GMM DIAGNOSTICS AND SPECIFICATION CALCULATIONS
Based on AP Chap. 11

- HORSE RACES
- TESTING MOMMENTS
- ESTIMATING ON ONE GROUP OF MOMENTS, TESTING ON ANOTHER

HORSE RACES

EXAMPLE: CHEN, ROLL & ROSS (1986) TEST WHETHER "MACROECONOMIC FACTORS" DRIVE OUT THE MARKET RETURN

$$m = b_1' f_1 + b_2' f_2$$

GIVEN FACTORS f_1, DO WE NEED $f_2 \Rightarrow b_2 = 0$

TWO WAYS TO TEST THIS

OTHER EXAMPLE MANAGED PORTFOLIO SUPERIOR TO PASSIVE BUY AND HOLD? MEAN-VARIANCE CONSTRUCTION

$m = a + b'R^w + b'_p R^P$

R^P : MANAGED PORTFOLIO

$H_0 : b_p = 0$

1. WALD TEST: $\hat{b}_2' \operatorname{var}\left(\hat{b}_2\right)^{-1} \hat{b}_2 \sim \chi^2_{\dim b_2}$

2. LR - TYPE TEST:

TJ_T (RESTRICTED) - TJ_T (UNRESTRICTED)

$\sim \chi^2$ (# RESTRICTIONS)

TESTING MOMENTS

$$g_T(b) = \begin{Bmatrix} g_{1T}(b) \\ \vdots \\ g_{KT}(b) \\ \vdots \\ g_{NT}(b) \end{Bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} g_{1T}(b) \\ \left.\begin{matrix} \\ \\ \end{matrix}\right\} \text{ADDITIONAL SET OF MOMENT CONDITIONS} \end{matrix}$$

ADDITIONAL SET OF MOMENT CONDITIONS

FOR INSTANCE PRICING ERRORS ON SMALL CAPS

$$H_0 : Eg\,(b) = 0$$

$$H_A : Eg_1\,(b) = 0, \;\; Eg\,(b) \neq 0$$

TEST STATISTIC

$$T g_T(\hat{b})' S^{-1} g_T(\hat{b}) - T g_{1T}(\hat{b}_1)' S_1^{-1} g_{1T}(\hat{b}_1)$$

$\sim \chi^2$ (# ELIMINATED MOMENT CONDITIONS)

PRESPECIFIED WEIGHTING MATRICES

INSTEAD OF (STATISTICALLY) OPTIMAL WEIGHTING MATRIX USE PRESPECIFIED WEIGHTING MATRIX

- ECONOMICALLY INTERESTING MOMENTS
- LEVEL PLAYING FIELD

OPTIMAL S MATRIX

CHANGES AS THE MODEL AND AS ITS PARAMETERS CHANGE

• COMPARISON OF MODELS NOT POSSIBLE VIA J-STATISTICS BECAUSE S MATRIX NOT THE SAME

NOTE: ONE CANNOT CLAIM BETTER FIT BECAUSE SMALLER J STATISTIC

- DIFFERENT WEIGHTING MATRIX
- DIFFERENT MOMENT CONDITIONS

⇒ USE OF WEIGHTING MATRIX THAT IS INVARIANT TO MODEL & PARAMETERS

- FOREGOING EFFICIENCY LIKE OLS VS GLS

- NEAR-SINGULAR S

S IS OFTEN NEAR-SINGULAR BECAUSE ASSET RETURNS ARE HIGHLY CORRELATED

ILLUSTRATIVE EXAMPLE

$$S = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$$

$$S^{-1} = \frac{1}{1-\rho^2} \begin{pmatrix} 1 & -\rho \\ -\rho & 1 \end{pmatrix}$$

CHOLESKI FACTORIZATION

$C'C = S^{-1}$

$$C = \begin{pmatrix} (1-\rho^2)^{-1/2} & -\rho(1-\rho^2)^{-1/2} \\ 0 & 1 \end{pmatrix}$$

THEN

$$\text{Min} \quad g_T' S^{-1} g_T$$

$$\text{Min} \quad (g_T' C')(C g_T)$$

Cg_T AS $\rho \to 1$ THEN FIRST ROW OF C BECOMES DOMINANT WITH TWO ELEMENTS WHICH ARE LARGE AND OF OPPOSITE SIGN

SOME PRESPECIFIED WEIGHTING MATRICES

HANSEN AND JAGANATHAN (1992)

ADVOCATE USING

$$W = E(xx')^{-1}$$

SECOND MOMENT MATRIX OF PAYOFFS

ECONOMICALLY INTERESTING DISTANCE MEASURE BETWEEN A "CANDIDATE DISCOUNT FACTOR" AND SPACE OF TRUE DISCOUNT FACTORS

DISTANCE BETWEEN y (MODEL FOR *m*) AND NEAREST VALID *m* IS SAME DISTANCE BETWEEN PROJ(y|$\underline{X}$) AND x^*

RECALL $\underline{X}$ IS SPACE OF ALL AVAILABLE PAYOFFS

$$x^* = \text{Proj}(m|\underline{X})$$

$$= p'(Exx')^{-1}x$$

$$\text{Proj}(y|\underline{X}) = E(yx')(Exx')^{-1}x$$

RECALL ALSO $\exists!\, x^* \in \underline{X}$:

$$p = E(x^* x) \qquad \forall x \in \underline{X}$$

$||\text{Proj}(y|\underline{X}) - x^*|| =$

$||E(yx') \, E(xx')^{-1}x - p' \, E(xx')^{-1}x|| =$

$||(E\,(yx') - p') \, E(xx')^{-1}x|| =$

$(E\,yx - p)' \, E(xx')^{-1} \, xx' \, E(xx')^{-1} \, x \, (E\,yx - p) =$

$(E\,yx - p)' \, (E\,(xx')^{-1} \, (E\,yx - p) = g_T' \, E(xx')^{-1} g_T$

SO MINIMIZING EUCLIDEAN DISTANCE BETWEEN x^* AND Proj(y|$\underline{X}$) IS EQUIVALENT TO MINIMIZING GMM CRITERION WITH MOMENT CONDITIONS $Eyx - p = 0$ AND $W = E(xx')^{-1}$

BECAUSE THE WEIGHTING MATRIX REMAINS INVARIANT, WE CAN COMPARE ASSET PRICING MODELS TO SEE HOW CLOSE THEY COME TO x^*

$$||\text{Proj}(y_1|\underline{X}) - x^*||$$

$$||\text{Proj}(y_2|\underline{X}) - x^*||$$

IMPORTANT NOTE: ONE HAS TO KEEP THE ASSET BASE X FIXED!

IDENTITY MATRIX

E(xx') IS ALSO OFTEN NEAR-SINGULAR WITH MANY ASSETS.

IDENTITY MATRIX AVOIDS (MOST) OF THESE PROBLEMS

ADVANTAGE OF Exx' IS THAT IT IS INVARIANT TO PORTFOLIO FORMATION

TAKE Ax INSTEAD OF x FOR NONSINGULAR MATRIX

A (NO INFORMATION IS LOST)

$$[E(yAx) - (Ap)]' E(Axx' A')^{-1} x [E(yAx) - Ap] =$$

$$[E(yx) - p]'A'(A')^{-1} E(xx')^{-1} A^{-1} x A[Eyx - p] =$$

$$[E(yx| - p]' E(xx')^{-1} [Eyx - p]$$

PROPERTY ALSO HOLDS FOR OPTIMAL S^{-1}

Hansen-Jagannathan Bounds
Based on AP Chap. 11 and CLM Chap 8

- Some structural models

Epstein-Zin-Weil

- Preferences that disentangle risk aversion and the elasticitity of IS

$$\frac{\Lambda_{t+1}}{\Lambda_t} = \left[\left(\frac{C^*_{t+1}}{C^*_t} \right)^{-\frac{1}{\psi}} \right]^{\theta} \left(R^m_{t+1} \right)^{\theta - 1}$$

where

$$\theta = \frac{1-\gamma}{1-1/\psi}$$

- With the same distributional assumptions

$$r^f_{t+1} = -\log\beta + \frac{1}{\psi} E_t \left(\triangle c_{t+1} \right) + \frac{\theta - 1}{2} \sigma^2_m - \frac{\theta}{2\psi^2} \sigma^2_c$$

Epstein-Zin-Weil

- High risk aversion does not need to imply a high rate: solve the risk-free rate puzzle
  - Unfortunately ψ is found to be small in experimental evidence

- Equity premium for risky assets including the market portfolio

$$E\left(r_{t+1}^{i} - r_{t+1}^{f}\right) + \frac{\sigma_i^2}{2} = \theta \frac{\sigma_{ic}}{\psi} + (1-\theta)\,\sigma_{im}$$

  the covariance σ_{im} may be high: solve the equity premium puzzle

  - However, the market-portfolio and consumption are linked through the budget constraint

Internal habit persistence

- Repeated exposure to a stimulus diminishes the response to it

- Common problem: high volatility of the risk-free rate

- Constantinides (1990) and Campbell and Cochrane:

$$U(C) = E_0 \left[\sum_{t=0}^{T} \beta \frac{(C_t - X_t)^{1-\gamma}}{1-\gamma} \right]$$

- Internal habit: X_t is a function of past own consumption

- Nests HARA utility with $\delta > 0$ and $\gamma > 0$ with relative risk aversion

$$\frac{\gamma}{1 - \delta / C}$$

Internal habit persistence

- It can solve the risk-free rate puzzle

$$r_{t+1}^{f} \simeq -\log\beta + \gamma E_t\left(\triangle c_{t+1}\right) - \left(ERA_t\right)^2 \frac{\sigma_c^2}{2}$$

- Skip the volatility problem by:
  - Constantinides assumes consumption is not $i.i.d.$
  - Campbell and Cochrane impose a highly nonlinear evolution of ERA_t which makes r_{t+1}^{f} constant

- Caveats:
  - It can not solve the equity premium puzzle
  - The way they get around the volatility problem is potentially counterfactual

External habit persistence

- Catching up with the Joneses

- Abel (1990):

$$U(C) = E_0 \left[\sum_{t=0}^{T} \beta \frac{(C_t / X_t)^{1-\gamma}}{1-\gamma} \right]$$

and

$$X_t = \left[C_{t-1}^{D} \overline{Y}_{t-1}^{1-D} \right]^{\alpha}$$

- It nexts both internal and external habit

Abel (1990)

TABLE 1—UNCONDITIONAL EXPECTED RETURNS
$\beta = 0.99$; $E\{x\} = 1.018$; $\text{VAR}\{x\} = (0.036)^2$

α	Stocks	Bills	Consols
A. Time-separable preferences ($\gamma = 0$)			
0.5	1.93	1.87	1.87
	[1.93]	[1.87]	[1.87]
1.0	2.83	2.70	2.70
	[2.83]	[2.70]	[2.70]
6.0	10.34	9.52	9.52
	[10.33]	[9.51]	[9.51]
10.0	14.22	12.85	12.85
	[14.13]	[12.72]	[12.72]
B. Relative consumption ($\gamma = 1$; $D = 0$)			
0.5	2.80	2.76	2.73
	[2.80]	[2.76]	[2.73]
1.0	2.83	2.70	2.70
	[2.83]	[2.70]	[2.70]
6.0	6.70	2.07	5.84
	[6.72]	[2.06]	[5.86]
10.0	14.73	1.59	13.16
	[14.95]	[1.55]	[13.32]
C. Habit formation ($\gamma = 1$; $D = 1$)			
0.86	33.56	4.53	35.25
0.94	6.83	3.48	7.44
1.00	2.83	2.70	2.70
1.06	8.43	1.93	7.40
1.14	38.28	0.93	35.16

Stylized facts: returns

- The average of the real return on the stock $(E(R_t^m))$ is usually higher than 5%. US 8.1%

- The average of the real risk-free rate is $(E\left(R_t^f\right))$ is usually lower than 3%. US 0.9%

- The average risk-premium is around 6%. US 7.2%

- The standard deviation of the stock return is usually higher than 17%. US 15%

- The standard deviation of the risk-free rate is usually lower than 2.5%. US 1.7%

Stylized facts: consumption and returns

- Average consumption growth is around 2.5%. US 2%
- Standard deviation of consumption growth lower than 2%. US 1.1%
- Very low autocorrelation
- The correlation between stock returns and consumption growth is around 0.23 for US data

Main problems of theoretical models

- The risk-premium they predict is too low for reasonable values of relative risk aversion: **the equity premium puzzle**

- The risk-free rate they predict is too high for reasonable values of the subjective discount factor and the elasticity of intertemporal substitution: **the risk-free rate puzzle**

A first pass at the equity premium

- In equilibrium with power utility we have that

$$E_t \left[\beta \left(\frac{\overline{Y}_{t+1}}{\overline{Y}_t} \right)^{-\gamma} R^m_{t+1} \right] = 1$$
$$E_t \left[\beta \left(\frac{\overline{Y}_{t+1}}{\overline{Y}_t} \right)^{-\gamma} R^f_{t+1} \right] = 1$$

  and any other return of course!

- Assuming joint lognormality of consumption and and returns and homoskedasticity

$$E \left(r^m_{t+1} - r^f_{t+1} \right) + \frac{\sigma^2_m}{2} = \gamma \sigma_{mc}$$

- Use data available to obtain $\widehat{\gamma} = 240$!!!!!

A first pass at the equity premium

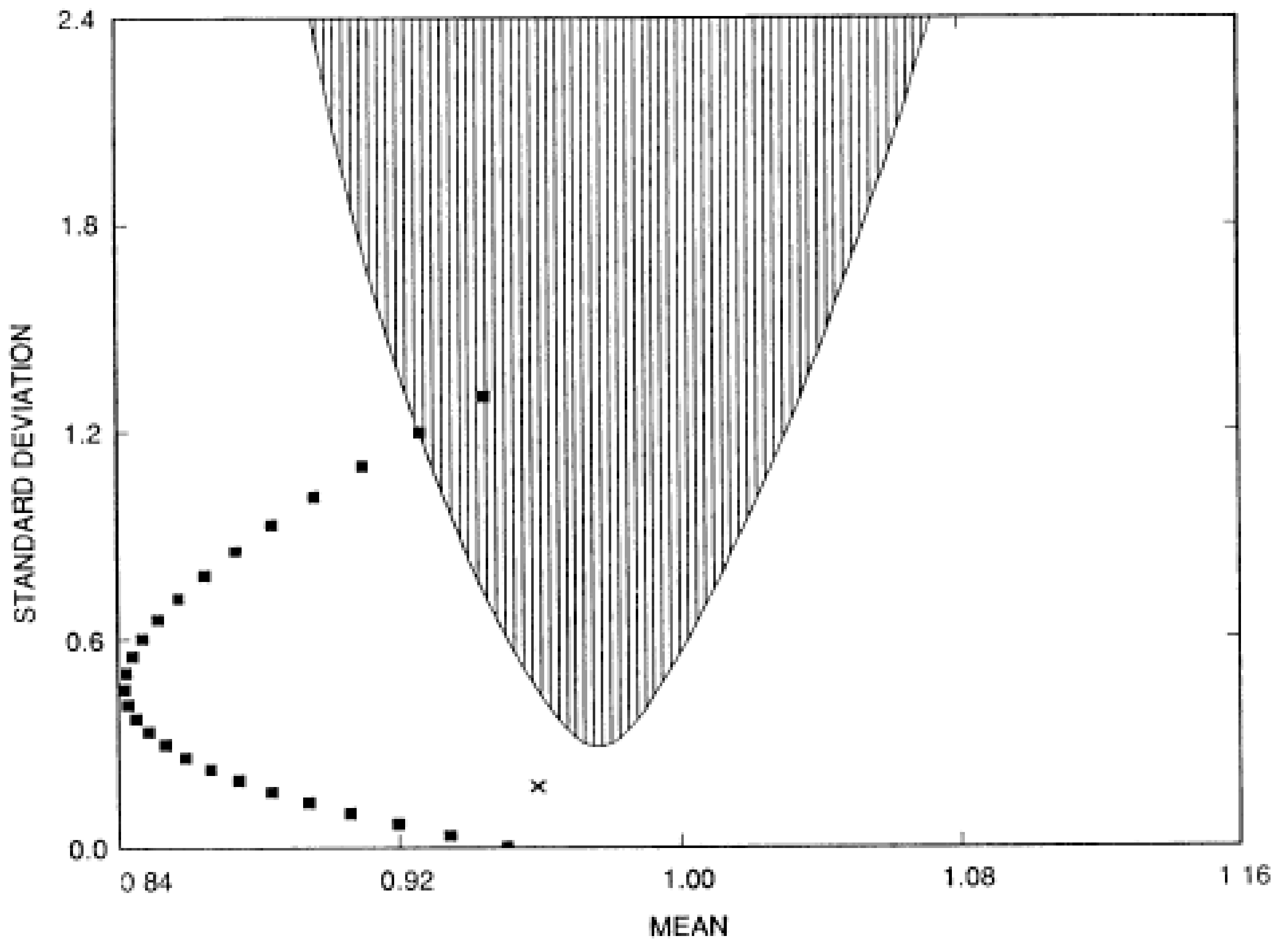
- The equity premium is too high to be razionalized with the standard model and reasonable (experimental evidence) γ

- What went wrong?

- – Is it consumption too smooth?

- – Is it the low correlation between consumption and stock returns?

Hansen and Jagannathan bounds



Low correlation

- Under the model assumptions and i.i.d consumption growth

  $$R^m_{t+1} = \alpha \frac{Y_{t+1}}{Y_t}$$

  which implies perfect correlation.

- Recall that

  $$E\left(r^m_{t+1} - r^f_{t+1}\right) + \frac{\sigma^2_m}{2} = \gamma\sigma_{mc}$$

  and for any other risky return

- With perfect correlation

  $$\sigma_{mc} = \sigma_m \sigma_c$$

  and $\widehat{\gamma} = 49$

Some quick answers

- Ritzs (1988) argues that a *peso* problem might explain the puzzle

- Problem:

  - It could potentially have an effect on both components of the equity premium

  - The puzzle is consistent accross countries: the extreme event should have been observed

- Kandel and Stambaug (1991) question experimental evidence and deny the puzzle

- Problem:

  - The risk-free rate puzzle

The risk-free

- From the FOC with the same distribution assumptions

$$r_{t+1}^{f} = -\log\beta + \gamma E_t\left(\triangle c_{t+1}\right) - \frac{\gamma^2\sigma_c^2}{2}$$

  - An increase in pacience lowers the rate

  - An increase in expected consumption growth lowers the rate. This effect depends on the elasticity of intertemporal substitution

  - An increase in variance lowers the rate. This *precaucionary effect* depends on risk aversion.

The risk-free rate puzzle

- For reasonble parameters of γ and β the implied rate is too high

  - For low γ the risk-free rate is too high (precaucionary effect is negligible and the risk-free rate is too high unless $\beta > 1$)

  - A high γ will lead to large variability of the risk-free rate

  - A reasonable value of the rate can only be obtained as a knife-edge case