Prove $\succsim$ complete & transitive $\iff$ $\succ$ asymmetric & negatively transitive

$\Rightarrow$

assume $\succsim$ complete & transitive

Recall $x \succ y$ iff $x \succsim y$ and $\neg(y \succsim x)$

$y \succ x$ not possible because this would imply $y \succsim x$ and $\neg(x \succsim y)$ which is the opposite as above.

Hence $x \succ y \iff \neg(y \succ x)$ so $\succ$ asymmetric

$x \succsim y$ and $y \succsim z \Rightarrow x \succsim z$

rewriting in terms of $\succ$ I have

$\neg(y \succ x)$ and $\neg(z \succ y) \Rightarrow \neg(z \succ x)$ so transitivity of $\succsim$ implies neg. trans. of $\succ$

$\Leftarrow$ assume $\succ$ asymmetric and negatively transitive.

$\succsim$ transitive by argument above.

$\neg(y \succ x)$ and $\neg(z \succ y) \Rightarrow \neg(z \succ x)$

rewriting I have $x \succsim y$ and $y \succsim z \Rightarrow x \succsim z$

need to show $\succsim$ complete

Suppose neither $x \succsim y$ nor $y \succsim x$ true for some $x, y \in X$

$\neg(x \succsim y) \Rightarrow y \succ x$

$\neg(y \succsim x) \Rightarrow x \succ y$ $\Rightarrow\Leftarrow$ since $\succ$ asymmetric