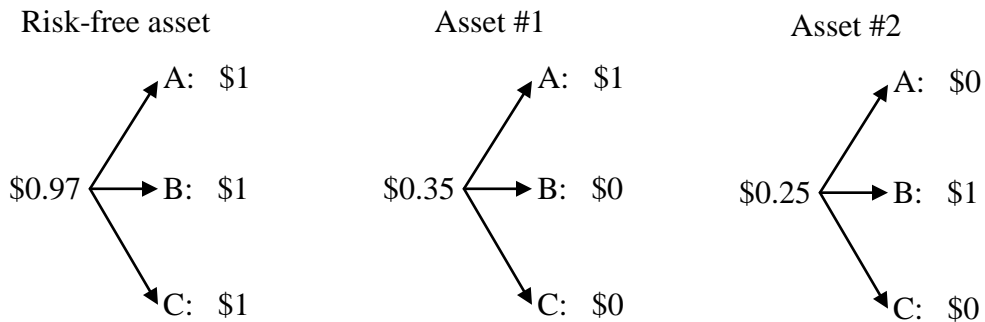
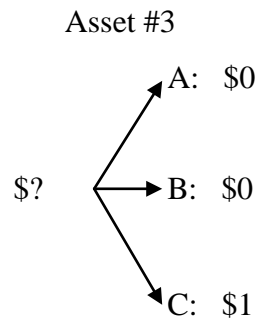


1. Assume that there are three future states of the economy. Let's label these states A, B and C. The risk-free asset pays \$1 in each of the three states and is currently selling for \$0.97. Asset #1 pays \$1 in state A and 0 in the other states. Asset #1 is currently selling for \$0.35. Asset #2 pays \$1 in state B and 0 in the other states and is currently selling for \$0.25. In diagrams, we have:



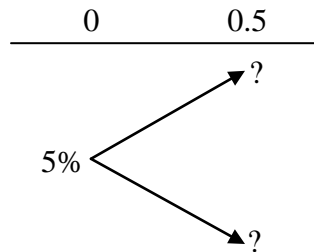
- a) What is the price of asset #3 that pays \$1 in state C and 0 in the other states?



- b) What are the *risk-neutral* probabilities of each of the states A, B and C?

2. Let's consider a lognormal model of interest rates with a semi-annual step size. The annualized drift from time 0 to time 0.5 is estimated to be 0.23. The standard deviation of monthly changes in log of 6-month interest rates is estimated to be 0.05. The current 6-month interest rate is 5% p.a.

- a. Build a 1-step binomial tree of 6-month interest rates.

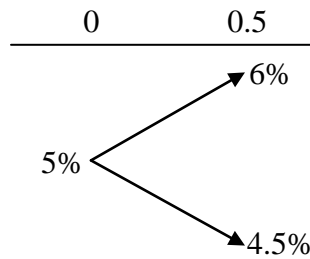


- b. Compute the per annum semi-annual compounding 1-year interest rate implied by the model

- c. Explain how you would estimate the drift m_2 from time 0.5 to time 1 if the 1.5-year interest rate is 7% p.a. semi-annual compounding.

3. Please list at least 2 strengths and 2 weaknesses of the lognormal model of interest rates. In your opinion, are these weaknesses serious? Explain briefly how these weaknesses can be overcome.

5. Consider the following binomial tree of risk-free 6-month interest rates where the risk-neutral probabilities of the 'up' and 'down' branches are 50%:



The implied 1-year semi-annual compounding risk-free interest rate from the above tree is 5.1236%.

- a) Consider a \$100 face, 10% semi-annual coupon, 1-year callable risk-free bond which can be called at time 0.5 for a call price of \$102. This bond has no default risk and liquidity risk. Based on the information given, what is the price of the bond?
- b) Consider **another** \$100 face, 10% semi-annual coupon, 1-year callable bond which can be called at time 0.5 only for a call price of \$102. If the bond's static spread is 0.34%, what price is it trading at?
- c) From your calculations in part a) and b) what is your best guess regarding the option-adjusted spread of the bond in part b)?

6. Interest rates are really low and the yield curve is currently flat at 1%. What is your best guess of the dollar duration of a \$100 face value, 1-year callable bond paying 10% semi-annual coupon rate, callable only at time 0.5 for a call price of \$102.