GMM ESTIMATION AND TESTING OF ASSET PRICING MODELS

9.0 INTRODUCTION

Suppose

$$m_{t+1} = \beta\left(C_{t+1} / C_t\right)^{-\gamma}$$

$$m_{t+1} = b' f_{t+1}$$

How do we estimate the parameters β and γ or the vector b?

Asset pricing model predicts:

$$E_t p_t = E_t m\left(data_{t+1}, parameters\right) x_{t+1}$$

This statement also holds unconditionally, so let us consider sample equivalent:

$$\frac{1}{T}\sum_{t=1}^{T} P_t = \frac{1}{T}\sum_{t=1}^{T}\left[m\left(data_{t+1}, par\right) x_{t+1}\right]$$

- GMM estimates the parameters by making these two sample means as close to each other as possible.
- GMM tests model by appraising how close sample means are.
- GMM versus MLE
  Basic references
  - Hansen (1982)
  - Hansen and Singleton (1982)
  - Hall (1992, 1999)
  - Ogaki (1992)
  - Matyas (1999)

9.1 GMM IN EXPLICIT DISCOUNT FACTOR MODELS

Approach popularized by Hansen and Singleton (1982).

Write discount factor as:

$$m_{t+1}(b)$$

For example:

$$m_{t+1}(b) = \beta(C_{t+1} / C_k)^{-\gamma}$$

$$b \equiv (\beta\gamma)'$$

Then

$$E_t p_t = E_t m_{t+1}(b) x_{t+1}$$
$$E_t\left(m_{t+1}(b) x_{t+1} - p_t\right) = 0$$

Define Errors

$$u_t(b) = m_{t+1}(b) x_{t+1} - p_t$$

Define Sample Mean

$$g_T(b) = \frac{1}{T}\sum_{t=1}^{T} u_t(b)$$

FORECAST ERRORS

$$u_{t+1} = m_{t+1} R_{t+1} - 1$$

Instrument orthogonal to u_{t+1}

$$EZ_t u_{t+1} = 0$$

Pricing Errors:

Consider Moment Conditions

$$g(b) = E[m_{t+1}(b) x_{t+1} - p_t]$$

$$E(m_{t+1}(b) x_{t+1}) - Ep_t$$

Is Pricing Error Diff. Model and Obs. Prices