A Habit-Based Explanation of the Exchange Rate Risk Premium

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Outline

• Introduction
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  – Currency risk premium
  – Contribution of this paper

• Model
  – Two-country external habit model

• Calibration
• Estimation
• Conclusion
INTRODUCTION
Uncovered Interest rate Parity (UIP)

When the interest rate of Home country is higher than Foreign country, investors expect depreciation of Home currency against Foreign currency.

\[ E_t(q_{t+1}) - q_t = r_t - r_t^* \]

- **NOTE**: UIP holds if and only if FX risk premium is zero
- We cannot test UIP directly because of the unobservable expectation term
- If rational expectation holds,
  \[ q_{t+1} = E_t(q_{t+1}) + \nu_t \quad \text{where} \quad E_t(\nu_t) = 0 \]
Test of UIP condition

- Testable equation
  \[ q_{t+1} - q_t = r_t - r_t^* - v_t \]

- Regression model
  \[ q_{t+1} - q_t = \alpha + \beta (r_t - r_t^*) + \epsilon_t \]

- The null hypothesis for UIP condition is \( \alpha = 0 \) and \( \beta = 1 \), but, in most of currency pairs, it is rejected.
- In some cases, \( \beta \) is negative (for example, USD/JPY)
- High interest rate currencies tend to be appreciate against low interest rate currencies

- UIP puzzle
- Carry trade is very profitable
Risk-based Explanation

- Recall: UIP test is the joint test of
  - Rational expectation
  - Zero risk premium

Failure of Rational Expectation?

or

Failure of zero risk premium?

“Risk-based Explanation”

Investors require risk premium for holding high-interest rate currency
Currency risk premium

- Adding risk premium in UIP regression

\[ q_{t+1} - q_t + rp_{t,t+1} = \alpha + \beta \left( r_t - r^*_t \right) + \varepsilon_t \]

- Risk premium must be time-varying and be negatively correlated with interest rate differential

- Can we replicate this feature with rational expectation asset pricing model?

External Habit Model (Campbell and Cochrane (1999))

Recall that the external habit model generates counter-cyclical risk premium
Verdelhan (2009) extends the external habit model to two-country setting and get negative $\beta$ with plausible structural parameters.

Key features of his model
- Pro-cyclical interest rates
- Counter-cyclical currency risk premium

Interest rate differentials and currency risk premium are negatively correlated
MODEL
Model

- Basic settings are same as CC
- Habit-based preference
  \[ E \sum_{t=0}^{\infty} \beta^t \frac{(C_t - H_t)^{1-\gamma} - 1}{1 - \gamma}, \]
- i.i.d. consumption growth
  \[ \Delta c_{t+1} = g + u_{t+1}, \text{ where } u_{t+1} \sim i.i.d. N(0, \sigma^2). \]
- Auto regressive surplus consumption ratio
  \[ s_{t+1} = (1 - \phi) \bar{s} + \phi s_t + \lambda(s_t)(\Delta c_{t+1} - g). \]
Model

- SDF

\[ M_{t+1} = \beta \left( \frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t} \right)^{-\gamma} = \beta e^{-\gamma \left[ g + (\phi - 1)(S_t - \bar{s}) + (1 + \lambda(s_t))(\Delta C_{t+1} - g) \right]} \]

- Sensitivity function

\[ \lambda(s_t) = \frac{1}{\bar{S}} \sqrt{1 - 2(s_t - \bar{s})} - 1, \text{ when } s \leq s_{\text{max}}, \ 0 \text{ elsewhere,} \]

\[ s_{\text{max}} = \bar{s} + (1 - \bar{S}^2)/2, \]

\[ \bar{S} = \sigma \sqrt{\frac{\gamma}{1 - \phi - B/\gamma}} \]

- In CC, they mainly focus on the case where \( B \) is zero
Pro-cyclical interest rates

- Risk-free interest rates become linear function of $s_t$

\[ r_t = \bar{r} - B(s_t - \bar{s}), \]

- When $B$ is negative, interest rates are low in bad times and high in good times
Counter-cyclical risk premium

- Expected log currency risk premium

\[ E_t(r_{t+1}^e) = \frac{1}{2} \text{Var}_t(m_{t+1}) - \frac{1}{2} \text{Var}_t(m_{t+1}^*) \]

A decreasing function of \( s_t \)?

- Recall

\[ M_{t+1} = \beta \left( \frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t} \right)^{-\gamma} = \beta e^{-\gamma \left[ g + (\phi - 1) (s_t - \bar{s}) + (1 + \lambda(s_t)) (\Delta c_{t+1} - g) \right]} \]

\[ \lambda(s_t) = \frac{1}{S} \sqrt{1 - 2(s_t - \bar{s})} - 1 \], when \( s \leq s_{\text{max}} \), 0 elsewhere,
Counter-cyclical risk premium

- Conditional variance of the log SDF becomes
  \[ \text{Var}_t(m_{t+1}) = \frac{\gamma^2 \sigma^2}{S^2} [1 - 2(s_t - \tilde{s})] \]
  decreasing function of \( s_t \)

- In order to get closed form expression, Verdelhan assumes that structural parameters are same in home and foreign countries.

- Expected currency risk premium becomes
  \[ E_t(r^e_{t+1}) = \frac{\gamma^2 \sigma^2}{S^2} (s^*_t - s_t) \]
  Counter-cyclical currency risk premium (key feature 2)
Solution to UIP failure

- The expected change in exchange rates is linear in the interest rate differential

\[ E_t(\Delta q_{t+1}) = \gamma \left( \frac{1 - \phi}{B} \right) [r_t - r^*_t] \]

- When \( B \) is negative, we can get negative \( \beta \)
CALIBRATION
Calibration

- Two countries (US and UK) share the same set of parameters
- Their endowment shocks are correlated
- 10,000 endowment shocks
- Assume $B$ is negative
# Calibration - result

<table>
<thead>
<tr>
<th>Simulation Results</th>
<th>Actual Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (%)</td>
<td>Std (%)</td>
</tr>
<tr>
<td>( \Delta c )</td>
<td>2.13</td>
</tr>
<tr>
<td>( r^f )</td>
<td>1.65</td>
</tr>
<tr>
<td>( \Delta q )</td>
<td>8.44</td>
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<tr>
<td>( pd )</td>
<td>344.17</td>
</tr>
<tr>
<td>( r^m )</td>
<td>5.63</td>
</tr>
<tr>
<td>( hpr^5 )</td>
<td>1.02</td>
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</table>

<table>
<thead>
<tr>
<th>Coef.</th>
<th>s.e.</th>
<th>Coef.</th>
<th>s.e.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_{\Delta q_t, \Delta c_t^+ - \Delta c_t} )</td>
<td>0.78</td>
<td>[0.01]</td>
<td>-0.04</td>
</tr>
<tr>
<td>( \rho_{r^m_{t+1} - r^f_t, pd_t} )</td>
<td>0.12</td>
<td>[0.01]</td>
<td>-0.14</td>
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<tr>
<td>( \rho_{r^m_{t+1} - r^f_t, r^f_t} )</td>
<td>-0.13</td>
<td>[0.01]</td>
<td>-0.03</td>
</tr>
<tr>
<td>( \alpha_{UIP} )</td>
<td>-0.99</td>
<td>[0.35]</td>
<td>-1.29</td>
</tr>
</tbody>
</table>
Real exchange rate is too volatile

- Consistent with the result of Brandt, Cochrane and Santa-Clara (2006)

\[ \sigma^2(\Delta q) = \sigma^2(m) + \sigma^2(m^*) - 2\rho(m, m^*)\sigma(m)\sigma(m^*) \]

- In order to get plausible equity premium, we need high variance of the SDF
- The correlation among consumption shocks across countries is low
- The variance of real exchange rates becomes high
Backus and Smith puzzle

- In complete market and with power utility, the correlation between the difference of consumption growth and changes of real exchange rate is equal to one (perfect correlation)

\[ \Delta q_{t+1} = -\gamma [\Delta c^*_t - \Delta c_{t+1}] \]

- In this model, correlation is not one, but still too great a correlation
Estimation

- Home country investor’s Euler equation
  \[ E_T[M_{t+1}R^e_{t+1}] = 0 \]

- Continuously-updating GMM

- Parameters: \( \gamma, \phi, \overline{S} \) (other structural parameters are fixed)

- 8 different sets of test assets consist of
  - Individual currencies
  - Currency portfolios (Lustig and Verdehan(2007))
  - 6 or 25 Fama French equity portfolios
  - CRSP VW stock market return
## Estimation Results

### Panel A: Individual Currencies

<table>
<thead>
<tr>
<th>Assets</th>
<th>8 C</th>
<th>8 C</th>
<th>8 C</th>
<th>8 P</th>
<th>8 P</th>
<th>8 P</th>
<th>6 FF +M</th>
<th>25 FF +M</th>
</tr>
</thead>
<tbody>
<tr>
<td>+6 FF +M</td>
<td>+25 FF +M</td>
<td>+6 FF +M</td>
<td>+25 FF +M</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( N )</td>
<td>16</td>
<td>23</td>
<td>42</td>
<td>8</td>
<td>15</td>
<td>34</td>
<td>7</td>
<td>26</td>
</tr>
<tr>
<td>( J )</td>
<td>10.38</td>
<td>22.60</td>
<td>57.93</td>
<td>3.43</td>
<td>14.79</td>
<td>3.43</td>
<td>12.20</td>
<td>38.56</td>
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<tr>
<td>( p )</td>
<td>0.66</td>
<td>0.31</td>
<td>0.03</td>
<td>0.63</td>
<td>0.25</td>
<td>0.63</td>
<td>0.02</td>
<td>0.02</td>
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<tr>
<td>( \gamma )</td>
<td>10.06</td>
<td>6.18</td>
<td>7.23</td>
<td>7.33</td>
<td>7.20</td>
<td>7.33</td>
<td>1.79</td>
<td>8.01</td>
</tr>
<tr>
<td>(0.09)</td>
<td>[2.60]</td>
<td>[1.27]</td>
<td>[1.80]</td>
<td>[0.75]</td>
<td>[1.04]</td>
<td>[0.75]</td>
<td>[3.18]</td>
<td>[1.41]</td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.98</td>
<td>0.95</td>
<td>0.99</td>
<td>0.81</td>
<td>0.98</td>
<td>0.81</td>
<td>0.89</td>
<td>0.97</td>
</tr>
<tr>
<td>(0.09)</td>
<td>[0.09]</td>
<td>[0.09]</td>
<td>[0.07]</td>
<td>[0.14]</td>
<td>[0.06]</td>
<td>[0.14]</td>
<td>[0.58]</td>
<td>[0.06]</td>
</tr>
<tr>
<td>( \bar{S} )</td>
<td>3.77</td>
<td>3.00</td>
<td>2.86</td>
<td>2.07</td>
<td>2.20</td>
<td>2.07</td>
<td>2.45</td>
<td>2.51</td>
</tr>
<tr>
<td>(0.42)</td>
<td>[0.43]</td>
<td>[0.05]</td>
<td>[0.56]</td>
<td>[0.25]</td>
<td>[0.56]</td>
<td>[4.45]</td>
<td>[0.11]</td>
<td></td>
</tr>
</tbody>
</table>

### Panel B: Currency Portfolios

### Panel C: Equity Portfolios
CONCLUSION
Conclusion

• Verdelhan (2009) proposes two-country, habit-based model which produces counter-cyclical risk premium and pro-cyclical real interest rate endogenously.

• In this model, implied UIP coefficient becomes negative.

• Estimation results support the negative UIP coefficient.