

Abstracts for the 3rd Annual Graduate Student Conference in Probability

May 1-3, 2009

hosted by
The Department of Statistics and Operations Research at UNC- Chapel Hill
and
The Department of Mathematics at Duke University

Friday, May 1st

9:30-9:50 am Ankit Gupta & Alexandra Chronopoulou

Ankit Gupta, University of Wisconsin- Madison

Two type stochastic model for concentration in yeast cell

We study the model proposed by Altschuler, Angenent and Wu for the dynamics of particles in a yeast cell. In this model there are two types of particles, A and B, which interact with each other. We are specifically interested in the clustering of particles on the membrane. Each cluster is called a clan. For any finite population size N , clan sizes follow a Markov Chain over the space of measures. We show that under suitable scaling these clan sizes follow a measure valued diffusion process in the infinite population limit. Moreover the ratio of A and B type particles converges to a constant for every clan. We also obtain the stationary distribution for clan sizes.

Alexandra Chronopoulou, Purdue University

Variations and Hurst index estimation for a Rosenblatt process using longer filters

Using Wiener chaos expansion and Malliavin calculus we study the behavior of the variations of the Rosenblatt process using longer filters. Since the Rosenblatt process is self-similar, has stationary increments, and exhibits long-range dependence, it is of great importance to estimate the parameter that describes this behavior: the Hurst/self-similarity index, H . Based on the quadratic variation of the filtered process we construct a class of strongly consistent estimators for H . Furthermore, we prove that the asymptotic distribution of the estimators is non-Gaussian with asymptotic variance that depends explicitly on the order and the length of the filter. We compare the numerical values of the asymptotic variances for various choices of filters, including finite-difference and wavelet-based filters. This is joint work with Ciprian Tudor (Sorbonne) and Frederi Viens (Purdue).

9:55-10:15 am Yaqin Feng & Cristina Tone

Yaqin Feng, University of North Carolina- Charlotte

Reaction-diffusion equations with extra parameters

We consider the time evolution of the particle field in \mathbb{R}^d . Assume that each particle together with its coordinates in the space \mathbb{R}^d has extra parameters (like age, mass, size, etc.). In the simplest model, this population can be described by non-linear KPP-type equation for the generating function of the number of particles in the fixed domain D and Laplace transform of the total masses in the same domain. From these equations one can derive the linear moment equations. The talk will present several limit theorems for the size of the population, its total mass for the “typical” particle. We will also discuss the intermittency phenomenon for the evolution in random media.

Cristina Tone, Indiana University

Central limit theorems for Hilbert-space valued random fields under a strong mixing condition

We want to study the asymptotic normality of the normalized partial sum of a Hilbert-space valued random field in the presence of the ρ' -mixing condition.

10:30-11:30 am **David Aldous: Keynote Address**

David Aldous, University of California- Berkeley

Spatial random networks

I will give an overview of ongoing research concerning networks linking random vertices in two-dimensional space, emphasizing two aspects.

(1) If we get to choose edges, subject to a given total length, then how efficient can we make the network in the sense of providing routes between vertices whose route-length is not much larger than straight-line distance, and what precise statistic is most useful for measuring this notion of “efficient”?

(2) There is a general class of “proximity graphs”, defined for arbitrary vertices, which are always connected. Applying to random points gives a class of random networks which are connected and have bounded mean degree. This class has scarcely been studied, but seems an appealing modeling alternative to the classical geometric random graphs for which one cannot have both connectivity and bounded mean degree.

1:00-1:20 pm Adina Oprisan & Dmytro Karabash

Adina Oprisan, University of Texas- Arlington

Large deviations for additive functionals of Markov processes

We study a class of empirical processes in $\mathcal{C}[0, \infty)$ associated with additive functionals of Markov processes that were shown to have a martingale decomposition representation. Based on a functional almost everywhere central limit theorem we obtain a large deviation result for these empirical processes.

Dmytro Karabash, Courant Institute of Mathematical Sciences

Brownian motion on manifolds with manifold time-space

Given two compact Riemannian manifolds M and N , the definition and interpretation of the Brownian manifold $B: M \rightarrow N$ will be given as an analogue of Lévy Brownian fields. The discussion of the case $M=N$, its uses, and open questions about this object will follow.

1:25-1:45 pm Arnab Ganguly & Georgios Fellouris

Arnab Ganguly, University of Wisconsin- Madison

Error analysis of the simulation method for a Jump Type Markov process

Here we try to deal with a special type of jump Markov process. These types of Markov processes arise in modeling system of chemical reactions where the state of the system, that is the vector of number of molecules of the reactant species, is given by some SDE, which we call the “exact equation.” There are algorithms to simulate from the exact equation but for a large system they are quite slow. So people have tried approximations to the exact equation and developed some faster simulation schemes called “The Tau Leap Method” and “The Midpoint Method.” I will discuss these two methods in my talk, and the main goal is to try to analyze the error arising from these approximations and also to compare the accuracy of The Tau Leap Method and The Midpoint Method.

Georgios Fellouris, Columbia University

Formulas for stopped Lévy processes at CUSUM stopping times

In this talk we discuss a general methodology for computing the expectation of the CUSUM-stopping time of a Lévy process, as well as the bivariate Laplace transform of this stopping time and the corresponding stopped process. For some special cases of Lévy processes we are able to obtain closed form expressions for these quantities.

1:50-2:30 pm Nathaniel Blair-Stahn & Mark Veillette

Nathaniel Blair-Stahn, University of Washington

Survival and limiting configurations in the two-type Richardson model, part 2

At last year’s conference I gave an expository talk about first passage percolation in connection with the two-type Richardson model, an interacting particle system simulating two species competing for space in two or more dimensions. This year I plan to summarize the important features of the two-type Richardson model and then describe in more detail some of the results I have proved about this process when the average growth rates of the two species are different. In particular, I will explain how to analyze the behavior of the process when one species initially occupies an infinite region, and how this analysis provides information about the growth of the process started from a finite initial configuration.

Mark Veillette, Boston University

First passage times of Lévy subordinators: moments and computation

Consider a non-decreasing Lévy process $\{D(s), s \geq 0\}$. The first-hitting time process $\{E(t) \ t \geq 0\}$ (which is sometimes referred to as an inverse subordinator) defined by $E(t) = \inf\{s : D(s) > t\}$ is a process which has arisen in many applications. The process E is, in general, non-Markovian with non-stationary and non-independent increments. In this talk, we give expressions for the Laplace transform of the mean function and the covariance function of E . In order to calculate these moments, one must invert a Laplace transform which depends on the characteristic exponent of the Lévy process D . We describe a numerical method for inverting this potentially complicated Laplace transform which is based on the Post-Widder inversion formula.

2:45-3:05 pm Ming Fang & Julius Esunge

Ming Fang, University of Minnesota

Fluctuations of branching random walks

It is known that the minimal displacement of the offspring in the n^{th} generation of branching random walks is asymptotically cn for some constant c . In this short talk, we will introduce a fluctuation property, i.e., the minimum of the maximal deviation of the paths of all walks from the line connecting 0 and cn . The fluctuation has order $n^{1/3}$. We will calculate a lower bound constant and an upper bound constant in the special case of binary branching and Gaussian walks, using moment methods and recursion derived from the tree structure. (We had hoped to get the same constant, but failed. Hence, this is incomplete work. Comments are very welcome!)

Julius Esunge, Louisiana State University

On evaluation points for stochastic integrals

Stochastic integrals have wide applications in many fields, particularly finance and economics. This talk will capture the essential role played by the choice of the evaluation point and how this makes for rich applications. We will touch on a recent idea of Ayed and Kuo, leading to the construction of a new type of stochastic processes.

3:10-3:30 pm Joseph Whitmeyer & Jian Song

Joseph Whitmeyer, University of North Carolina- Charlotte

Models of dissemination through pairwise contact

We present two Markov models of the spreading of rumors or, more generally, the dissemination of some attribute through random pairwise contact between particles. Specifically, we determine the limiting distribution as the population becomes large for the time to universal possession of the attribute by the population. For the first model, all particles are identical but one initially possesses the attribute. The limiting distribution is the convolution of two double exponential distributions. For the second model, there are two kinds of particles: active particles, which can spread the attribute, and passive particles, which can only acquire the attribute. Here, the limiting distribution is simply a double exponential distribution.

Jian Song, University of Kansas

Feynman-Kac formula for heat equation driven by fractional white noise

In this paper, we shall extend the Feynman-Kac formula for the heat equation with fractional noise

$$\begin{cases} \frac{\partial u}{\partial t} = \frac{1}{2} \frac{\partial^2 u}{\partial x^2} + u \frac{\partial^2 W}{\partial t \partial x} \\ u(0, x) = f(x) \end{cases}$$

where $W(t, x)$ is a fractional Brownian sheet with Hurst parameters H_1 in time and H_2 in space, respectively. The Feynman-Kac formula is

$$u(t, x) = \mathbb{E}^B \left[f(B_t^x) \exp \left(\int_0^t \int_{\mathbb{R}} \delta(B_{t-r}^x - y) W(dr, dy) \right) \right]$$

where B_t^x is a Brownian motion independent of W , \mathbb{E}^B denotes the expectation with respect to the Brownian motion B_t^x , and δ denotes the Dirac delta function.

3:35-4:15 pm Paul Varkey & Matt Cecil

Paul Varkey, University of Illinois- Chicago

What I believe about what you believe about what I believe, and so on ad infinitum, and, why it matters

In many strategic interactions, modeled usually in a game-theoretic framework, the players may be uncertain about the pertinent aspects of the game including the state of the world, other players' uncertainties, others' uncertainties about others' uncertainties, and so on. In principle, this uncertainty can be captured by maintaining beliefs (expressed as probability distributions) about the world and other players, beliefs about others' beliefs, beliefs about others' beliefs about others' beliefs, and so on. This *explicit* representation leads to an *infinite hierarchy* of beliefs, which is clearly unapplicable in practice. The goal, then, as outlined by Harsanyi is to construct a rich space of all possible (universal) *types* for the players' that induce these infinite hierarchies of beliefs *implicitly*. Most early constructions of universal type spaces were done under particular topological assumptions of the underlying state space. This was followed, more recently, by purely measure-theoretic constructions (and, hence, were *topology-free*). This talk will provide an overview of this enterprise (known in the community as the Harsanyi program) with an emphasis on the measure-theoretic construction of a universal type space. The talk will be adequately supported with examples that

illustrate the strategic usefulness of expressing infinite hierarchies of beliefs, in general, along with the particular advantages of using a measure-theoretic approach for this purpose.

Matt Cecil, University of Connecticut

Heat kernel measures on path and loop groups

A \mathbb{R}^d -valued Brownian motion can be shown to have a continuous version, and therefore defines a measure, Wiener measure, on the space of continuous paths starting at the origin. One could also use this method to define a measure on the path space on a connected Lie group G . We will describe different measures, heat kernel measures, which are the law of the endpoint of a path group-valued ‘Brownian motion.’ Here we consider the path and loop groups as infinite dimensional groups. Construction of these heat kernel measures for a general non-compact Lie group G is joint work with B. Driver.

4:30-5:30 pm **Russell Lyons: Keynote Address**

Russell Lyons, Indiana University

Asymptotic enumeration of spanning trees via traces and random walks

Methods of enumeration of spanning trees in a finite graph and relations to various areas of mathematics and physics have been investigated for more than 150 years. We will review the history and applications. Then we will give new formulas for the asymptotics of the number of spanning trees of large graphs. A special case answers a question of McKay (1983) for regular graphs. The general answer involves a quantity for infinite graphs that we call “tree entropy,” which we show is a logarithm of a normalized (Fuglede-Kadison) determinant of the graph Laplacian for infinite graphs. Proofs involve new traces and the theory of random walks.

Saturday, May 2nd

8:45-9:05 am Justin Grieves & Balaji Raman

Justin Grieves, University of Tennessee- Knoxville

Moderate deviation of intersection of ranges of random walks in the stable case

Physicists are interested in the shapes of polymer strands, which we can model using random walks. By measuring the number of intersections of a group of strands, we can get a good idea about whether the strands are condensed or spread apart. In this talk, we will discuss techniques and new results for calculating the large deviation of the intersection of ranges of random walks in the case where the walks are the domain of attraction for a stable process.

Balaji Raman, University of Connecticut

Volatility of Eurodollar futures and Gaussian HJM term structure models

The relation between Gaussian HJM term structure models and LIBOR futures is well documented (for instance Musiela and Rutkowski (2005)). Different models for Eurodollar futures price process can be obtained by varying the volatility coefficients of the underlying term structure model. Here we come up with the most suitable model for Eurodollar futures by matching the theoretical volatility pattern with the realized volatility. We focus on volatility coefficients whose respective integrated volatilities are bounded and deterministic. The choices of coefficients are determined by a plot of monthly quadratic variation for the available futures data. An n -dimensional version of Ho-Lee and Vasicek were amongst the coefficients considered. The realized volatility approach falls short when we consider a model for futures price with the underlying term structure being governed by a two-dimensional Brownian motion with each coefficient corresponding to a different volatility pattern. This led us to work with the traditional approach of maximum likelihood estimation. The MLE approach also yields similar results when we consider the above mentioned volatility coefficients. Further, this technique allows us to consider a model which can be treated as an alternative to the Ho-Lee constant volatility and the Vasicek exponent volatility when the underlying Brownian motion is of one-dimension. We call this a *Delta Model*.

9:10-9:30 am Philip Kilanowski/Marko Samara & Fangfang Wang

Philip Kilanowski/Marko Samara, The Ohio State University

A dynamical version of the Kratky-Porod model of semi-flexible polymers

The Kratky-Porod model of semi-flexible polymers in thermal equilibrium is understood in the chemical physics literature to be the continuum version of the discrete, freely-rotating chain. We begin by representing the freely rotating chain via a process of random rotations and rigorously prove that the chain, suitably re-scaled, converges to a process that can be described explicitly in terms of a spherical Brownian motion. This result gives a rigorous

mathematical definition of the Kratky-Porod model. The representation also allows us to describe the dynamics of the Kratky-Porod polymer in terms of a stochastic differential equation driven by an Ornstein-Uhlenbeck sheet. Using this equation we investigate the behavior of the polymer in both the weakly-bending and freely jointed limits.

Fangfang Wang, University of North Carolina- Chapel Hill

Statistical analysis of volatility component models

The volatility component models have received much attention recently, not only because of their ability to capture complex dynamics via a parsimonious parameter structure, but also because it is believed that they can handle well structural breaks or non-stationarities in asset price volatility. The talk studies the distributional properties of various volatility component models. Sufficient conditions for the existence and/or uniqueness of (strictly) stationary (ergodic) solutions with mixing property to the volatility component models are derived. Hence, the talk revisits the component models from a statistical perspective and attempts to explore the stationarity and mixing properties of the underlying processes. There is a clear need for such an analysis, since any discussion about non-stationarity presumes we know when component models are stationary. As it turns out, this is not the case and the purpose of the paper is to rectify this. We also look into the amplifying behavior of the maximum likelihood estimates of recently proposed volatility component models and establish their local consistency and asymptotic normality.

9:35-10:15 am Ivan Corwin & Johannes Ruf

Ivan Corwin, Courant Institute of Mathematical Sciences

Traffic jams, polymer growth, and random matrices

What happens when rush hour traffic comes up against the afternoon joy riders? How long will it take to get home? We answer these and related questions by considering a classical model for traffic known as the Totally Asymmetric Simple Exclusion Process (TASEP). We relate this two-sided density version to a directed polymer growth model known as Last Passage Percolation with two-sided boundary conditions. Considering the distribution of last passage times we demonstrate a connection to the largest eigenvalue distribution for perturbed Wishart (Covariance) Ensembles of Random Matrix Theory. We fully classify the types of current fluctuations which exist when two different traffic densities come into contact.

Johannes Ruf, Columbia University

Optimal trading strategies under arbitrage

Explicit formulas for optimal trading strategies in terms of minimal required initial capital are derived to replicate a given terminal wealth in a continuous-time Markovian context. To achieve this goal this talk does not assume the existence of an equivalent local martingale measure. Instead a new measure is constructed under which the dynamics of the stock price processes simplify. It is shown that delta hedging does not depend on the “no free lunch with vanishing risk” assumption. However, in the case of arbitrage the problem of finding an optimal strategy is directly linked to the non-uniqueness of the partial differential equation corresponding to the Black-Scholes equation. The recently often discussed phenomenon of “bubbles” is a special case of the setting in this talk.

10:30-11:30 am **Daniel Stroock: Keynote Address**

Daniel Stroock, Massachusetts Institute of Technology

Gaussian measures in infinite dimensions

I will begin by explaining why there is no “standard Gauss measure” on an infinite dimensional Hilbert space. I will then describe the Wiener-Segal-Gross school method of dealing with this fact. If time permits, I will close with an interesting result of Irving Segal’s which highlights out the tension which arises from the fact that, on the one hand, Gaussian measures would like to live on Hilbert spaces while, on the other hand, they cannot do so unless the Hilbert space is finite dimensional.

1:00-1:20 pm Aaron Smith & Xuemiao Hao

Aaron Smith, Stanford University

Markov chains on left-regular bands

It is fairly well known that random walks on groups have a much nicer structure than random walks in general, obtained by exploiting the algebraic structure. In this talk, I will discuss how to exploit a less well-known algebraic structure, the left-regular band, to obtain information about a different broad class of random walks, including the riffle shuffle, Tsetlin library, and Ehrenfests’ Urns. This is a primarily expository talk on the work of Aguiar, Bridigare, Brown, Diaconis, Hanlon, Mahajan and Rockmore, with a few recent observations and open questions towards the end.

Xuemiao Hao, University of Iowa

Asymptotic tail probability of the maximum exceedance over a renewal threshold and its application in insurance mathematics

Motivated by the observations that many problems in applied fields, including corporate finance, insurance risk, and production systems, can be reduced to the study of the maximum exceedance of a sequence of random variables over a renewal threshold, we derive a unified asymptotic formula for the tail probability of such a maximum exceedance for both light-tailed and heavy-tailed cases. We also consider an application of our result in the study of the ruin probability in a Lévy insurance risk model with taxation.

1:25-1:45 pm Russ Thompson & Helena Kauppila

Russ Thompson, Cornell University

Comparison theorems for random walks on quotients of finitely generated groups

We review Pittet and Saloff-Coste's result that the long time behavior of the probability of return for a random walk on a Cayley graph is a quasi-isometric invariant. Comparison theorems for random walks on finitely generated groups will also be covered, and we will present extensions of these results to quotients of finitely generated groups by subgroups of infinite index.

Helena Kauppila, Columbia University

Optimal consumption with investment in incomplete semimartingale markets

We study consumption and investment decisions in incomplete semimartingale markets. The agent's utility is modeled by a time-inhomogeneous felicity function and is of the type introduced by Hindy, Huang, and Kreps. These utilities are chosen because they satisfy important economic considerations such as intertemporal substitution. We use a stochastic representation result to arrive at an appropriate set of dual variables and apply minimax arguments to establish a relationship with the utility optimization problem and a related dual problem. The dual variables are no longer adapted processes, but rather processes with adapted densities. We present explicit solutions to the optimization problem in a complete market.

1:50-2:30 pm Péter Mester & Maxim Bichuch

Péter Mester, Indiana University

Percolation with two robust clusters

An example of a two dimensional invariant percolation is given which produces two infinite clusters both having critical probability less than 1. This is joint work with Olle Häggström and the content is part of what appeared in the recently published paper "Some two-dimensional finite energy percolation processes."

Maxim Bichuch, Carnegie Mellon University

An optimal portfolio of correlated futures with small transaction costs

We consider an agent who invests in two correlated futures contracts and a money market and consumes in order to maximize the utility of consumption over an infinite planning horizon in the presence of proportional transaction costs $\lambda > 0$. The utility function is of the form $U(c) = c^p/p$ for $0 < p < 1$. We provide a heuristic derivation and a rigorous viscosity solution argument of the asymptotic expansion of the value function in powers of $\lambda^{1/3}$.

2:45-3:05 pm Yunjiang Jiang & Hongzhong Zhang

Yunjiang Jiang, Stanford University

A new total variation distance bound on Kac Random Walk

We show that the classical Diaconis Saloff-Coste bound of $O(n^{2n})$ for the Kac walk on S^n starting at a fixed point on the sphere can be improved to $O(n^2 \log n)$ using conditioning and truncation, as well as the classical L^2 spectral gap bound on total variation distance. The method used shows that total variation distance is not significantly affected if the starting distribution does not have an L^2 density, hence the same approach should work for the other models, such as the 3-dimensional Kac collision model, or the nonuniform Kac walk.

Hongzhong Zhang, City University of New York

Drawdowns and drawups in a finite time horizon

In this work we derive the probability that a drawdown of a units precedes a drawup of equal units in a biased random walk model and its continuous equivalent, a drifted Brownian motion model in the presence of a finite time horizon. A drawdown is defined as the difference of the historical maximum and its present value, while a drawup is defined as the difference of the present value and its historical minimum. We then generalize our underlying process dynamics to

general diffusion processes and derive the Laplace transform of the probability that the drawdown precedes a drawup of *unequal* units. And in a Brownian motion model, we provide a probability density using analytic inversion of the Laplace transform. Finally, we discuss two main applications of these results in finance and the detection of transient signals. In particular, first the results are applied to the problem of determining the probability that there is a given percentage drop of an investor's wealth from its historical minimum preceding a specific percentage rise of his/her wealth from its historical minimum when the investor has a finite window of opportunity and the wealth process is a Geometric Brownian motion. Another application of interest is the detection and identification of two-sided alternatives in the drift of a Brownian motion process when each alternative represents the direction of the signal and the signal is only available up to a specific given moment in time.

3:10-3:30 pm Nicos Georgiou & Li Chen

Nicos Georgiou, University of Wisconsin- Madison
Soft edge results for longest increasing paths on the planar lattice

For two-dimensional last-passage time models of weakly increasing paths, interesting scaling limits have been proved for points close to the axis (the hard edge). For strictly increasing paths of Bernoulli(p) marked sites, the relevant boundary is the line $y = px$. We call this the soft edge to contrast it with the hard edge. We prove laws of large numbers for the maximal cardinality of a strictly increasing path in the rectangle $[p^{-1}n - xn^a] \times [n]$ as the parameters a and x vary. The results change qualitatively as a passes through the value $1/2$.

Li Chen, Oregon State University

The malfunction probability and surplus ruin probability for non-profit organizations- interesting models and open problems

The rate of consumption of non-profit organizations (NPOs) is a non-decreasing process since NPOs are expected to provide more good work during each time cycle. If they are able to cover the increasing spending on projects they promise, they are considered functioning. On the other hand, NPOs face many risks such as injuries to their employees, volunteers, and clients, property damage, etc. These unwelcome risks inevitably add to NPOs' consumption even though NPOs purchase insurances to reduce the cost of certain risks. They also invest in bonds and stocks, trying to raise money for their goals. However investments also open up a door for them to lose money. Assume that at time $T(0)$, a NPO starts to carry out its projects. Let $T(1)$ denote the deadline that the NPO has promised to complete the set of projects before a new series of projects are established. I define the term "malfunction probability" (this shares similarity with "malfunction" in the medical sense) as $P(t < T(n))$, where t is the time when the NPO runs out of its money and $T(n)$ is the deadline to implement the n^{th} set of projects. If the NPO invests in a risky asset whose price follows a geometric Brownian motion within the process of carrying out periodic projects, i.e. with the increasing consumption pattern, by examining the wealth and total funding size of the NPO, optimal investing strategies are discussed in order to minimize the "malfunction probability." If the NPO invests with the surplus from the previous time cycle $[T(n), T(n + 1)]$, the surplus ruin probability decays for the NPO with investment that has small volatility, assuming that the risky asset follows a geometric Brownian motion. My future work will include tax exemptions and risky relations with large corporate donors in the model.

3:35-4:15 pm Weijun Xu & Joshua Tokle

Weijun Xu, Harvard University
Eigenvalues for Wishart matrices

It is well known that the eigenvalue distribution for large hermitian matrices with independent identically distributed entries converges to a semi-circle, no matter what the original distribution is. This "central limit theorem" is also true for some matrices with dependent entries, e.g. the Wishart ensembles, but with a different limit distribution. I will introduce Pastur's proof for the case of Wishart matrices, and talk about my current work on improving this result to a short scale.

Joshua Tokle, University of Washington
Transition densities of symmetric α -stable processes

For $\alpha \in (0, 2)$, a symmetric α -stable process is a discontinuous Lévy process with heavy tails. Such processes are useful for modeling phenomena in physics and finance that share these properties. In the last year, much progress has been made in establishing sharp two-sided bounds for the transition densities—or heat kernels—of stable processes killed upon exiting domains. In this talk I'll survey the basic theory of stable processes (from the central limit theorem to the Dirichlet fractional Laplacian), and I'll give an account of this recent work on heat kernel estimates, including some new results.

4:30-5:30 pm **David Aldous: Remarks on Teaching**

David Aldous, University of California- Berkeley

Remarks on teaching an undergraduate “Probability in the Real World” course

Even if one’s own research in probability is just math theory, most of us believe that math probability does say some correct things about the real world. But what? What predictions of theory would you bet \$100 on? What evidence would stand up to cross-examination in a court of law? I’ll talk about my attempt to teach a “no spherical cows” course.

Sunday, May 3rd

8:45-9:05 am Ravi Srinivasan & Mykhaylo Shkolnikov

Ravi Srinivasan, Brown University

Complete integrability in Burgers turbulence

Burgers turbulence is a non-equilibrium model that appears in a number of probabilistic contexts, particularly in stochastic coalescence. We show that under certain conditions the statistical evolution takes the remarkably simple form of a Lax pair. This equation admits a highly non-trivial solution derived by Groeneboom in the 1980’s in connection with universality in nonparametric statistics. This is joint work with Govind Menon.

Mykhaylo Shkolnikov, Stanford University

Metastability in mean field models

The talk will give an outline of recent results on metastable behaviour in mean field models. More precisely, methods for computing the sharp asymptotics of metastable exit times for local minima of Gibbs measures of such models will be presented. Part of the talk will consist of new results obtained by the speaker in joint work with Amir Dembo.

9:10-9:30 am Ali Al-sharadqah & Matthew Turner

Ali Al-sharadqah, University of Alabama- Birmingham

Fitting circles to scattered data: parameter estimates have no moments

Fitting circle to observed points is one of the basic tasks in computer vision, pattern recognition, nuclear physics and others. We study a nonlinear regression problem of fitting a circle (or a circular arc) to scattered data. We present the most popular fitting methods (geometric fit, Kasa fit, Pratt fit, and Taubin fit). Under any standard assumptions on the statistical distribution of errors that are commonly adopted in the literature, we prove the center and the radius estimates have no moment for Pratt, Geometric, Taubin fits. We also discuss methodological implications of this fact.

Matthew Turner, University of Tennessee- Knoxville

Stochastic integration with respect to stable and tempered stable random measures on $\mathbb{R}_+ \times \mathbb{R}^d$

A (tempered) stable random measure on $\mathbb{R}_+ \times \mathbb{R}^d$ is a random measure Z where, for each $A \in \mathcal{B}_0(\mathbb{R}_+ \times \mathbb{R}^d)$, $Z(A)$ is an (tempered) α -stable random variable, $0 < \alpha < 2$. Stable processes have a desirable scaling property at the expense of infinite p -moments for $p \geq \alpha$, which can be remedied by tempering the tails of the distribution while preserving local scaling. The stochastic integral with respect to an α -stable and tempered α -stable random measure on $\mathbb{R}_+ \times \mathbb{R}^d$ is developed with applications in mind to SPDE’s with non-Gaussian noise depending upon time and space.

9:35-9:55 am Hailin Sang & Kunwoo Kim

Hailin Sang, University of Cincinnati

Variable bandwidth kernel density estimation with clipping procedures

It is shown that the McKay (1993) and Jones, McKay and Hu (1994) modifications of Abramson’s (1982) variable bandwidth kernel density estimator satisfies optimal asymptotic properties for estimating densities with four or six uniformly continuous derivatives, uniformly on bounded sets where the preliminary estimator of the density is bounded away from zero.

Kunwoo Kim, University of Illinois- Urbana-Champaign

Effect of friction on noise

Sticky Brownian motion behaves exactly like Brownian motion away from 0, but it spends *positive* time at 0. We first consider one- dimensional SDE’s where we can see a sticky behavior or a *delay* near 0. By *tuning* friction (drift

term) and noise, we can obtain sticky Brownian motion weakly. If time permits, we will consider a noisy mechanical system where we can see a delay.

10:00-10:40 am Arijit Chakrabarty & Sergio Almada

Arijit Chakrabarty, Cornell University

Effect of truncation on heavy-tailed models

There are a lot of situations where heavy-tailed models have proven to be a good fit, and at the same time there is a natural upper bound on the possible values, and hence the need to study truncated heavy-tailed models. This talk is on understanding such models. It turns out that depending upon the truncation level and the tail, one can differentiate between two regimes. To be specific, the difference between the two regimes shows up in the central limit behavior and the decay rate of large deviation probabilities. Time permitting, I shall also talk on the statistical problem of deciding the truncation regime from data.

Sergio Almada, Georgia Institute of Technology

A view towards heteroclinicity of a dynamical system perturbed by small noise

We give some extension of the exit of a domain problem for an Ito diffusion. The particular setup is motivated by the dynamics of an heteroclinic network. We require the use of a non-smooth change of variables, which lead us to use reversibility properties of diffusions. In order to deal with technicalities, some enlargement of filtration results will be used instead of the anticipative calculus approach.

10:55-11:15 am Meredith Burr & Chia Ying Lee

Meredith Burr, Tufts University

Weak convergence of stochastic integrals driven by continuous time random walks

Continuous time random walks (CTRWs) serve as models in many of the sciences and economics. This talk con-

siders sequences of CTRWs of the form $X^n(t) = \sum_{i=1}^{N_{nt}} \frac{\xi_i}{a_n}$ which are known to converge in distribution to a limit

$Y(t)$ in the space $D[0, \infty)$, of right continuous functions with left limits, endowed with the Skorohod topology. The ξ_i are assumed to be i.i.d. and independent of the i.i.d. inter-jump times τ_i determining the counting process $N_{nt} = \max\{m \geq 0 : \sum_{i=1}^m \tau_i \leq nt\}$. An interesting question is whether a stochastic integral driven by the scaled CTRW $X^n(t)$ converges in distribution to a stochastic integral driven by the limit process $Y(t)$. Following Kurtz and Protter, we call the sequence of processes X^n good if whenever $(H^n, X^n) \Rightarrow (H, Y)$, then there exists a filtration (\mathcal{F}_t) such that Y is an (\mathcal{F}_t) -semimartingale and $\int H^n dX^n \Rightarrow \int H dY$, where convergence is again in the Skorohod topology. For simplicity, the talk will show that the sequence of CTRWs X^n is good in the simplest case where the ξ_i are in the domain of attraction of an α -stable random variable, $\alpha \in (1, 2]$, and the inter-jump times τ_i are deterministic, i.e. $N_{nt} = \lfloor nt \rfloor$.

Chia Ying Lee, Brown University

Randomization of forcing in large systems of PDE for improvement of energy estimates

In many physical applications, one is interested in computing an energy associated with a system of linear PDE driven by localized forcing. For large systems, where direct computation is prohibitive, we study its related stochastic PDE, and use a change of basis method to estimate the energy to high accuracy, and with significant reduction in computational cost. We establish vast improvements for simple convection-diffusion equations, and also good improvements for the 2D wave equation.

11:20-12:00 pm Bobby Reiner & Jason Miller

Bobby Reiner, University of Michigan

Inference in the presence of Volterra noise

Volterra processes represent a rich class of continuous Gaussian random fields capable of displaying short and long memory properties and possessing varying degrees of roughness of paths. The classical Brownian motion and the more recently popularized fractional Brownian motion are two such processes. The focus of this work is on establishing existence and uniqueness of solutions to stochastic differential equations driven by Volterra processes, the development of inference for them, and establishing properties for the estimators that are derived. Since continuous observation of a process is not practical nor possible in many situations, estimation based on a more realistic discrete observation scheme is also discussed.

Jason Miller, Stanford University

Thick points of the Gaussian free field

Let $U \subseteq \mathbb{C}$ be a bounded domain with smooth boundary and let F be an instance of the continuum Gaussian free field on U with respect to the Dirichlet inner product $\int_U \nabla f(x) \cdot \nabla g(x) dx$. The set $T(a; U)$ of a -thick points of F consists of those $z \in U$ such that the average of F on a disk of radius r centered at z has growth $\sqrt{a/\pi} \log \frac{1}{r}$ as $r \rightarrow 0$. We show that for each $0 \leq a \leq 2$ the Hausdorff dimension of $T(a; U)$ is almost surely $2 - a$ and that with probability one $T(a; U)$ is empty when $a > 2$. Furthermore, we prove that $T(a; U)$ is invariant under conformal transformations in an appropriate sense. The notion of a thick point is connected to the Liouville quantum gravity measure with parameter γ given formally by $\Gamma(dz) = e^{\sqrt{2\pi}\gamma F(z)} dz$ considered by Duplantier and Sheffield. This is joint work with Xiaoyu Hu and Yuval Peres.

12:05-12:45 pm Ricardo Restrepo & Yun Xue

Ricardo Restrepo, Georgia Institute of Technology

Linear dependence of binary random vectors of fixed weight

We consider the evolution of a random collection of vectors in $(\mathbb{F}_2)^n$ of weight k . Our main interest is to establish the threshold of linear dependence in such a case. To catch up with the lower bound established by Calkin (2003), we compare the process of formation of linear dependences with the evolution of certain point process over labeled hypertrees.

Yun Xue, Michigan State University

Fractal and smoothness properties of space-time Gaussian models

Spatio-temporal models are now widely used for inference in statistics and many applied areas. In such contexts, interests are often in the fractal nature of the sample surfaces and in the rate of change of the spatial surface at a given location in a given direction. In this paper we apply the theory of Yaglom (1957) to construct a large class of space-time Gaussian models with stationary increments, establish bounds on the prediction errors and determine the smoothness properties and fractal properties of this class of Gaussian models. For the prediction error, we switch to consider the conditional variance, and use a Fourier transformation to finally get the lower bound. Our results can directly be applied to analyze the stationary space-time models introduced by Cressie and Huang (1999) and Stein (2005), respectively. (This talk is based on a joint paper with Professor Yimin Xiao.)

12:50-1:10 pm Cristina Canepa & Ruoting Gong

Cristina Canepa, Carnegie Mellon University

Space-time Poisson processes applied to default data

We analyze the defaults of different firms using a marked space-time Poisson process. Our approach is based on the similarities between the occurrences of earthquakes and occurrences of firm defaults in economy. Similarities are given by the fact that earthquakes take place in clusters, one big earthquake triggers other offsprings and the same thing happens in economy, when one big default triggers others. We use several techniques from statistical literature in seismology to default data. A modified version of the Epidemic-type Aftershock Sequence (ETAS) model introduced by Ogata (1998) incorporating time and magnitude is tested. Defaults in economy are clustered not only in time but also in other parameters which are to be defined/tested. For instance, it is obvious that when the state of economy is bad, there will be more defaults, so we take an index of the economy as one coordinate of the “space.” Another coordinate considered is some index of the market liquidity (e.g. 3 month interest rates). We model the occurrence rate density of defaults in “space” and time as the sum of *two terms*: one representing the *independent* or spontaneous default background rate (in a healthy market, the most inefficient firms should default; these can be seen as ‘beneficial’ defaults), and the *other term* representing the defaults *induced* by previous ones (triggered not only by their own inefficiency but also by the inefficiency of others; these can be seen as ‘bad’ defaults).

Ruoting Gong, Georgia Institute of Technology

Viscosity and Principal-Agent problem

We develop a stochastic control system from a continuous-time Principal-Agent model in which both the principal and the agent have imperfect information and different beliefs about the project. We attempt to optimize the agent’s utility function under the agent’s belief. Via the corresponding Hamilton-Jacobi-Bellman equation we prove that the value function is jointly continuous and satisfies the Dynamic Programming Principle. These properties directly lead to the conclusion that the value function is a viscosity solution of the HJB equation. Uniqueness is then also established.