delta \epsilon^*)$</td>
<td>Correlation of FX growth and consumption growth differentials</td>
<td>.15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho(v^i_d, v^j_d)$</td>
<td>Autocorrelation of price-dividend ratio</td>
<td>.89</td>
<td>.82</td>
<td>.92</td>
</tr>
<tr>
<td>$E(v^i_d)$</td>
<td>Average price-dividend ratio</td>
<td>3.3</td>
<td>2.9</td>
<td>2.9</td>
</tr>
<tr>
<td>$E(r^i_d - r^j)$</td>
<td>Average excess return</td>
<td>5.5</td>
<td>6.5</td>
<td>5.3</td>
</tr>
<tr>
<td>$\sigma(r^i_d)$</td>
<td>Volatility of excess return</td>
<td>17.1</td>
<td>22.8</td>
<td>19.1</td>
</tr>
<tr>
<td>$E(r^i)$</td>
<td>Average risk-free rate</td>
<td>1.5</td>
<td>1.6</td>
<td>1.3</td>
</tr>
<tr>
<td>$\sigma(r^i)$</td>
<td>Volatility of risk-free rate</td>
<td>1.5</td>
<td>2.9</td>
<td>1.2</td>
</tr>
<tr>
<td>$\rho(r^i_d - r^j, r^j_d - r^j)$</td>
<td>Correlation of excess returns</td>
<td>.67</td>
<td>.60</td>
<td></td>
</tr>
<tr>
<td>$\rho(v^i_d, v^j_d)$</td>
<td>Correlation of price-dividend ratios</td>
<td>.77</td>
<td>.93</td>
<td></td>
</tr>
<tr>
<td>$\rho(\Delta d, \Delta d^i)$</td>
<td>Correlation of dividend growth</td>
<td>-.03</td>
<td>-.07</td>
<td></td>
</tr>
<tr>
<td>$\rho(r^i_d, r^j_d)$</td>
<td>Correlation of risk-free rates</td>
<td>.65</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>$E(r^i_d - r^j + \Delta \epsilon) / E(r^j_d)$</td>
<td>Foreign Sharpe ratio (domestic units)</td>
<td>.20</td>
<td>.31</td>
<td>.23</td>
</tr>
<tr>
<td>$\rho(r^i_d - r^j, \Delta \epsilon) / E(r^j_d)$</td>
<td>Correlation of excess returns, domestic units</td>
<td>.63</td>
<td>.60</td>
<td>.54</td>
</tr>
<tr>
<td>$\rho(r^i_d - r^j, r^j_d - \Delta \epsilon)$</td>
<td>Correlation of excess returns, local units, and FX growth</td>
<td>.04</td>
<td>-.03</td>
<td>.00</td>
</tr>
<tr>
<td>$\sigma(\Delta d)$</td>
<td>Volatility of consumption growth</td>
<td>1.4</td>
<td>2.9</td>
<td>2.5</td>
</tr>
<tr>
<td>$\sigma^2(x) / \sigma^2(\Delta \epsilon) \times 100$</td>
<td>Share of predictable consumption variance</td>
<td>. . .</td>
<td>. . .</td>
<td>8.2</td>
</tr>
<tr>
<td>$\sigma(\Delta \epsilon)$</td>
<td>Volatility of dividend growth</td>
<td>16.9</td>
<td>6.9</td>
<td>11.9</td>
</tr>
<tr>
<td>$\rho(\Delta \epsilon, \Delta \epsilon^*)$</td>
<td>Correlation of consumption growth</td>
<td>.28</td>
<td>.35</td>
<td></td>
</tr>
</tbody>
</table>

Note.—Data are quarterly real per capita and are from 1971:01 to 1998:04, as in Brandt et al. (2006). Means and variances are annualized and multiplied by 100. All variables are in log units and are measured in terms of local real consumption, “local units,” unless noted otherwise. Domestic (foreign) units stands for variables measured in terms of the same real domestic (foreign) consumption good. FX denotes log exchange rate growth. The model calibration is reported in table 1. All simulated variables are time aggregated to an annual frequency.

rate volatility can be offset by decreasing the persistence of the predictable component (right panel). In this way, the volatility of the stochastic discount factors falls more than their international correlation.

### D. Other Moments of Returns in International Financial Markets

In table 2 we show that our baseline calibration is able to match key features of international financial markets. In comparing our results to
the data, we have assumed the United States to be the home country and the United Kingdom to be the foreign country. We have already documented the ability of the model to produce highly correlated pricing kernels and exchange rates that are as volatile as they are in the data. Here we show that our symmetric calibration implies an average growth of the U.S. dollar vis-à-vis the British pound equal to zero, which is not too far from the 1.3 percent sample average obtained in the post–Bretton Woods era.

Another well-documented empirical feature of the exchange rate growth is its lack of serial correlation and predictability. Our model replicates this finding. In general, system (4) suggests that exchange rate growth contains a predictable component proportional to the cross-country difference of the price-dividend ratios. When the long-run components are highly correlated, however, almost all the volatility of the exchange rate is explained by i.i.d. news to consumption. This explains why, consistent with the data, neither the lagged exchange rate growth rate nor the lagged difference of the price-dividend ratios is correlated with exchange rate movements in our model. As we can see from equation (5), the perfect correlation of the long-run components is also responsible for the extremely high correlation between the log exchange rate growth and the cross-country difference of the consumption growth rates.

We now turn our attention to stock markets. Our model generates persistent and volatile price-dividend ratios. When investors are concerned about the intertemporal distribution of risk, the price-dividend ratio fluctuations are considered very risky, resulting in high risk premia and low average levels for price-dividend ratios. As documented in table 2, our model produces excess returns and risk-free rates whose average and volatility are precisely in line with those observed in the U.S. and U.K. data.

We push our analysis one step further to demonstrate that the model can also match the average correlation of excess returns in the two countries, in spite of the low calibrated correlation of dividend growth rates. In the data, the high correlation of the returns is driven by the high correlation of the price-dividend ratios. This is exactly what happens in our model. We regard this finding as further validation of the existence of highly cross-country-correlated predictable components of consumption and dividend growth. When cash flow growth rates are i.i.d., the price-dividend ratios are constant and returns are as cross-country correlated as the growth of the dividends—a counterfactual

---

2 The correlation \(\rho(\Delta c, \Delta c^*)\) generated by the model is smaller than one because time-aggregated annual consumption growth is a nonlinear function of the 12 monthly consumption growth rates.
implication. By employing Epstein and Zin preferences and by allowing a small but highly persistent and highly cross-country-correlated component to also be responsible for the dynamics of consumption and risky cash flows, we obtain a new mechanism for the comovement of stock prices across countries.

Our model produces excess correlation of real risk-free rates. System (4) shows that the approximate solution for the risk-free rates in this economy is a linear function of the predictable component of consumption growth alone. As a consequence, the correlation of real risk-free rates entirely reflects the perfect correlation of the $\lambda$’s. In Section III this problem is mitigated as we assume a less-than-perfect correlation of the long-run components.

Up to this point we have focused on returns expressed in local units, $r^{US}$ for the United States and $r^{UK}$ for the United Kingdom. In table 2, however, we also report statistics for returns expressed in domestic units. For a U.S.-based investor, foreign returns expressed in domestic units are $r^{UK}_{t+1} + \Delta e_{t+1}$. Similarly, for a U.K.-based investor, returns equal $r^{US}_{t+1} - \Delta e_{t+1}$. Expressing returns in domestic units is necessary in order to study the model’s implications for the international investment opportunity sets of both U.S.- and U.K.-based investors. Foreign assets turn out to be attractive for two crucial reasons. First, exchange rate movements, $\Delta e_t$, appear to be uncorrelated with foreign asset price fluctuations, that is, $\text{corr}(\Delta e_t, r^*_t) \approx 0$. Second, the cross-country correlation of returns expressed in local units, $\text{corr}(r^1_t, r^2_t)$, is about 60 percent. As table 2 suggests, our model succeeds in replicating both of these pieces of empirical evidence and enables us to account for foreign Sharpe ratios expressed in domestic units.

In the lower part of table 2 we report some statistics for consumption and dividend growth rates for the United States and United Kingdom that confirm that our calibration is consistent with the empirical evidence on international quantities.

III. Estimating International Long-Run Risks

The task of measuring long-run risks from consumption data is even harder in small samples. However, the ability to estimate the predictable components of consumption growth at each point in time plays a crucial role in the testing of our model against the data. In this section we quantify the finite sample detectability issue of the slowly moving conditional means of consumption growth in the cross section of countries and we explore one way to construct an observable time series of long-run risks using the predictive information contained in asset prices.

We restrict our attention to annual consumption since longer-term consumption growth rates are more likely to contain a better signal and
less noise regarding the low-frequency component of consumption. We focus on the United States and the United Kingdom for a number of reasons both practical and historical. First of all, the identification of a highly persistent time series requires as long a sample as possible. From this standpoint, the United States and the United Kingdom are probably the best examples of countries with a long history of accurate consumption data. Also, the United States and the United Kingdom have had a tradition of financial integration for at least a large part of the post–World War II period, as documented by Taylor (1996), Quinn (1997), and Obstfeld (1998), among others. This is important because in assessing the predictions of the model for the dynamics of international asset prices, we need to ensure that U.S. and U.K. agents considered each other’s assets as part of their own investment opportunity sets.

Description of the data.—Our common sample covers the years from 1929 to 2006. Data on U.S. consumption of nondurables and services, gross domestic product, and population are from the National Income and Product Accounts of the Bureau of Economic Analysis. Value-weighted market returns, yields on 3-month Treasury bills, dividends, and dividend yields for the United States are from the Center for Research in Security Prices. Consumer price index (CPI) inflation and the spread between BAA and AAA corporate bonds (our measure of the default premium) were obtained from the Web site of the Federal Reserve Bank of St. Louis. The U.K. consumption series for the years 1963–2006 were obtained from the U.K. Office of National Accounts. For earlier data, we referred to Mitchell (1979). Long time series for the U.K. gross domestic product, FTSE returns, yields on 3-month Treasury bills, dividend yields, population, and CPI inflation were obtained from the Web site of Global Financial Data. All variables have been demeaned.

A. Consumption-Only Information

Kalman filter approach.—We document that detecting predictive components of consumption growth with the postulated properties of the previous section is an extremely difficult task also in the cross section of countries. We start our analysis by using a Kalman filter to derive a recursive representation of the likelihood function for the state space model in the system of equations (3) and to estimate the relevant parameters governing the consumption dynamics in the United States and in the United Kingdom. Our ultimate goal is to test the null of high persistence and high cross-country correlation of the predictive components through a likelihood ratio test. On the basis of our findings, we cannot reject that U.S. and U.K. predictive components are highly persistent and perfectly cross-country correlated at conventional signif-
risks for the long run 167

Specifically, a correlation of 1 and a persistence of 0.965 (which is the monthly equivalent of 0.65 at an annual frequency) cannot be rejected at a 5 percent level of significance (these results are reported in the technical appendix available in the electronic version of this paper).

Monte Carlo experiment.—What is the minimum sample size needed to sharply identify the long-run components and reject the random walk model? How would our estimation differ if we were to increase the number of countries and include more consumption time series? We address these questions by simulating the dynamics of consumption growth in $N$ countries with the following process:

$$
\Delta c_t = x_{t-1} + \sigma \varepsilon_{t,t},
$$

$$
x_t = P x_{t-1} + \varphi \sigma R^{1/2} \varepsilon_{x,t},
$$

where $P$ is a diagonal matrix that collects the persistence parameters $\rho^c$ on the main diagonal and has zeros on the off-diagonal, and $R$ is the correlation matrix of the shocks to the predictive components. We assume that all predictive components share the same autoregressive coefficient and that there is only one correlation coefficient, which is common to all pairs of $x$’s. We also impose that the country-specific shocks to consumption growth are uncorrelated across countries and that the average consumption growth rate is zero. Notice that all these restrictions make the estimation exercise simpler than it would be with actual data. In a small-sample environment, these simplifying assumptions are important: our results should be interpreted as providing an upper bound on the ability of an econometrician to identify the long-run risk components in consumption growth.

We set the autoregressive coefficient of the predictive components to 0.8547 and the volatility parameter $\sigma$ to 0.2355. These choices correspond to the annualized monthly frequency persistence of 0.987 and volatility of 0.0068 that we assumed in our calibrations. We set the identification parameter $\varphi = 0.34$. This corresponds to an $R^2$ of about 40 percent, which is an upper bound on the monthly predictability of consumption growth. We simulate 40,000 independent samples of size ranging from 50 to 200 years. We let the number of simulated consumption series be one, two, and five and the correlations of the predictive components be 0.9, 0.95, and 1, respectively. In each sample we estimate the equations in system (6).

The results of the estimation of $\varphi$ are reported in table 3. A number of interesting observations emerge. First, with only one consumption series it is virtually impossible to reject the pure random walk hypothesis. This is in essence the message contained in Bansal and Yaron (2004). Second, a sample size of 80 appears to be enough to reject the pure
random walk hypothesis with two consumption growth series affected by the same predictive component. Third, even a small departure from perfect correlation of the predictive components significantly hampers the task of rejecting the null.

The results of table 3 have important consequences for our empirical investigation. The only countries for which we have relatively stable long samples of about 80 years of consumption data are the United States and the United Kingdom. For any other developed country the number of reliable annual data points shrinks to about 50. Thus, unless the predictive components are perfectly correlated, consumption data alone are not enough to identify the low-frequency dynamics of international consumption movements.

### B. Consumption and Price Information

**Predictive regressions approach.**—Bansal et al. (2006) document that variables such as lagged price-dividend, consumption-output ratios, risk-free rates, lagged consumption growth, and default premia should contain a direct measure of long-run risk at each date and state. On the basis of this finding, we take the projection of consumption growth on the aforementioned set of variables to be our measure of long-run risk. In principle, there are multiple ways of constructing the conditioning information set for the predictive regressions. In our empirical analysis, we consider the following alternatives:

\[
\Delta c_i^t = p d_{t-1}^i \beta_i^1 + \epsilon_{c,1,i}^t
\]  

(7)

\[
\Delta c_i^t = [p d_{t-1}^i, r_{t-1}^i] \beta_2^t + \epsilon_{c,2,i}^t
\]  

(8)
TABLE 4
Predictive Regressions: The Three Ingredients

<table>
<thead>
<tr>
<th></th>
<th>United States (1)</th>
<th>United Kingdom (2)</th>
<th>United States (3)</th>
<th>United Kingdom (4)</th>
<th>United States (5)</th>
<th>United Kingdom (6)</th>
<th>corr($x^{us}$, $x^{uk}$) (7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price-dividend and</td>
<td>7.19</td>
<td>3.47</td>
<td>.16</td>
<td>.08</td>
<td>.768</td>
<td>.787</td>
<td>.579</td>
</tr>
<tr>
<td>risk-free</td>
<td>(.00)</td>
<td>(.03)</td>
<td>(.076)</td>
<td>(.074)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All predictive</td>
<td>9.07</td>
<td>6.85</td>
<td>.39</td>
<td>.27</td>
<td>.672</td>
<td>.759</td>
<td>.758</td>
</tr>
<tr>
<td>variables</td>
<td>(.00)</td>
<td>(.00)</td>
<td>(.074)</td>
<td>(.074)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price-dividend only</td>
<td>7.29</td>
<td>6.01</td>
<td>.08</td>
<td>.07</td>
<td>.885</td>
<td>.726</td>
<td>.849</td>
</tr>
<tr>
<td></td>
<td>(.00)</td>
<td>(.01)</td>
<td>(.065)</td>
<td>(.099)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VAR</td>
<td>11.71</td>
<td>4.20</td>
<td>.32</td>
<td>.15</td>
<td>.869</td>
<td>.765</td>
<td>.717</td>
</tr>
<tr>
<td></td>
<td>(.00)</td>
<td>(.00)</td>
<td>(.058)</td>
<td>(.075)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note.—Data are annual and go from 1930 to 2006. Columns 1 and 2 report the $F$-statistics and the associated $p$-values for the test that all the coefficients of the predictive regressions are equal to zero for the corresponding row model for the United States and the United Kingdom. Columns 3 and 4 report the $R^2$ for the predictive regressions for the United States and the United Kingdom. Columns 5 and 6 report the estimated autoregressive coefficients of the predictive components of U.S. and U.K. consumption growth. Column 7 reports the estimated cross-country correlation of the predictive components along with their bootstrapped confidence interval for the 1971–2006 period.

Regression (7) focuses on the explanatory ability of price-dividend ratios only. Regression (8) also incorporates the risk-free rate, an additional measure of long-run risk according to our model (see system [4]). Regression (9) provides a more accurate fit by including additional control variables that are shown to have predictive power for consumption.

In table 4, we report summary statistics supporting the key features of the long-run components highlighted in our theoretical model. First, the long-run components exist, as the $F$-tests consistently reject the null that consumption growth rates are i.i.d. Second, the long-run components are small and explain between 7 percent and 32 percent of annual consumption growth variance. Third, the long-run components are very persistent: the monthly equivalents of our estimates of $\rho^{us}_x$ and $\rho^{uk}_x$ range between 0.967 and 0.989. Fourth, their international correlation can be as high as 0.879 in the post–Bretton Woods sample.

An additional way of constructing the time series of observable long-run risks in the two countries consists in deriving a measure for the price-consumption ratios, $v_c$ and $v^{*}_c$. According to system (4), they should

\[ \Delta c^*_t = [pd_{t-1}, r_{f,t-1}, \Delta c_{t-1}, \Delta c_{y,t-1}, \text{default}_{t-1}] \beta^*_t + e_{c,*,t} \]  \hspace{1cm} (9)

\[ \forall i \in \{\text{US, UK}\}. \]
contain direct information about the predictive components of consumption growth. As a way of building these series, we consider the expected discounted value of consumption growth over the infinite horizon, \( \sum_{j=0}^{\infty} E_i \kappa_1 \Delta c_{t+j} \). The discount factor \( \kappa_1 \) is related to the unconditional mean of the price-consumption ratio as in Campbell and Shiller (1988). We set \( \kappa_1 = 0.965 \), a value consistent with the average wealth-consumption ratio estimates in Lustig, Van Nieuwerburgh, and Verdelhan (2009). The results are basically unaltered for a wide range of alternative values. Expectations are computed recursively from an unrestricted VAR(1) on the vector \( z_t = [\Delta c_t, \rho d_t, r_f] \). Given this assumption, it follows that

\[
\sum_{j=0}^{\infty} E_i \kappa_1 \Delta c_{t+j} = A'(I - \kappa_1 A)^{-1} z_t \quad \forall i \in \{\text{US, UK}\},
\]

where \( A' \) is the estimated matrix of vector autoregression (VAR) coefficients. The last row of table 4 reports the summary statistics for this approach, providing further evidence in favor of the presence of highly cross-country-correlated and slowly moving predictive components of consumption growth rates.

**Correlations and volatilities.**—In figure 3 we plot the time series of \( \Delta c_t^{\text{US}} \) and \( \Delta c_t^{\text{UK}} \), the fitted values of \( \Delta c_t^{\text{US}} \) and \( \Delta c_t^{\text{UK}} \) from regression (9). The other regressions produce similar results. Several interesting facts need to be noticed. First, the correlation between the predictable components...
of consumption growth seems to change over time. Indeed, it ranges from a historical low value of 0.22 from the beginning of the sample until 1941 to an all-time high level of 0.90 in the last 20 years. Second, there appears to be a negative relationship between the correlation of the predictable components of consumption and the volatility of the growth of the dollar-pound exchange rate. In particular, if we divide the post–Bretton Woods sample into two parts, we find that the lowest exchange rate volatility occurred in the years in which the correlation of long-run growth prospects was the highest.

To further investigate this negative relationship, we construct a time-varying measure of annual correlations between the fitted values $\tilde{x}_{t}^{US}$ and $\tilde{x}_{t}^{UK}$ and the time-varying volatilities of exchange rate movements. For the construction of time-varying second moments, we employ exponential smoothing. Specifically, the date $t$ correlation, $\rho_{t}^{i}$, is obtained as follows:

$$ q_{12,t} = \varrho q_{12,t-1} + (1 - \varrho) \tilde{x}_{t}^{US} \tilde{x}_{t}^{UK}, $$
$$ q_{1,t} = \varrho q_{1,t-1} + (1 - \varrho)(\tilde{x}_{t}^{US})^{2}, $$
$$ q_{2,t} = \varrho q_{2,t-1} + (1 - \varrho)(\tilde{x}_{t}^{UK})^{2}, $$
$$ \rho_{t}^{i} = q_{12,t} / \sqrt{q_{1,t} q_{2,t}}, $$

where $\tilde{x}_{t}$ is the demeaned counterpart of $\tilde{x}_{t}^{i}$ and $q_{1,0}$, $q_{2,0}$, and $q_{12,0}$ are variances and covariance computed on a presample of 15 years. The parameter is set to 0.95. Similarly dynamic variances of exchange rate movements can be computed via exponential smoothing as

$$ \sigma_{t}^{2} = \varrho \sigma_{t-1}^{2} + (1 - \varrho)(\Delta \epsilon_{t})^{2}. $$

Table 5 reports the time-series regressions of exchange rate volatilities on a constant and on the measures of cross-country correlations of long-run risks obtained from the four predictive regression approaches described above. The negative relationship between volatilities and correlations is a robust finding across all the alternative methods of measuring long-run risks. This is even more apparent from looking at figure 4. The fluctuations in the British pound–U.S. dollar exchange rate have become less and less volatile over time, just as the correlation of long-run growth perspectives has steadily increased. This finding provides further empirical validation of our theoretical model.

Column 3 of table 5 reports the variance of the difference of the predictive components as a fraction of the overall exchange rate volatility. This should give a sense of the amount of predictability of exchange rate fluctuations that is contained in the long-run risks. These numbers are typically very low, confirming that log exchange rate growth is difficult to forecast.
TABLE 5
Correlations and Volatilities

<table>
<thead>
<tr>
<th></th>
<th>$R^2$</th>
<th>$\beta$</th>
<th>$\text{Var (}\Delta\epsilon\text{)/}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>Price-dividend and risk-free</td>
<td>.929</td>
<td>-2.456</td>
<td>.007</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[.013]</td>
<td></td>
</tr>
<tr>
<td>All predictive variables</td>
<td>.610</td>
<td>-9.809</td>
<td>.004</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[.077]</td>
<td></td>
</tr>
<tr>
<td>Price-dividend only</td>
<td>.792</td>
<td>-5.996</td>
<td>.003</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[.074]</td>
<td></td>
</tr>
<tr>
<td>VAR</td>
<td>.924</td>
<td>-7.419</td>
<td>.061</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[.092]</td>
<td></td>
</tr>
</tbody>
</table>

Note.—Columns 1 and 2 report the $R^2$ and the slope coefficient in the regression of the exchange rate’s volatility on the cross-country correlation of long-run risks, respectively. The numbers in parentheses are the bootstrapped probabilities of observing a nonnegative slope. Column 3 displays the fraction of exchange rate growth variance that is accounted for by the variance of the difference of the predictive components of consumption growth. The sample is 1971–2006.

C. Long-Run Risk, Consumption, and Euler Equations

In this subsection we use the set of predictive variables for consumption outlined above to test a set of moment restrictions resulting from our model (eqq. [2]–[3]). In particular, we focus on the Euler equations for domestic and foreign stock market returns and risk-free rates for U.S.- and U.K.-based investors, on the first two moments of exchange rate growth, and on the correlation of exchange rate movements and cross-country consumption growth differences (also known in the literature as the Backus and Smith [1993] correlation). We estimate a common set of preference parameters by forcing U.S. and U.K. agents to share the same risk aversion coefficient and elasticity of intertemporal substitution.

In table 6 we show the results of our estimation by using different sets of predictive variables to filter long-run consumption risks. We do not report the results obtained through the VAR approach since the implied predictive components are highly correlated with the equity price-dividend ratios within each country (during the post–Bretton Woods sample, their correlations are 0.991 and 0.925 for the United States and the United Kingdom, respectively). In what follows, hence, we use only the parsimonious regressions (7)–(9).

Focusing on the entire sample may be inappropriate when it comes to testing the ability of our model to reproduce the average volatility of the exchange rate and the validity of some of the Euler equations. Exchange rates have abandoned the nominal rigidities regime imposed by the Bretton Woods system in the early 1970s, and as a consequence of that, the degree of volatility of this series changes dramatically. Fur-
thermore, an extensive literature (reviewed by Quinn [1997]) documents that the degree of financial markets’ openness between the United States and the United Kingdom increased only after the late 1960s. For all these reasons, we find it appropriate to restrict our attention to the 1971–2006 sample.

In table 6 we report the results of two alternative approaches. In the first case, we take the time series of long-run risks as exogenously given and use the moment conditions to estimate the preference parameters of the two countries’ investors (cols. 1–3). Second, we test the entire model by jointly estimating the set of preference parameters and coefficients governing the dynamics of the predictable components of consumption growth in the United States and the United Kingdom (cols. 4–6). In the latter case, we also include the set of least-squares orthogonality conditions to identify the additional parameters of the model. We further assume that the predictive components of consumption growth rates have exactly the same persistence.

Columns 1–3 of table 6, labeled as Conditional Estimation, are two-country counterparts of the exercise conducted by Bansal et al. (2006), whose focus is on U.S. Euler equation restrictions alone. Compared to their results, our estimated risk aversion coefficient is sizably smaller and the intertemporal elasticity of substitution is greater than one. We regard this as an important result, which is driven primarily by the information contained in exchange rate movements. The explanation is as follows. Risk aversion and persistence of the predictive component of consumption growth play complementary roles in contributing to risk premia. At an annual frequency, the persistence parameter suffers a downward bias from its monthly counterpart due to temporal aggre-
<table>
<thead>
<tr>
<th></th>
<th>Conditional Estimation</th>
<th>Joint Estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{P}{D} )</td>
<td>( \frac{P}{D}, R_f )</td>
</tr>
<tr>
<td>( \psi )</td>
<td>4.094</td>
<td>1.371</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>4.359</td>
<td>3.402</td>
</tr>
<tr>
<td>(2)</td>
<td>[3.181, 9.575]</td>
<td>[2.282, 7.376]</td>
</tr>
<tr>
<td>( \rho_x )</td>
<td>\ldots</td>
<td>\ldots</td>
</tr>
<tr>
<td>(3)</td>
<td>\ldots</td>
<td>\ldots</td>
</tr>
<tr>
<td>( \rho(\tilde{x}^{US}, \tilde{x}^{UK}) )</td>
<td>\ldots</td>
<td>\ldots</td>
</tr>
<tr>
<td>(4)</td>
<td>\ldots</td>
<td>\ldots</td>
</tr>
<tr>
<td>Wald statistic</td>
<td>\ldots</td>
<td>\ldots</td>
</tr>
<tr>
<td>( p )-value</td>
<td>\ldots</td>
<td>\ldots</td>
</tr>
<tr>
<td>( \gamma - 1/\psi )</td>
<td>.000</td>
<td>.008</td>
</tr>
<tr>
<td>( p )-value</td>
<td>.000</td>
<td>.000</td>
</tr>
</tbody>
</table>

Note.—Each column reports the parameters estimated using GMM on the 1971–2006 sample. All GMMs have the following set of moment conditions in common: Euler equations for domestic stock market returns and risk-free rates (4), Euler equations for foreign stock market returns and risk-free rates expressed in local units (4), first two moments of the exchange rates’ growth (2), and the covariance between exchange rates’ growth and cross-country consumption growth differences (1). The weighting matrix is the identity matrix. The GMMs in cols. 1–3 are conditional on the ordinary least squares estimates of the regression of consumption growth of the previous section. For the results reported in cols. 4–6, all models’ parameters are estimated jointly by adding the appropriate set of orthogonality restrictions. The numbers is brackets below each estimate are bootstrapped 95 percent confidence intervals. Wald statistics test the null of no predictability in consumption growth rates in both countries. The row \( \gamma - 1/\psi \) \( p \)-value reports the \( p \)-value for the one-tailed null \( H_0 : \gamma \leq 1/\psi \).
gation issues. In a single-country setting, risk aversion has to increase to generate large risk premia. To prevent the risk-free rate from being excessively low, the intertemporal elasticity of substitution drops below one. In a multicountry setting, the modest degree of volatility of exchange rate movements prevents risk aversion from becoming too large. This limits the downward pressure on the estimated intertemporal elasticity of substitution.

When we turn our attention to the joint estimation of the predictive components of consumption growth and the preference parameters, the results improve even further. The estimated preference parameters are in line with the numbers that we suggested in the earlier sections. As we jointly estimate all the model’s parameters, the persistence of the predictive components increases. This improves the ability of the model to satisfy the Euler equation restrictions. The model is not rejected by the data at least at the 1 percent level of significance, the only exception being for the case in which the set of predictors is limited to the price-dividend ratio and the risk-free rate. A Wald test on the null that consumption growth rates are jointly i.i.d. processes is overwhelmingly rejected. This provides further validation of the presence of long-run risks in consumption dynamics.

In table 6 we also report the \( p \)-values for the null hypothesis that consumers have a preference for late resolution of uncertainty, that is, \( \gamma \leq 1/\psi \). We typically reject the null at conventional levels of significance. According to equation (5), this implies that a currency should depreciate not only in response to positive short-run news but also following good news for the long run. We have repeated the same set of exercises for the 1931–2006 sample. Not surprisingly, the presence of multiple exchange rate regimes and the lower degree of financial integration of the first part of the century leads to an overwhelming rejection of the model and to an increased amount of uncertainty surrounding the estimated coefficients. Furthermore, the estimated coefficients of risk aversion are low, as the model tries to match a lower average exchange rate volatility (these results are reported in the online technical appendix).

### D. Long-Run Risk, Dividends, and Returns

In this subsection we implement a generalized method of moments (GMM) estimation that directly compares the time series of returns and exchange rates implied by our model, described by (3)–(4), with those observed in the data. In order to estimate the dividends process reported in (3), we proceed as follows. We jointly estimate the following two equations on the entire sample:
### TABLE 7

**Predictive Regressions on Dividends**

<table>
<thead>
<tr>
<th></th>
<th>$\Delta c_{i-1}$</th>
<th>$pd_{i-1}$</th>
<th>$cy_{i-1}$</th>
<th>default</th>
<th>$r_{i-1}$</th>
<th>$\lambda$</th>
<th>$R^2_{it}$</th>
<th>Likelihood Ratio Test</th>
<th>$\rho_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>.455</td>
<td>.011</td>
<td>-.063</td>
<td>1.289</td>
<td>-.045</td>
<td>2.653</td>
<td>.071</td>
<td>13.28</td>
<td>.673</td>
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<td></td>
<td>(.150)</td>
<td>(.006)</td>
<td>(.043)</td>
<td>(.739)</td>
<td>(.134)</td>
<td>(1.052)</td>
<td>(.010)</td>
<td>(.074)</td>
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<td>United Kingdom</td>
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<td>.032</td>
<td>-.088</td>
<td>. . . .</td>
<td>-.226</td>
<td>1.666</td>
<td>.129</td>
<td>1.460</td>
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<td>(.128)</td>
<td>(.020)</td>
<td>(.050)</td>
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<td>(.531)</td>
<td>(.580)</td>
<td>(.691)</td>
<td>(.074)</td>
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</table>

Note.—In cols. 1–6, we report the maximum likelihood estimates of the parameters in (10). Newey-West adjusted standard errors are reported in parentheses. Column 7 shows the implied for dividend growth. Column 8 reports the likelihood ratio tests ($p$-values in parentheses) with respect to the unrestricted system (11). Column 9 reports the estimates of the persistence of the predictable component. Data are annual and demeaned. The sample ranges from 1929 to 2006.

\[
\Delta c_i^t = \beta_0^c \Delta c_{i-1}^{t-1} + \beta_1^c pd_{i-1}^{t-1} + \beta_2^c \Delta cy_{i-1}^{t-1} + \beta_3^c default_{i-1}^{t-1} + \beta_4^c r_{i-1}^{t-1} + \varepsilon_{c,t}^{t-1}
\]

\[
\Delta d_i^t = \nu(\beta_0^d \Delta c_{i-1}^{t-1} + \beta_1^d pd_{i-1}^{t-1} + \beta_2^d \Delta cy_{i-1}^{t-1} + \beta_3^d default_{i-1}^{t-1} + \beta_4^d r_{i-1}^{t-1}) + \varepsilon_{d,t}^{t-1}
\]

for all countries $i = \{\text{US, UK}\}$. The estimates are reported in columns 1–6 of table 7 for three different combinations of the predictive variables. The results suggest that predictability of dividends growth exists in both the United States and the United Kingdom, regardless of the set of predictors. The $R^2$s are lower than those reported in table 4, but most of the estimated coefficients are significantly different from zero.

We regard system (10) as the restricted version of the following system of equations:

\[
\Delta c_i^t = \beta_0^c \Delta c_{i-1}^{t-1} + \beta_1^c pd_{i-1}^{t-1} + \beta_2^c \Delta cy_{i-1}^{t-1} + \beta_3^c default_{i-1}^{t-1} + \beta_4^c r_{i-1}^{t-1} + \varepsilon_{c,t}^{t-1}
\]

\[
\Delta d_i^t = \phi_0^d \Delta c_{i-1}^{t-1} + \phi_1^d pd_{i-1}^{t-1} + \phi_2^d \Delta cy_{i-1}^{t-1} + \phi_3^d default_{i-1}^{t-1} + \phi_4^d r_{i-1}^{t-1} + \varepsilon_{d,t}^{t-1}
\]

The results of the likelihood ratio tests for the null that $\phi_j^d = \lambda \beta_j^c$ for all $i = \{\text{US, UK}\}$ and for all $j = \{0, 1, 2, 3\}$ (table 7) indicate that in both countries the null cannot be rejected at least at the 1 percent level of significance, which appears to be consistent with our theoretical assumption of a common long-run component in the dynamics of consumption and dividends. On the grounds of this evidence, we assume henceforth that the consumption and cash flows of each of the two...
countries can be predicted by the same linear combination of variables. This will enable us both to keep our econometric specification close to the theoretical model and to deal with a smaller set of parameters in the estimation.

We estimate our pricing model focusing only on the post–Bretton Woods sample. All GMMs have the following set of moment conditions in common: the average residual between the log excess returns, the log risk-free rate, the log exchange rate growth, and the squared log exchange rate growth implied by the model and those observed in the data are zero (six restrictions). As in the previous section, we also add the Backus and Smith correlation, for a total of seven restrictions.

Our main results are summarized in table 8. As in the previous section, we first estimate the preference parameters, taking as given the dynamics of quantities in each country. We report our conditional estimates of the preference parameters in columns 1–3 so that each column refers to a specific set of predictive variables. Through this empirical methodology, the uncertainty about each estimate is significantly reduced. This result is particularly evident for the intertemporal elasticity of substitution. As in the previous section, there is a statistically significant preference for early resolution of uncertainty.

In columns 4–6 of table 8, we report the results of the joint estimation of systems (10) and (4) over the 1971–2006 sample. Two things should be noted. First, the confidence intervals of the intertemporal elasticity of substitution are still tight around two, consistent with the results in columns 1–3. Second, our estimates of the relative risk aversion coefficient confirm that the introduction of exchange rates into the estimation procedure can mitigate the bias issues noted by Bansal et al. (2006).

According to the Wald test, the assumption that all the cash flow growth rates are i.i.d. is rejected also in the post–Bretton Woods sample. The point estimates of the persistence of the long-run components, $\rho_s$, are in line with those obtained in the rest of our empirical exercises. Furthermore, all bootstrapped confidence intervals are relatively small given our short sample, suggesting a tight identification of our parameters. Turning our attention to the $J$-statistic, we document a significant improvement when we jointly estimate preference and quantity parameters. In all three cases, the model cannot be rejected at the 1 percent level of significance.

In table 9, we report key moments of exchange rate and U.S. and U.K. returns obtained when the larger set of predictive variables for consumption is used. At our point estimates, the volatility of exchange rate growth is 16.6 percent and hence is very close to that observed in the data. As in the data, the exchange rate in our model displays almost no serial correlation. Furthermore, the less than perfect correlation of
## Table 8

### GMM Estimation with Dividends

<table>
<thead>
<tr>
<th>Conditional Estimation</th>
<th>Joint Estimation</th>
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<tr>
<td></td>
<td>$P/D$</td>
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<tr>
<td></td>
<td>(1)</td>
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<tr>
<td>$\psi$</td>
<td>2.188</td>
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<tr>
<td>$\gamma$</td>
<td>5.066</td>
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<tr>
<td>$\rho_x$</td>
<td>.981</td>
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<tr>
<td>$\rho(x^{US}, x^{UK})$</td>
<td>.879</td>
</tr>
<tr>
<td>Wald-statistic $p$-value</td>
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<td>$\gamma - 1/\psi$ $p$-value</td>
<td>.000</td>
</tr>
<tr>
<td>$J$-statistic $p$-value</td>
<td>.160</td>
</tr>
</tbody>
</table>

Note.—Each column reports the parameters estimated using GMM on the 1971–2006 sample. Data are annual. All GMMs have the following set of moment conditions in common: first moment for stock market returns and risk-free rate (4), first two moments of the exchange rates’ growth (2), and the covariance between exchange rates’ growth and cross-country consumption growth differences (1). The weighting matrix is the identity matrix. The first three GMMs are conditional on the ordinary least squares estimates reported in table 7. In cols. 4–6 we report results of the joint GMM estimation of (4) and (10). The appropriate orthogonality restrictions are added. The numbers in brackets below each estimate are bootstrapped 95 percent confidence intervals. Our Wald statistics are computed imposing all the restrictions required to test the null of no predictability in consumption and dividend growth rates in both countries. The row $\gamma - 1/\psi$ $p$-value reports the $p$-value for the one-tailed null $H_0: \gamma \leq 1/\psi$. 


our estimated long-run components breaks the equality between exchange rate growth and cross-country consumption growth differentials pointed out by Backus and Smith (1993). Since our long-run components are very persistent and our agents have a preference for the timing of resolution of uncertainty, even small cross-country differences in the realization of the long-run shocks can sizably reduce the correlation between consumption differentials and exchange rate movements. This results in the ability of our model to produce a Backus and Smith correlation as low as 0.23.

Turning our attention to the equity markets, we point out that the model replicates almost exactly the degree of international correlation of U.S. and U.K. excess returns. The equity premia implied by the model are sizable although smaller than their empirical counterparts. The model-implied Sharpe ratios are lower than those in the data. For the United States, in particular, the Sharpe ratio falls below 20 percent because our model overestimates the 1974 U.S. stock market crash. In the post-1974 period, indeed, the U.S. Sharpe ratio implied by the model goes up to about 21 percent, and hence it is closer to the data. Column 7 of table 9 displays that the model can generate large excess returns and, at the same time, low risk-free rates.

IV. Concluding Remarks

By disentangling the intertemporal elasticity of substitution from the reciprocal of the coefficient of risk aversion and by allowing for a per-

5 In our estimation exercises we impose symmetry across the United States and the United Kingdom for parsimony. In 1974, however, the fall of the stock market was two times more severe in the United Kingdom than in the United States.
sistent and highly cross-country-correlated forecastable component of consumption growth, it is possible to explain a large set of international finance puzzles using a frictionless consumption-based asset pricing model. In our analysis, the dichotomy between international quantities and prices documented by the international finance literature disappears after we account for the intertemporal composition of consumption risk. Consumption and cash flow dynamics are in line with the data, and at the same time the equity premium puzzle is solved within each country. Exchange rate fluctuations resemble those that we observe for major industrialized countries, and the international investment opportunity set reflects the equity risk premia and the degree of correlation suggested by the data.

Future developments of this line of research should address the deeper economic question of the origin of the predictable components of consumption growth in the context of a fully specified production economy. In particular, it will be important to study the predictions of the model for the dynamics of current accounts, portfolio allocations, and international investment flows. Furthermore, the introduction of stochastic volatility could allow the model to replicate the empirical evidence on currency excess returns studied by Lustig and Verdelhan (2007) and Lustig et al. (2008).

Appendix

GMM on Euler Equations Restrictions

The predictable components of consumption growth rates are constructed according to the regressions (7)–(9). Innovations to consumption growth and its low-frequency components are computed as follows:

\[ e_i^{t+1} = \Delta c_i^{t+1} - x_i^t, \]
\[ e_s^{t+1} = x_i^t - \rho x_i^{t-1}. \]

We then use a log-linearized version of the model (eqq. [2]–[3]) in order to recover time series of stochastic discount factors and exchange rates. We calibrate the mean of consumption growth to .02 and the subjective discount factor to \( \delta = .99 \) for both the United States and the United Kingdom. The GMM procedure is applied to the sample counterparts of the following set of Euler equations:

\[ E[M_{i,t+1}^i R_{j,t+1}^j] - 1 = 0 \quad \forall i = \{US, UK\} \text{ and } \forall j = \{m,f\}, \]
\[ E[M_{i,t+1}^{US} R_{j,t+1}^{UK} \exp(\Delta e_{i+1}^j)] - 1 = 0 \quad \forall j = \{m,f\}, \]
\[ E[M_{i,t+1}^{UK} R_{j,t+1}^{US} [\exp(\Delta e_{i+1}^j)]^{-1}] - 1 = 0 \quad \forall j = \{m,f\}, \]

and of the following exchange rate moment conditions:
\[ E[M_{t+1}^{US}/M_{t+1}^{UK} - \exp(\Delta e_{t+1})] = 0, \]
\[ E[(M_{t+1}^{US}/M_{t+1}^{UK})^2 - \exp(\Delta e_{t+1})]^2] = 0, \]
\[ E[[M_{t+1}^{US}/M_{t+1}^{UK} - \exp(\Delta e_{t+1})] \exp(\Delta e_{t+1}^{UK} - \Delta e_{t+1}^{US})] = 0. \]

When preference parameters and coefficients governing the dynamics of the predictable components of consumption growth are jointly estimated, we introduce also the appropriate set of orthogonality conditions.

**GMM on the Reduced Form of the Model**

The innovations to consumption growth and its predictable component can be computed as in the previous section. The innovation to cash flows is
\[ \epsilon_{d,t+1} = \Delta d_{t+1} - \lambda x_{t+1} i, \quad \forall i \in \{\text{US, UK}\}. \]

The log-linearized version of the model (eqs. [3]–[4]) is then used to construct the sample counterparts of the following moment conditions:
\[ E[(r_{m,t+1} - \tilde{r}_{f,t+1}) - (r_{i,t+1} - r_{f,t+1})] = 0, \quad \forall i \in \{\text{US, UK}\}, \]
\[ E[\tilde{r}_{f,t} - r_{f,t+1}] = 0, \quad \forall i \in \{\text{US, UK}\}, \]
\[ E[M_{t+1}^{US}/M_{t+1}^{UK} - \exp(\Delta e_{t+1})] = 0, \]
\[ E[(M_{t+1}^{US}/M_{t+1}^{UK})^2 - \exp(\Delta e_{t+1})]^2] = 0, \]
\[ E[[M_{t+1}^{US}/M_{t+1}^{UK} - \exp(\Delta e_{t+1})] \exp(\Delta e_{t+1}^{UK} - \Delta e_{t+1}^{US})] = 0, \]

where \( \tilde{r}_{m,t+1} \) and \( \tilde{r}_{f,t} \) are the model predictions for the market returns and the risk-free rate. When preference parameters and the coefficients governing the dynamics of the predictable components of consumption growth are jointly estimated, we introduce also the appropriate set of orthogonality conditions.

**Bootstrap Methodology**

All bootstrapped confidence intervals are generating by following the procedure outlined in Hall and Horowitz (1996) and in greater detail in Horowitz (2006). This bootstrap procedure applies to GMM estimation involving dependent data. The bootstrap was implemented by block sampling 12 consecutive observations at a time with replacement.

**References**


