Incongruence Between the Statistical Theory and Substantive Application of Growth Mixture Models in Psychological Research

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Outline

Act I: Character Introduction
- The latent curve model
- The growth mixture model
- The prototypical empirical application

Act II: The Challenge
- Methodological concerns with applications of growth mixture models
- Theoretical concerns with applications of growth mixture models

Act III: The Dénouement
- Doing better science with and without growth mixture models.

The Latent Curve Model

Unconditional Model

Matrix expression: Marginal PDF:
\[ y_i = \Lambda \eta_i + \epsilon_i \]
\[ f(y_i) = \phi(\Lambda \alpha, \Lambda \Psi \Lambda' + \Theta) \]

Conditional Model

Matrix expression: Marginal PDF:
\[ y_i = \Lambda \eta_i + \epsilon_i \]
\[ f(y_i \mid x_i) = \phi(\Lambda \alpha + \Lambda \Gamma x_i, \Lambda \Psi \Lambda' + \Theta) \]

Key Assumptions

- iid random effects (exchangeability / single population)
- Normally distributed random effects and residuals
  - Implies marginal normality of repeated measures
- Properly specified mean and covariance structure
- Linear relationships between repeated measures and exogenous predictors
- Simple random sample
- Data missing at random
The Growth Mixture Model

- Elaborates the LCM by allowing latent classes, relaxing assumption of single population

### Unconditional Model

**Marginal PDF:**

\[ f(y_i) = \sum_{k=1}^{K} \pi_k \phi_k \left( \Lambda a_k, \Lambda \Psi \Lambda' + \Theta_k \right) \]

### Conditional Model

**Marginal PDF:**

\[ f(y_i | x_i) = \sum_{k=1}^{K} \pi_k \phi_k \left( \Lambda a_k + \Lambda \Gamma x_i, \Lambda \Psi \Lambda' + \Theta_k \right) \]

\[ \pi_k(x_i) = \frac{\exp(\alpha_{ik} + \gamma' x_i)}{\sum_{k=1}^{K} \exp(\alpha_{ik} + \gamma' x_i)} \]

### Variants on the Growth Mixture Model

**Identical functional form:**

\[ f(y_i) = \sum_{k=1}^{K} \pi_k \phi_k \left( \Lambda a_k, \Lambda \Psi \Lambda' + \Theta_k \right) \]

**Homogeneous class covariance matrices:**

\[ f(y_i) = \sum_{k=1}^{K} \pi_k \phi_k \left( \Lambda a_k, \Lambda \Psi \Lambda' + \Theta \right) \]

**No random effects (latent trajectory class analysis):**

\[ f(y_i) = \sum_{k=1}^{K} \pi_k \phi_k \left( \Lambda a_k, \Theta \right) \]

### Caveats to Assumptions

- Observed variables can be binary, ordinal or counts, but random effects must be normal.
- Complex samples can be handled given clustering information and sampling weights
- Some nonlinear effects can indeed be modeled
Applications of Growth Mixture Models

- Number of applications of growth mixture models is accelerating.
- An imperfect index: citation counts for


The Prototypical Application


- Content is aggressive/deviant behavior or substance use
- School-based saturation samples and convenience samples
- Median consent rate 75%, median attrition rate 25%
- Ad hoc measurement of outcome variable:
  - Ordinal item, sum or average of ordinal items, logged sum of counts
- Number of latent classes estimated by BIC: 2 to 6, mode of 4
- Latent classes directly interpreted as types and used to draw policy implications.

Tenability of Model Assumptions

<table>
<thead>
<tr>
<th>Assumptions of the Model</th>
<th>Typical Application</th>
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<tbody>
<tr>
<td><strong>Conditional normality</strong></td>
<td>clear floor effects, typically poor measurement, distributions likely to be skewed in any case.</td>
</tr>
<tr>
<td><strong>Properly specified model</strong></td>
<td>rarely evaluated for 1-class model</td>
</tr>
<tr>
<td><strong>Linearity of relationships</strong></td>
<td>never evaluated</td>
</tr>
<tr>
<td><strong>Simple random sample</strong></td>
<td>never random, nesting within school</td>
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<tr>
<td><strong>Missing at random</strong></td>
<td>Non-response, non-random attrition.</td>
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**When the Assumptions are Wrong**

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**Conditional Normality**

- A mixture of normals is necessarily non-normal (except in degenerate cases)

- A non-normal distribution does not necessarily arise from a mixture.

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**Conditional Normality**

- 500 Datafiles Generated From a Single Population Latent Curve Model (with N=200 or N=600 each)

- Three Distributional Conditions

- When marginal distributions were normal, minimum BIC occurred with 1 class 100% of the time

- When marginal distributions were nonnormal, minimum BIC occurred with 2 classes 100% of the time (spurious classes)

- Spurious latent classes served to approximate the nonnormal repeated measures via a normal mixture.
When the Assumptions are Wrong

Assumptions of the Model: Consequence if Wrong:

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Properly Specified Covariance Structure

- For an unconditional GMM, the implied aggregate covariance matrix for the repeated measures is:

\[
\Sigma(\pi, \theta) = \sum_{k=1}^{K} \sum_{k'=1}^{K} \pi_k \pi_{k'} (\Lambda_k a_k - \Lambda_{k'} a_{k'}) (\Lambda_{k'} a_k - \Lambda_{k'} a_{k'})^T + \sum_{k=1}^{K} \pi_k (\Psi_k' \Lambda_k' + \Theta_k)
\]

- Given this, the estimation of spurious latent classes can "compensate" for improper specification of the within-class structure.
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Linearity

- 500 Datafiles Generated From a Single Population Latent Curve Model (N=600 each)
- Exogenous variable nonlinearly predicts the intercept

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**Simple Random Sample**

- Most empirical applications include some nesting not taken account of either through fixed grouping variables or random effects.
- For a small number of groups, group differences in change over time may emerge as latent classes (i.e., omitted known grouping variable compensated for by a latent grouping variable).
- For a larger number of groups, latent classes may discretely approximate a continuous distribution of random effects.

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**Missing at Random**

- Most studies have lower than optimal consent rates and some attrition.
- Those not participating or dropping out may reside in particular regions of the population distribution (e.g., the “worst” cases in the upper tail).
- The observed distributions will then be distorted.
- GMMs fit to these observed distribution may not recover true latent class structure.

**Missing at Random: Non-Response**

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- GMMs fit to these observed distribution may not recover true latent class structure.
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The Methodological Challenge

- Aside from true population subgroups, latent classes can represent:
  - Non-normality
  - Misspecification of within-class model
  - Nonlinear effects
  - Complex sample
  - Non-response / Non-random attrition

  *Typically more than one of these will be present.*

Improving Methods

- Most of these issues cannot be fixed by changing the statistical model.
- What is needed:
  - Better measurement (interval level)
  - Diagnostics for checking conditional normality, detecting misspecification of the covariance structure, and visualizing potentially nonlinear relationships.
  - More rigorous sampling procedures
  - Sound methodology
  - More careful interpretation

Theoretical Concerns

- Even if methodology can be improved, theoretical reasons for skepticism remain.
The “Selling” of Growth Mixture Models

- Applied researchers are convinced that LCMs give them one trajectory while growth mixture models give them multiple trajectories.

Modeling Heterogeneity with the GMM

A typical study identifies and then predicts the latent classes.

- Worst case: assignment by modal probability then prediction.
- Best case: Prediction done in the model itself.

\[ \pi_{ik} = \frac{\exp(\alpha_{ik} + \gamma'_{ik} x_{ik})}{\sum_{k=1}^{K} \exp(\alpha_{ik} + \gamma'_{ik} x_{ik})} \]

Modeling Heterogeneity with the GMM

- If we predict only which of the four trajectories an individual belongs to, we limit ourselves to this taxonomy of four.

- But do we really believe these four trajectories represent a definitive taxonomy of heterogeneity in change over time?

Returning to the Latent Curve Model

- The conditional LCM can capture more heterogeneity in patterns of change.

**Conditional Model**

Level 1: \[ y_{li} = \eta_{li} + \eta_{2li} \lambda_{li} + \epsilon_{li} \]

Level 2: \[ \eta_{li} = \alpha_{1i} + \gamma_{1i} x_{li} + \gamma_{12i} x_{2i} + \gamma_{13i} x_{3i} + \zeta_{li} \]

\[ \eta_{2li} = \alpha_{2i} + \gamma_{21i} x_{li} + \gamma_{22i} x_{2i} + \gamma_{23i} x_{3i} + \sigma_{2i} \]

- If these two predictors are dichotomous, we obtain four trajectories, if continuous, we obtain an infinite number of trajectories.
**Comparison**

- The GMM gives us four discrete trajectory types to predict.
- The conditional LCM gives us a potentially infinite family of model-implied trajectories, of which four are plotted.

**Another option**

- One other possibility in the GMM is to predict both class membership and random variability within classes.
- However this partitions the effects of the predictors into within- and between-class portions, making interpretation difficult.
- In practice this is rarely done.
- When done, the results are usually interpreted poorly.

**Summary of Challenges**

- The chief methodological challenge to the application of GMMs is that one does not know what the latent classes represent.
- The chief theoretical challenge is that it isn’t clear that a finite set of trajectory classes well ever sufficiently capture heterogeneity in change in the population.

One must then ask, under what circumstances are growth mixture models scientifically useful?
What a Scientifically Useful Application Might Look Like

- Let’s consider the application of these models to a radically different kind of data that does not share the limitations of many psychological data sets.
  - World Bank data on average life expectancy within countries.
  - Initial hypothesis: two trajectory classes will emerge representing developed and developing nations, respectively.

Tenability of Model Assumptions

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<th>Life Expectancy Application</th>
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<td><strong>Conditional normality</strong></td>
<td>Dependent variable is measured on a ratio scale, no apparent floor or ceiling effects</td>
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<td><strong>Properly specified model</strong></td>
<td>Will begin with an unrestricted finite mixture</td>
</tr>
<tr>
<td><strong>Linearity of relationships</strong></td>
<td>Can use model diagnostics if this appears risky</td>
</tr>
<tr>
<td><strong>Simple random sample</strong></td>
<td>The biggest challenge: sample units likely spatially correlated, sample IS population</td>
</tr>
<tr>
<td><strong>Missing at random</strong></td>
<td>No, but too little to matter: 198 of 212 countries provided data (93% “consent” rate), 5% or less missing each year.</td>
</tr>
</tbody>
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A Look at the Data

![Average Life Expectancy](AverageLifeExpectancy.png)

How Many Latent Classes?

- Comparative fit of unrestricted multivariate normal mixture models with 1 to 5 classes:
How well do the models reproduce the data?

Choosing among models

- We expected to find two classes and indeed saw the greatest increase in model fit with the addition of the second class.
- The minimum BIC was obtained with 4 classes and this model subdivides high and low trajectories in a compelling way.

Who’s clustering where?

2 Class Model:
- Class 1: France, Sweden, USA, Lebanon, Nepal, Philippines...

Who’s clustering where?

2 Class Model:
- Class 1: France, Sweden, USA, Lebanon, Nepal, Philippines...
- Class 2: Iraq, Kenya, Estonia, Ireland, Haiti, Ethiopia...
Who’s clustering where?

2 Class Model:
- Class 1: France, Sweden, USA, Lebanon, Nepal, Philippines...
- Class 2: Iraq, Kenya, Estonia, Ireland, Haiti, Ethiopia...

This classification is absurd
Ordinarily, however, we would have no way of knowing this

Who’s clustering where?

4 Class Model:
- Class 1: Botswana, Burundi, Congo, Zambia, Kenya, Zimbabwe, Lesotho, Liberia...
- Class 2: Armenia, Belarus, Estonia, Lithuania, Latvia, Romania, Saudi Arabia, Portugal...
- Class 3: France, Sweden, Iceland, Japan, Singapore, United States...
Who’s clustering where?

4 Class Model:
• Class 1: Botswana, Burundi, Congo, Zambia, Kenya, Zimbabwe, Lesotho, Liberia...
• Class 2: Armenia, Belarus, Estonia, Lithuania, Latvia, Romania, Saudi Arabia, Portugal...
• Class 3: France, Sweden, Iceland, Japan, Singapore, United States...
• Class 4: Antigua, Panama, Argentina, Taiwan, Fiji, Paraguay, Kuwait, Czech Republic...

This classification is more sensible

How was this exercise scientifically useful?
• The exploratory mixture analysis showed that my naïve initial hypothesis was too simplistic.
• The class of countries experiencing decreases in life expectancy after 1987 shows a qualitative difference from the other class trajectories.
• Confidence in these results is bolstered by knowing what the latent classes are not representing (e.g., assumption violations).
• Nevertheless, an expert might say these results are still overly simplistic (e.g., 4 “types” of countries...)

What about the old way?
• With better theory, we can pursue confirmatory analyses.
• For this data, we might specify a conditional LCM:
  • Use status as developed nations, transitional economies, and sub-Saharan as time-invariant predictors.
  • Include GDP, conflict, and HIV prevalence as a time-varying predictors, possibly with lagged effects.
• Although a less complex statistical model, the LCM would likely capture and explain more heterogeneity in patterns of change than the GMM.
Conclusions

- The statistical theory behind GMMs is incongruent with most applications.
  - Methodological problems range from measurement to sampling to model specification.
  - At a theoretical level, GMMs lack verisimilitude.
- When methodological problems are minimized, GMMs can reveal unanticipated heterogeneity in patterns of change.
- Hypothesized heterogeneity in patterns of change is likely better evaluated with conditional LCMs.
- Indirect applications of mixtures also hold much promise.