



UNC  
KENAN-FLAGLER  
BUSINESS SCHOOL

# Swaptions

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# Nov 30<sup>th</sup>?

## The existing schedule

- Nov. 30: Treasury Bond Futures
- Dec. 7: CDO Pricing
- Dec. 14: Review
- Note: Project 1 due Nov. 30. No Project 2. HW 3 due Dec 14.

## Alternative

- Nov. 30: Continuous time affine term structure models (optional)
- Dec. 7: Treasury Bond Futures and/or CDO Pricing
- Dec. 14: Complete Dec 7 lecture and Review





# CoCo's in the news

- **C**ontingent **C**onvertible Bonds are suggested as a possible solution for bank insolvency during financial crises.
- Convert automatically into equity when a bank's finances deteriorate below a predetermined level – such as capital ratio level.



# CoCo's in the news

The pricing of CoCo's shares something with typical single name CDS contract – a fixed leg and a contingent leg.

- On the fixed side, coupon until maturity or until capital ratio trigger.
- On the contingent side, switch to equity. The value of the contingent leg is the share value of distressed bank.

# 0 – Plan

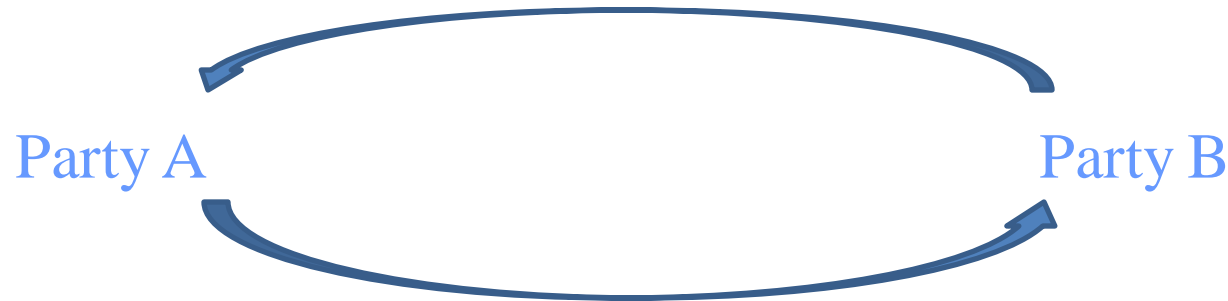


1. Review of Interest Rate Swaps
2. What is a swaption?
3. Why swaption?
4. The uses of swaption



# 1 – Review of Interest Rate Swaps

A receives a 2-yr bond  
paying 6%, face \$1000  
issued by B

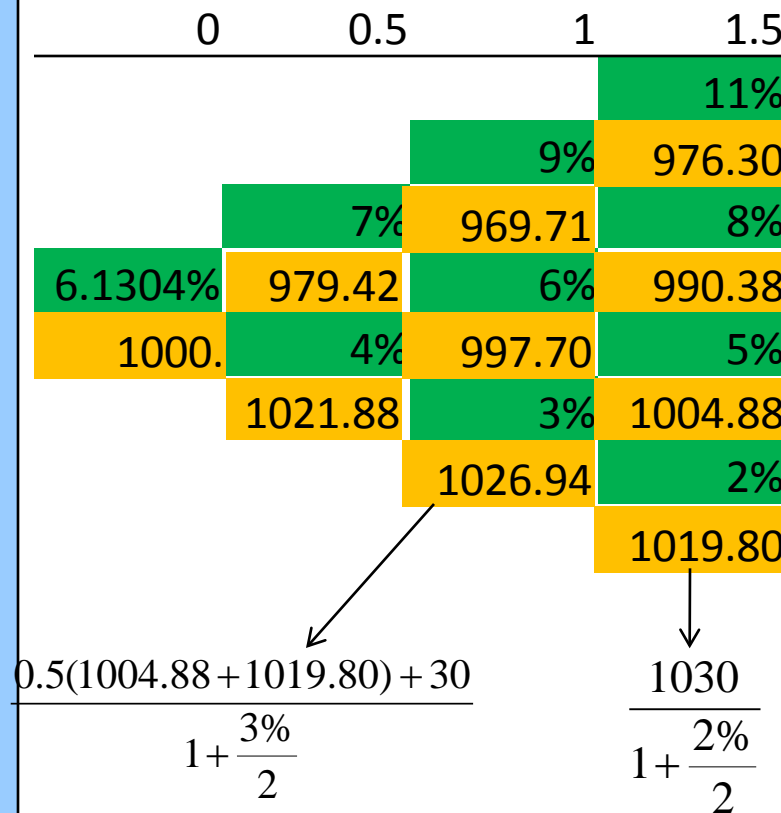


B receives a 2-yr FRN  
Paying 6-m LIBOR, face  
\$1000, issued by A

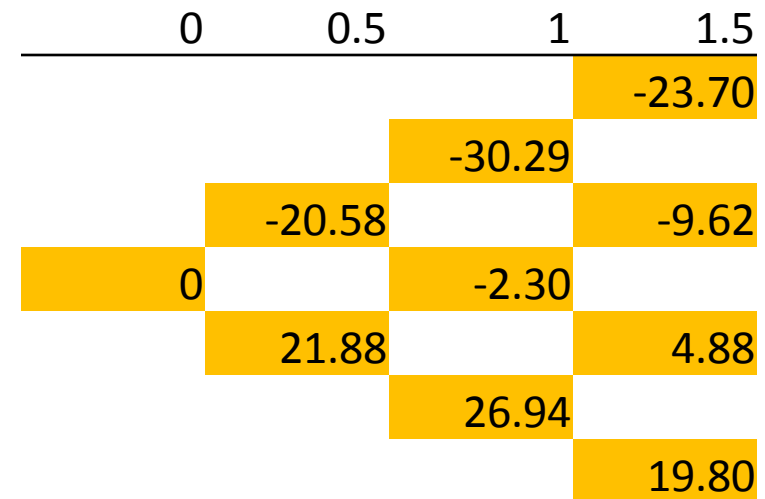


# 1 – Review of Interest Rate Swaps

- Within a binomial tree:



## Swap values





# Swaps

- A swap is an exchange of fixed interest rate payments for floating interest rate payments over some period of time.
- Fixed-rate payer (1) long position in a floating-rate bond and (2) short position in a fixed rate bond



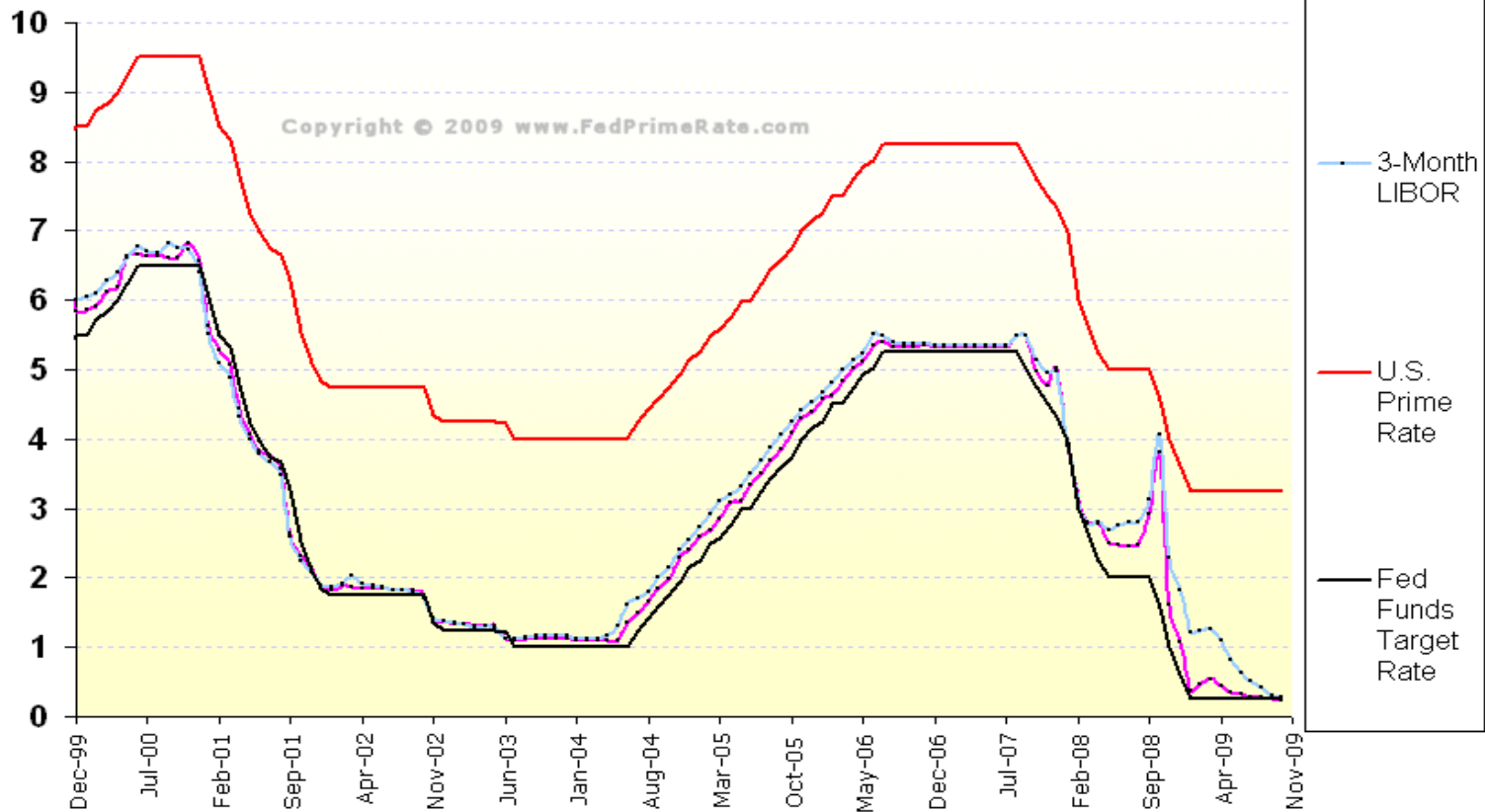
# 1 – Context

- At the beginning of 2004, speculating that the Fed will raise their target Fed funds rate, Sadi Ozelge, manager of Risk-neutral Ltd, decided to buy \$100 million face value of risk-free FRN paying semi-annual coupons
- Coupons of the FRN are tied to the 6-month Treasury rates



# 1-Context

**Fed Funds Target Rate vs. U.S. Prime Rate vs.  
1-Month LIBOR vs. 3-Month LIBOR,  
December 1, 1999 - October 30, 2009**





# 1 – Context

- 18 Sep 2007: the Fed reduced their Target by 50 basis points with possibility of further cuts
- This worried Sadi and he was looking for ways to hedge the risk of rates declining for at least the next 2 years
- What can he do?



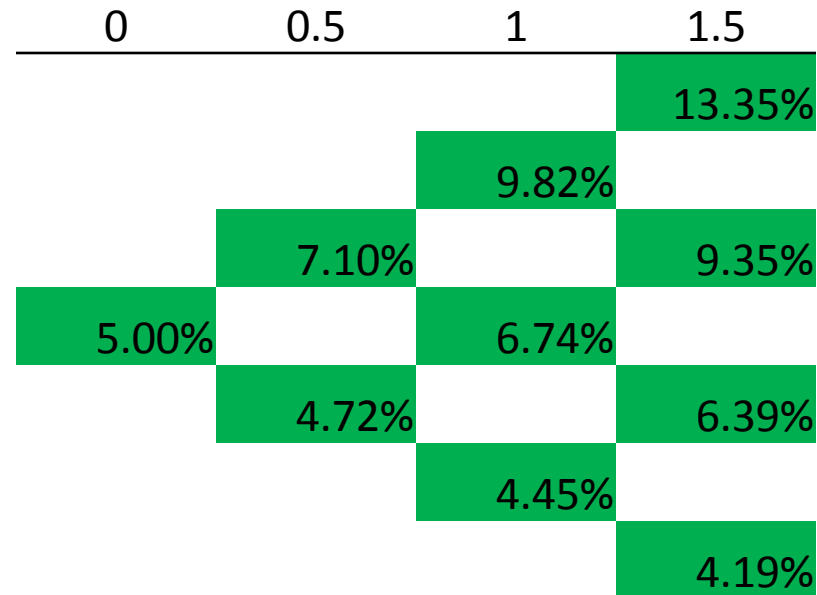
# Swaptions (“swap options”)

- Option to enter into a swap at an “off-market” rate
- Types:
  - “payer”: right to enter into a pay fixed, receive floating swap
  - “receiver” right to enter into a receive fixed, pay floating swap
  - European, American, Bermudan (modified American) exercise style (European style exercise most common)



## 2 – Valuing a floor contract

- Given the following 6-month interest rate tree:



- How could we value the floorlets, assuming a strike rate of 4.75%?



# First and Second Floorlets

- The first floorlet: The first floorlet protects the first coupon payment, 6 months from today. Note that 5.0% is higher than the floor rate of 4.75%. This means the value of the first floorlet is simply zero.
- The second floorlet: The second floorlet protects the second coupon payment, 1 year from today which is function of the 6-month interest rate 6 months from today.



## Second Floorlet (cont)

- From the interest rate tree, we know that the interest rate could be either 7.10% in which case there will be no payment from the floor seller; or 4.72% which would trigger the floor contract since 4.72% is smaller than the floor rate of 4.75%.
- In such a scenario, the floor seller would have to make a payment equal to  $0.5 \times \$100 \text{ million} \times (4.75\% - 4.72\%) = \$15,000$ .



- Note that the \$15,000 payment is due at the end of time 1. The PV of payment at the end of time 0.5:  $15,000 / (1 + 4.72\% \times 0.5) = 14,654.16$
- Now take a step back to time 0. With 50% chance (risk-neutral) that floor will be worth nothing (interest 7.10%) and 50% chance that the floor contract will be worth \$14,654.16. PV is  $[.5 * (0 + 14,654.16) / (1 + 5\% / 2)] = 7184$



## 2 – Valuing a floor contract

- The total cost of the floor contract = sum of the costs of all the floorlets =  
 $0 + 7,184 + 35,156 + 32,030 = \$74,370$
- Sadi thinks that this is too expensive – how could he reduce the cost?
  1. ?
  2. ?
  3. ?



## 2 – Valuing a floor contract

- The total cost of the floor contract = sum of the costs of all the floorlets =  
 $0 + 7,184 + 35,156 + 32,030 = \$74,370$
- Sadi thinks that this is too expensive – how could he reduce the cost?
  1. Reduce maturity of floor contract from 2 years to 1.5
  2. Partial coverage of face value
  3. Reduce strike rate
  4. Sell cap contract to pay for floor contract



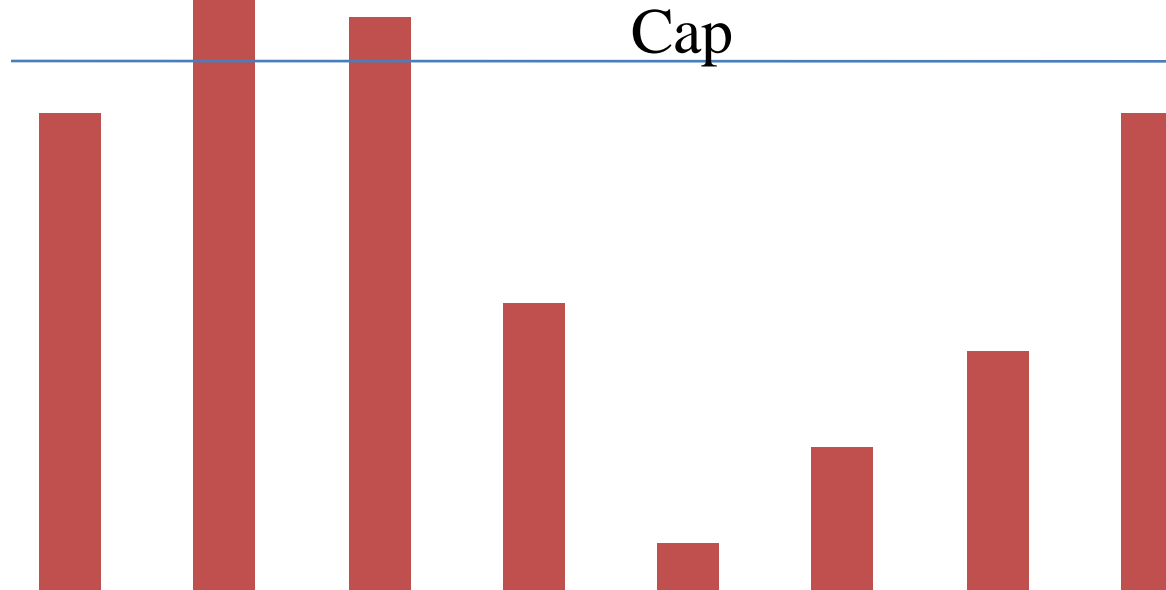
## 2 – Valuing a cap contract

- What is a cap contract?
  - By selling a cap contract, Sadi agrees to give the part of the coupon that is above a strike rate (or a cap rate, or a ceiling rate) to the buyer of the cap contract
  - The buyer of the cap contract can use it as an insurance against rising interest rates if he/she borrows floating

# 2 – Valuing a cap contract



Coupon payments from a FRN





## 2 – Valuing a cap contract

- Valuing a cap contract is very similar to valuing a floor contract
- Again, a 2-year semi-annual cap consists of 4 caplets – which can be valued separately
- What does the cost of the cap depend on intuitively?
  1. ?
  2. ?
  3. ?



## 2 – Valuing a cap contract

- Valuing a cap contract is very similar to valuing a floor contract
- Again, a 2-year semi-annual cap consists of 4 caplets – which can be valued separately
- What does the cost of the cap depend on intuitively?
  1. The maturity of the cap
  2. The strike rate
  3. Interest rate volatility



## 2 – Valuing a cap contract

- The fourth caplet of a 2-year semi-annual cap with a cap rate of 10%

	0	0.5	1	1.5	2
				0.133535	1.676728
			0.098237	1.571784	
		0.071006	0.749098	0.093508	0
	0.05	0.361707	0.067359	0	
0.176442	0.047186		0	0.063855	0
		0	0.044484	0	
			0	0.041888	0
				0	



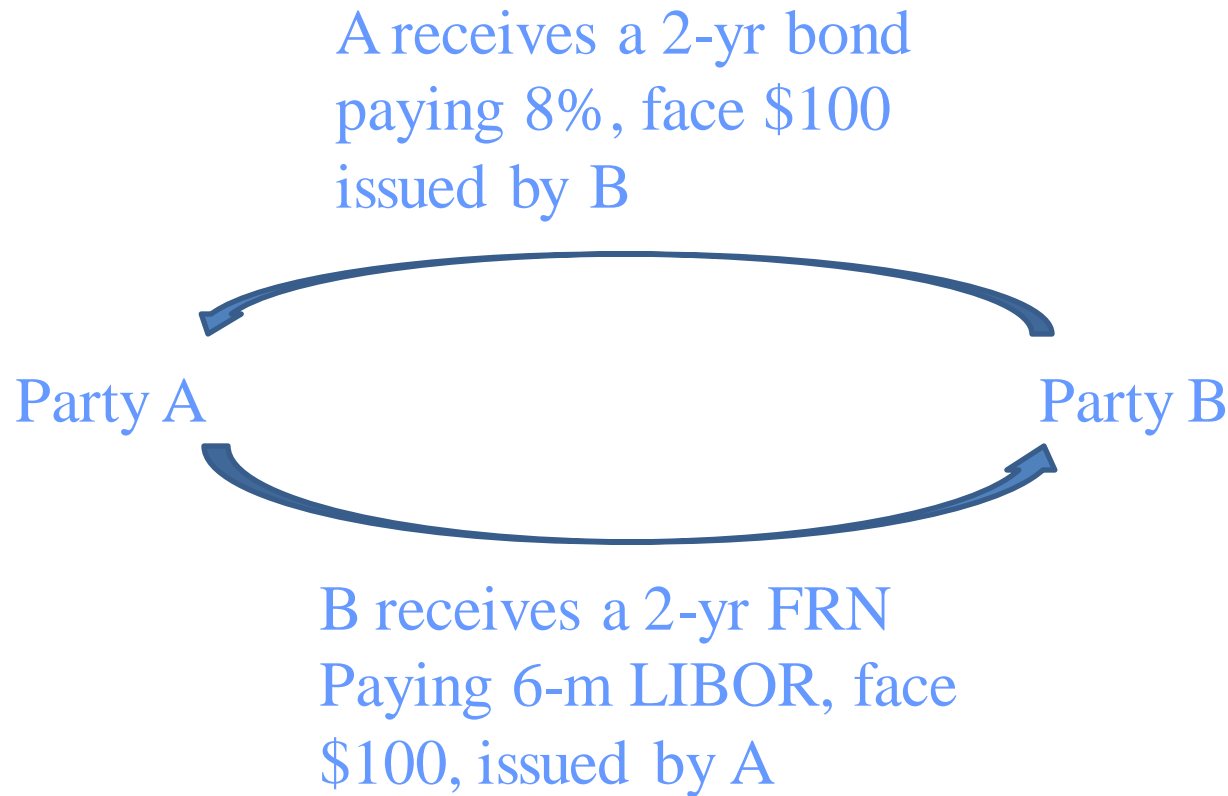
## 2 – Collar and reverse collar

- If we buy a floor and sell a cap like Sadi did, we enter into a reverse collar
  - A reverse collar limits the floating coupons we earn within certain bands
  - Depending on how we design the bands, the cost of the reverse collar can be 0 or even negative
- If we buy a cap and sell a floor, we enter into a collar
  - If we are borrowers, a collar limits the interest we pay within certain bands



## 2 – What is a swaption?

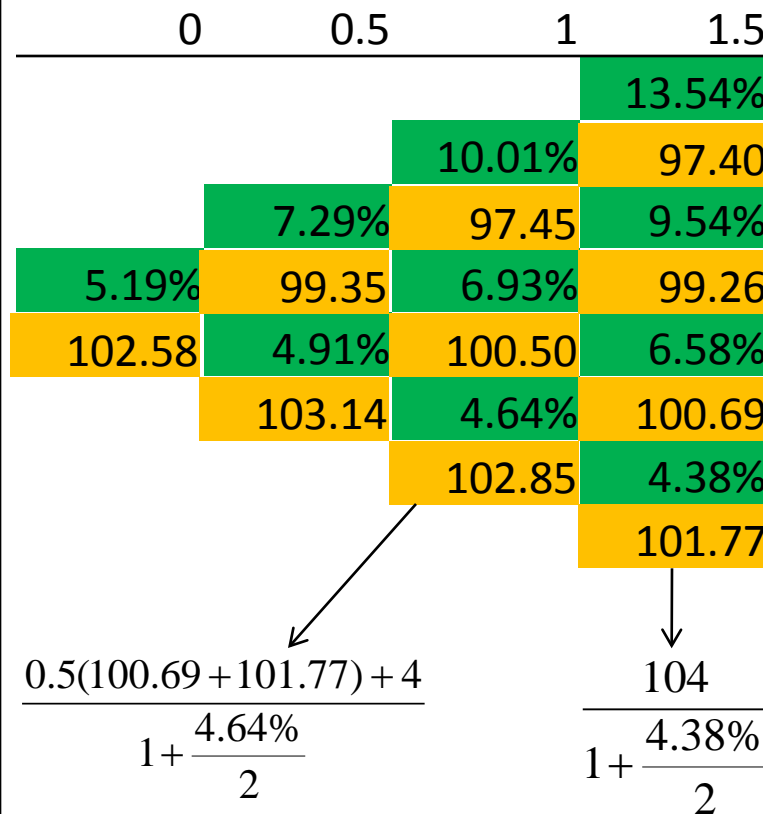
- Swaption = option to enter into a swap with a pre-specified fixed rate  $K$ 
  - Receiver swaption: option to enter into a swap to receive the fixed rate  $K$  (and pay the floating)
  - Payer swaption: option to enter into a swap to pay the fixed rate  $K$  (and receive the floating)
- You pay a premium for the swaption at time 0 and are not required to pay anything when entering the swap



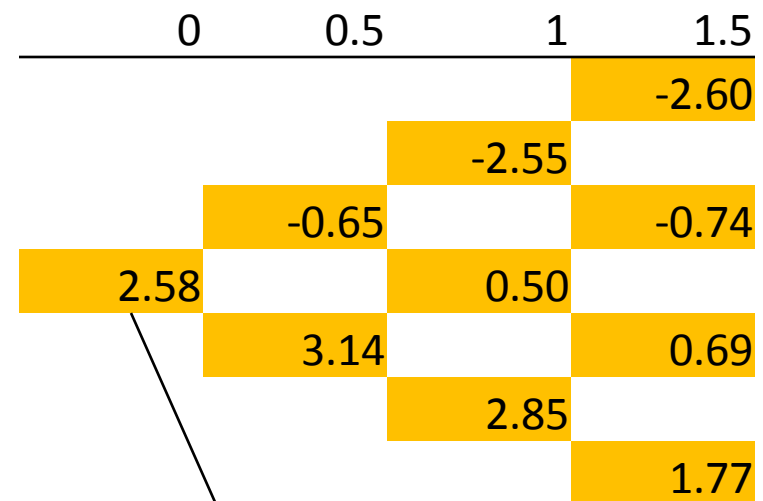
- The value of a swap = the value of the fixed rate bond - \$100.



- Within a binomial tree:



### Swap values

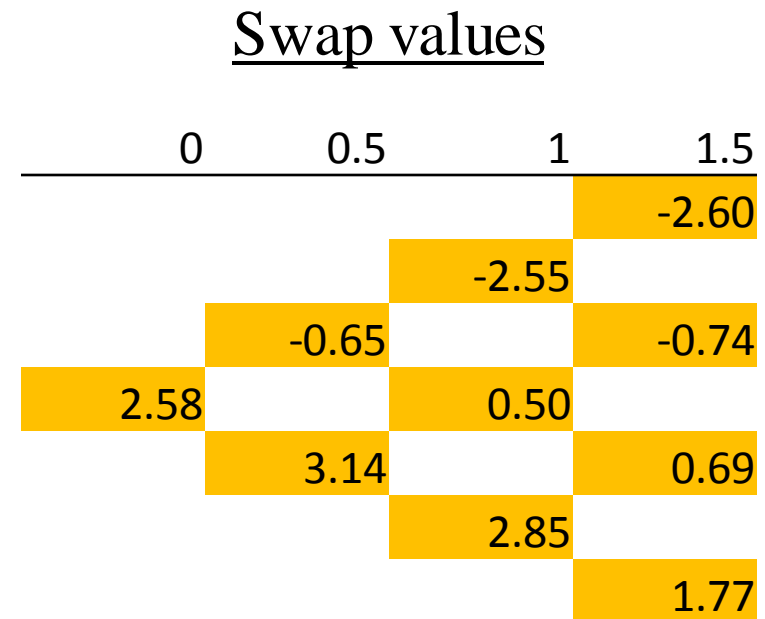
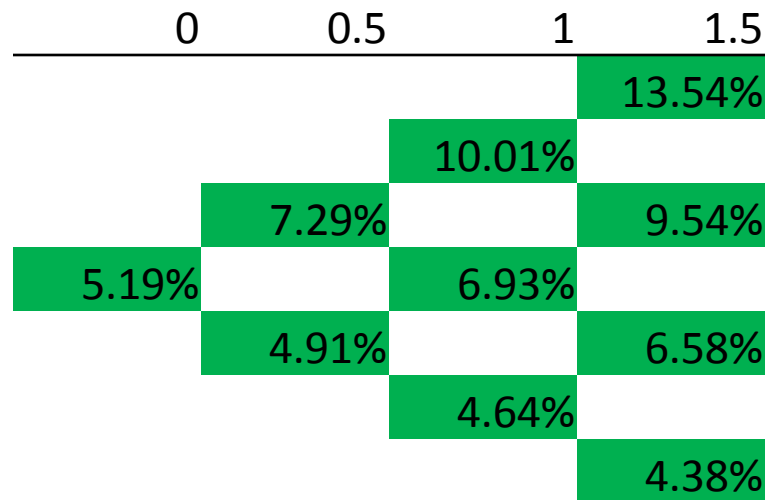


102.58 - 100



## 2 – Why swaption?

The swap values here are for fixed-receiver or fixed payer?



- Receiver swaption participates in the positive and stay away from negative values





## 3 – Swaption Valuation

- For simplicity, I have assumed that the maturity of the swaption is the same as the maturity of the swap (both 2 years)
- In general, the maturity of the swaption can be smaller than the maturity of the underlying swap
- How can we redo our previous valuation if the maturity of the swaption is 1 year?



## 3 – Swaption Valuation

- We've been dealing with American swaption → the option to enter the swap can be exercised anytime before the maturity of the swaption
- For European swaption → the option can only be exercised on the maturity date
- Bermudan swaption → the option can only be exercised on certain dates prior to maturity
- What are the effects on valuation?



## 4 – Uses of swaptions

Can you home-make:

- a callable bond from a non-callable bond and a swaption?
- A cancelable swap?



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Can you home-make:

- a callable bond from a non-callable bond and a swaption?
- A cancelable swap?