Econ 710: Advanced Microeconomic Theory I | Recitation #5
Fall 2007: Krishna/Petranka | September 28, 2007

1 Properties of the Demand Correspondence and the Indirect Utility Function - Informal

Note: Unless otherwise stated, we assume that $u(\cdot)$ is a continuous utility function representing a locally nonsatiated preference relation.

1.1 Marshallian Demand - $x(p, w)$

1.1.1 Properties of the Marshallian Demand

- Homogeneous of degree 0 in (p,w)
- Walras's law
- Convexity/uniqueness
- Empirically verifiable

1.1.2 What about the Law of Demand?

The law of demand states that demand and price move in opposite directions.

1.2 Indirect Utility Function - $v(p, w) = u\big(x(p, w)\big)$

1.2.1 Properties of the Indirect Utility Function

- Homogeneous of degree zero
- Strictly increasing in w and nonincreasing in p_l for any l
- Quasiconvex
- Continuous in p and w

2 Properties of the EMP and the Hicksian (Compensated) Demand Correspondence

2.1 EMP

2.2 Hicksian

2.2.1 Properties of the Hicksian Demand curve

- Homogeneous of degree zero in p
- No excess utility
- Convexity/uniqueness
- Empirically unverifiable

3 Relationships between concepts (MWG p. 75)

- $v(p, w) = u\big(x(p, w)\big)$
- $h(p, u) = x\big(p, e(p, u)\big)$
- $x(p, w) = h\big(p, v(p, w)\big)$
- $e(p, u) = e\big(p, v(p, w)\big)$
- $v(p, w) = v\big(p, e(p, u)\big)$
- Or, rewriting the last 2 lines,
    * $u = v(p, w)$
    * $w = e(p, u)$
- Or, rewriting another way,

    * $u = v\big(p, e(p,u)\big)$
    * $w = e\big(p, v(p,w)\big)$
- $h_i(p,u) = \frac{\partial e}{\partial p_i}$
- Roy's identity
- Slutsky equation

3.1 Example

Suppose the indirect utility function, V, equals:

$$V(p, I) = \frac{I^2}{4p_x p_y}$$

1. Find the Marshallian demands for good x and y.

2. Find the expenditure function.

3. Find the Hicksian demands.

4 Linear Algebra

4.1 Inner Products

I'm sure you're all used to seeing the inner product as the dot product. ie:

$$(p|x) = p_1x_1 + p_2x_2 + p_3x_3 \ldots$$

Technically, this is just one possible way to represent an inner product. The true definition of an inner product is:

An inner product on a vector space, V, over a field, K (which must be the real or complex numbers), is a function $(,) : VxV \rightarrow K$ *such that, for all* $k_1, k_2 \in K$ *and* $v_1, v_2, v, w \in V$*, the following properties hold:*

1. $(k_1v_1 + k_2v_2, w) = k_1(v_1, w) + k_2(v_2, w)$ (linearity)
2. $(v, w) = \overline{(w, v)}$ where $\overline{}$ denotes complex conjugation (complex symmetry)
3. $(v, v) \geq 0$ and $(v, v) = 0$ iff $v = 0$ (positive definite)

It is important to note that the dot product is just 1 example of a function with these properties. Technically, any function we construct with these properties is an inner product.

Example: Show that the dot product satisfies requirements 1 and 3 over the vector space, $\Re^n$.

4.2 Riesz Representation Theorem

For every continuous linear functional, f, on a Hilbert space, H, there is a unique $u \in H$ *such that* $f(x) = (x|u)$ *for all* $x \in H$

Recall the definition of a linear functional:

Let V and W be vector spaces over the same field, F. A linear functional is a function, $T : V \rightarrow W$ (where K is 1-dimensional) such that:

1. $T(v + w) = T(v) + T(w)\ \forall v, w \in V$
2. $T(\lambda v) = \lambda T(v)\ \forall v \in V,$ and $\lambda \in F$

Example: Show the Riesz representation theorem works with the dot product inner product over $\Re^n$

5 Hyperplanes

A hyperplane in $\Re^n$ is the $(n-1)$-dimensional affine subspace of $\Re^n$ formed by all the $n-$vectors that satisfy a linear equation in n unknowns. A vector $p \neq 0$ in $\Re^n$ and a scalar α define the hyperplane $H(p, \alpha)$ given by:

$$H(p, \alpha) = \{x = (x^1, \ldots, x^n) \in \Re^n; (p|x) = \sum_{i=1}^{n} p_i x^i = \alpha\}$$

Using this definition, notice that we can partition $\Re^n$ into two regions:

$$H^+(p, \alpha) = \{x = (x^1, \ldots, x^n) \in \Re^n; \sum_{i=1}^{n} p_i x^i \geq \alpha\}$$

$$H^-(p, \alpha) = \{x = (x^1, \ldots, x^n) \in \Re^n; \sum_{i=1}^{n} p_i x^i \leq \alpha\}$$

We say two sets are separated by a hyperplane if one lies in H^+ and one lies in H^-. i.e. we say a hyperplane, $H(p, \alpha)$ separates two sets X and Y in $\Re^n$ if, for all x in X and y in Y, we have $(p|x) \leq \alpha \leq (p|y)$.

A hyperplane $H(p, \alpha)$ is a *supporting hyperplane* for a set X if it contains a point on the boundary of X and the whole set lies on the same side of $H(p, \alpha)$. Equivalently, $H(p, \alpha)$ supports X if either $\alpha = \inf\{(p|x); x \in X\}$ or $\alpha = \sup\{(p|x); x \in X\}$

One interesting thing to notice about hyperplanes is that given any two vectors, x' and x'' in $H(p, \alpha)$, $(p|(x' - x'')) = (p|x') - (p|x'') = \alpha - \alpha = 0$. Therefore, p is orthogonal to any line segment in $H(p, \alpha)$.