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FACIAL EXPRESSION. *See* Nonverbal Communication.

FACTOR ANALYSIS refers to a broad family of multivariate linear models designed to examine the interrelations among a set of continuously distributed manifest variables as a function of a smaller set of unobserved latent factors. This technique is often used to examine the common influences believed to give rise to a set of observed measures (measurement structure) or to reduce a larger set of measures to a smaller set of linear composites for use in subsequent analysis (data reduction). Spearman first introduced factor analysis as a method for understanding the common influences thought to underlie multiple measures of intelligence (1904), and Thurstone extended Spearman's model with the development of the common factor model (1935).

Factor analysis is based on the premise that the measured variables are a linear additive function of the unobserved latent factors, and that these factors give rise to the pattern of observed correlations among the measures. The fundamental equation for the factor model is:

$$y_j = \lambda_{j1}\xi_1 + \lambda_{j2}\xi_2 + \ldots + \lambda_{jm}\xi_m + \varepsilon_j$$

where y_j represents the j^{th} of p continuously distributed variables measured on a sample of n independent subjects, λ_{jm} represents the factor loading (or partial regression coefficient) relating variable j to the m^{th} factor ξ_m, and ε_j represents the influence that is unique to variable y_j and is independent of all ξs and all other εs. The variance of the observed measure is thus an additive combination of the variance associated with the set of underlying factors (the communality) and the variance associated with the unique factor (the uniqueness). The uniqueness can be further broken down into specific variance (the systematic variance unique to that particular measure) and random error variance.

The above equation represents the structure of the measured variables as a function of the underlying latent factors. Alternatively, the correlational structure among the measured variables can be derived from the equation in matrix terms such that

$$R = \Lambda\Phi\Lambda' + \Theta$$

where R is the $p \times p$ symmetric correlation matrix of p-measured variables, Λ is the $p \times m$ matrix of factor loadings λ, Φ is the $m \times m$ symmetric correlation matrix of the latent factors, and Θ is the $p \times p$ diagonal matrix of unique variances ε. If Φ is a diagonal matrix of ones (an identity matrix), then the factors are orthogonal, or uncorrelated. If, instead, Φ is a symmetric correlation matrix, then the factors are oblique, or correlated. The matrix of factor loadings Λ is called the factor pattern matrix, and the matrix of zero-order correlations between the measures and the factors is called the factor structure matrix. The factor pattern and factor structure matrices are equal only if the factors are orthogonal; if the factors are oblique the factor pattern matrix is usually consulted for purposes of interpretation.

There are two related approaches to the factor model, component analysis and common factor analysis. Component analysis is not a true factor analytic model because the resulting components are direct linear combinations of the measured variables. However, it is often considered with the common factor model, given several shared similarities. In component analysis, it is assumed that all variables are measured without error and that all observed variance among the measures is available for factoring. Thus, Θ is set to zero and the full correlation matrix R_f (containing values of ones on the diagonal) is factored. In contrast, the common factor model assumes that the observed variables are measured with error and that less than the observed variance is available for factoring. Thus, Θ is not constrained to zero and the reduced correlation matrix R_r (containing values less than one on the diagonal) is factored. The diagonal elements of R_r are communality estimates and represent the proportion of variance of each measured variable shared with the set of underlying factors.

Basic Steps in Factor Analysis

Once an appropriate sample and set of measures have been obtained, there are generally four major decision points when using factor analytic models: method of factor extraction, number of factors to extract, communality estimation, and factor rotation.

Factor Extraction. One method of factor extraction used for both the component and the common factor model is the method of principal factors. Principal factors is based on the computation of the eigenvalues and vectors (or characteristic roots and vectors) of the correlation matrix to maximize the variance of each successively extracted factor. Principal factoring can be applied either to the full correlation matrix R_f that results in principal components analysis, or to the reduced correlation matrix R_r that results in principal axis factoring. A more recently developed and commonly used technique for factoring R_r is maximum likelihood estimation, in which model parameters are estimated with the highest likelihood of having produced the observed correlation matrix. Additional extraction techniques include minimum residual analysis, alpha factoring, and image analysis.

Number of Factors. The next step is to determine the number of factors to be extracted. The Kaiser-Guttman Rule suggests that the number of factors should correspond to the number of eigenvalues of the full correlation matrix R_f that exceed one, although this criteria is criticized given the tendency to overextract factors. The scree plot is a graph of each eigenvalue plotted in descending order, and the number of factors is determined at the point where an appreciable "bend" occurs in the plot. The maximum likelihood goodness-of-fit test evaluates the magnitude of the residuals that exist in R after extracting a given number of factors, and it tests whether additional factors are necessary to meaningfully reduce the size of the residuals. Finally, cross-validation can be used in which two independent samples with two sets of measures from the same domain are examined simultaneously to evaluate the optimal number of factors to extract from each sample. Determining the appropriate number of factors to extract is often thought to be the most important decision in factor analysis.

Communality Estimation. For the common factor model, communality estimates must be obtained for placement on the diagonal of the reduced correlation matrix R_r. Methods for estimating communalities typically utilize an initial estimate that may then be iteratively updated during the computation of the factor solution. Initial estimates include the largest correlation of each variable with all other variables and the squared multiple correlation of each measure with all other measures. The method used for communality estimation becomes less important given larger numbers of measured variables.

Factor Rotation. The factors extracted from R_r can be rotated to aid in substantive interpretation. Factor rotation is possible because, for any one factor solution that fits the data to a specific degree, there will exist an infinite number of equally good solutions, each represented by a different factor loading matrix. Rotations can be orthogonal (e.g., varimax, quartimax, and equimax), in which the rotated factors are uncorrelated, or oblique (e.g., promax, direct oblimun, and orthoblique), in which the rotated factors are correlated. The goal of factor rotation is to achieve Thurstone's simple structure (1935), and the selection of an appropriate rotation is usually based on theory and the interpretability of the resulting solution.

Factor Score Estimation. Once the factor analysis is completed, factor scores can be estimated that represent the scores that would have been observed for an individual if the latent factors could be measured directly. The common factor model is considered indeterminate because there are more model parameters estimated than pieces of information observed. Thus, factor scores cannot be computed directly (as is possible in component analysis), but must be estimated using methods such as the regression method, the Bartlett method, the Anderson-Rubin approach, or simple unit weighting. Factor scores can then be used in subsequent analysis, such as multiple regression or MANOVA.

Variants of the Factor Model

The models described above are often termed exploratory factor analysis (EFA) and are applied when strong theory is lacking and the observed data are freely explored in search of meaningful patterns among the observations. In contrast, confirmatory factor analysis (CFA) provides formal statistical tests of a priori hypotheses about the specific factor structure thought to underlie the set of observed measures (Jöreskog, 1969). Unlike the EFA model, in which all measured variables relate to all latent factors, the CFA model imposes explicit restrictions on the factor pattern matrix so that the measured variables relate with some (or usually just one) latent factors but do not relate with others. Test statistics and goodness-of-fit indices evaluate the adequacy of these imposed restrictions as a function of the observed sample data. One type of structural equation model is a CFA model in which one or more of the latent factors are specified as dependent measures in a system of regression equations.

There are many techniques related to the factor model that share the general goal of examining similarities among observations. Correspondence analysis is a nonlinear principal-components analysis of raw categorical response data, or co-occurrence matrices of paired preferences. Profile analysis is used when a set of measures is gathered from multiple groups of individuals and the similarity of profiles of means is examined across the groups. Cluster analysis classifies previously unclassified observations into discrete groups based on two or more distance or similarity measures among the observations. Multidimensional scaling graphically maps a set of observed distance or similarity measures between pairs of items onto one or more underlying dimension, and multidimensional unfolding is a method for scaling individual preferences among a set of ranked stimuli.

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FALSE MEMORY. False memories occur when people remember events differently from the way they happened or, in the most extreme case, remember events that never happened at all. These memory errors differ from the most studied types of memory errors, those involving forgetting. Forgetting is an error of omission, arising when a person tries to remember some bit of prior knowledge (a name, a fact, a previous occurrence), but it does not come to mind. False memories are errors of commission, because details, facts, or events come to mind, often vividly, but the remembrances fail to correspond to prior events.

The vicissitudes of memory were studied by European psychologists near the beginning of the twentieth century, especially by William Stern in Germany and Alfred Binet in France, but the first landmark study was by Frederic Bartlett, a British psychologist. In his famous book *Remembering* (1932), Bartlett emphasized the constructive nature of memory. He argued that recollection of experiences is a reconstructive process driven by schemas, or general organizational schemes. The basic idea is that specific experiences and their details may not be remembered, but overall themes are. When people try to recover memories, they are guided by these schemas or themes and fill in details that are consistent with the schemas, but that may actually be quite wrong.

A key concept to understanding false memories is recoding. People do not directly record experience as a faithful copy of the outside world, but rather recode it in terms of their knowledge of the world (their schemas). In one famous experiment, people looked at ambiguous shapes and were told to remember them. For two groups of people, the shapes were given different labels (such as, for one shape, either broom or rifle).