GMM DIAGNOSTICS AND SPECIFICATION CALCULATIONS
Based on AP Chap. 11

- NESTED MODELS

- TESTING MOMMENTS

- ESTIMATING ON ONE GROUP OF MOMENTS, TESTING ON ANOTHER
- NON-NESTED MODELS

NESTED MODELS

EXAMPLE: CHEN, ROLL & ROSS (1986) TEST WHETHER "MACROECONOMIC FACTORS" DRIVE OUT THE MARKET RETURN

$$m = b_1' f_1 + b_2' f_2$$

GIVEN FACTORS f_1, DO WE NEED $f_2 \Rightarrow b_2 = 0$

OTHER EXAMPLE: MANAGED PORTFOLIO SUPERIOR TO PASSIVE BUY AND HOLD?

MEAN-VARIANCE CONSTRUCTION

$m = a + b'R^w + b'_p R^P$

R^P : MANAGED PORTFOLIO

$H_0 : b_p = 0$

FINAL ILLUSTRATIVE EXAMPLE: EPSTEIN-ZIN PREFERENCES AND CRRA PREFERENCES

Two Testing procedures

- 1. WALD TEST

$$\hat{b}_2' \operatorname{var}\left(\hat{b}_2\right)^{-1}\hat{b}_2 \sim \chi^2_{\dim b_2}$$

- 2. LR - TYPE TEST:

TJ_T(RESTRICTED) - TJ_T (UNRESTRICTED)
$\sim \chi 2$ (#RESTRICTIONS)

TESTING MOMENTS

$$g_T(b) = \begin{bmatrix} \left.\begin{matrix} g_{1T}(b) \\ \vdots \\ g_{KT}(b) \end{matrix}\right\} g_{1T}(b) \\ \left.\begin{matrix} \vdots \\ g_{NT}(b) \end{matrix}\right\} \text{ADDITIONAL SET OF MOMENT CONDITIONS} \end{bmatrix}$$

FOR INSTANCE PRICING ERRORS ON SMALL CAPS

$$H_0 : Eg\ (b) = 0$$

$$H_A : Eg_1\ (b) = 0,\ \ Eg\ (b) \neq 0$$

TEST STATISTIC

$$T g_T(\hat{b})' S^{-1} g_T(\hat{b}) - T g_{1T}(\hat{b}_1)' S_1^{-1} g_{1T}(\hat{b}_1)$$

$\sim \chi^2$ (# ELIMINATED MOMENT CONDITIONS)

NON-NESTED MODELS

MOST SDF MODELS ARE NON-NESTED

FORMAL GMM TESTS FOR NON-NESTED MODELS ARE VERY DIFFICULT TO IMPLEMENT, SEE SINGLETON (1985) AND GHYSELS AND HALL (1990)

PRESPECIFIED WEIGHTING MATRICES

INSTEAD OF (STATISTICALLY) OPTIMAL WEIGHTING MATRIX USE PRESPECIFIED WEIGHTING MATRIX

- ECONOMICALLY INTERESTING MOMENTS
- LEVEL PLAYING FIELD

OPTIMAL S MATRIX

CHANGES AS THE MODEL AND AS ITS PARAMETERS CHANGE

• COMPARISON OF MODELS NOT POSSIBLE VIA J-STATISTICS BECAUSE S MATRIX NOT THE SAME

NOTE: ONE CANNOT CLAIM BETTER FIT BECAUSE SMALLER J STATISTIC

- DIFFERENT WEIGHTING MATRIX
- DIFFERENT MOMENT CONDITIONS

⇒ USE OF WEIGHTING MATRIX THAT IS INVARIANT TO MODEL & PARAMETERS

- FOREGOING EFFICIENCY LIKE OLS VS GLS

- NEAR-SINGULAR S

  S IS OFTEN NEAR-SINGULAR BECAUSE ASSET RETURNS ARE HIGHLY CORRELATED

ILLUSTRATIVE EXAMPLE

$$S = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$$

$$S^{-1} = \frac{1}{1-\rho^2} \begin{pmatrix} 1 & -\rho \\ -\rho & 1 \end{pmatrix}$$

CHOLESKI FACTORIZATION

$C'C = S^{-1}$

$$C = \begin{pmatrix} (1-\rho^2)^{-1/2} & -\rho(1-\rho^2)^{-1/2} \\ 0 & 1 \end{pmatrix}$$

THEN

$$\text{Min} \quad g_T' S^{-1} g_T$$

$$\text{Min} \quad (g_T' C')(C g_T)$$

Cg_T AS $\rho \to 1$ THEN FIRST ROW OF C BECOMES DOMINANT WITH TWO ELEMENTS WHICH ARE LARGE AND OF OPPOSITE SIGN

SOME PRESPECIFIED WEIGHTING MATRICES

HANSEN AND JAGANATHAN (1992)

ADVOCATE USING

$$W = E(xx')^{-1}$$

SECOND MOMENT MATRIX OF PAYOFFS

ECONOMICALLY INTERESTING DISTANCE MEASURE BETWEEN A “CANDIDATE DISCOUNT FACTOR” AND SPACE OF TRUE DISCOUNT FACTORS

DISTANCE BETWEEN y (MODEL FOR m) AND NEAREST VALID m IS SAME DISTANCE BETWEEN PROJ(y|$\underline{X}$) AND x^*

RECALL $\underline{X}$ IS SPACE OF ALL AVAILABLE PAYOFFS

$$x^* = \text{Proj}(m|\underline{X})$$

$$= p'(Exx')^{-1}x$$

$$\text{Proj}(y|\underline{X}) = E(yx')(Exx')^{-1}x$$

RECALL ALSO $\exists!\, x^* \in \underline{X}$:

$$p = E(x^* x) \qquad \forall x \in \underline{X}$$

$$||\text{Proj}(y|\underline{X}) - x^*|| =$$

$$||E(yx')\, E(xx')^{-1}x - p'\, E(xx')^{-1}x|| =$$

$$||(E\,(yx') - p')\, E(xx')^{-1}x|| =$$

$$(E\, yx - p)'\, E(xx')^{-1}\, xx'\, E(xx')^{-1}\, x\, (E\, yx - p) =$$

$$(E\, yx - p)'\, (E\,(xx')^{-1}\, (E\, yx - p) = g_T'\, E(xx')^{-1} g_T$$

SO MINIMIZING EUCLIDEAN DISTANCE BETWEEN x^* AND Proj(y|$\underline{X}$) IS EQUIVALENT TO MINIMIZING GMM CRITERION WITH MOMENT CONDITIONS $Eyx - p = 0$ AND $W = E(xx')^{-1}$

BECAUSE THE WEIGHTING MATRIX REMAINS INVARIANT, WE CAN COMPARE ASSET PRICING MODELS TO SEE HOW CLOSE THEY COME TO x^*

$$\|\mathrm{Proj}(y_1|\underline{X}) - x^*\|$$

$$\|\mathrm{Proj}(y_2|\underline{X}) - x^*\|$$

IMPORTANT NOTE: ONE HAS TO KEEP THE ASSET BASE X FIXED!

IDENTITY MATRIX

E(xx') IS ALSO OFTEN NEAR-SINGULAR WITH MANY ASSETS.

IDENTITY MATRIX AVOIDS (MOST) OF THESE PROBLEMS

ADVANTAGE OF Exx' IS THAT IT IS INVARIANT TO PORTFOLIO FORMATION

TAKE Ax INSTEAD OF x FOR NONSINGULAR MATRIX

A (NO INFORMATION IS LOST)

$$[E(yAx) - (Ap)]' E(Axx' A')^{-1}x[E(yAx)- Ap] =$$

$$[E(yx) - p]'A'(A')^{-1}E(xx')^{-1}A^{-1}x A[Eyx - p] =$$

$$[E(yx| - p]' E(xx')^{-1}[Eyx - p]$$

PROPERTY ALSO HOLDS FOR OPTIMAL S^{-1}

Hansen-Jagannathan Bounds
Based on AP Chap. 11 and CLM Chap 8

- How to compare various models without observing the SDF.
- Recall the equations:

$$1 = E_t m_{t+1} R_{t+1}$$

$$1 = \left(E_t m_{t+1}\right)\left(E_t R_{t+1}\right) + Cov_t\left(m_{t+1}, R_{t+1}\right)$$

Let us fix the mean of $Em_{t+1} = M$ and let us consider the implied SDF $m_{t+1}(M)$ which we can write as:

$$m_{t+1}(M) = M + \left[R_{t+1} - E\left(R_{t+1}\right)\right]\beta_M$$

Therefore $\iota = ME\left(R_{t+1}\right) + Cov\left(R_{t+1}, m_{t+1}(M)\right)$

$$\iota = ME\left(R_{t+1}\right) + E\left(R_{t+1} - \mathrm{ER}_{t+1}\right)\left(m_{t+1}(M) - M\right)$$

$$\iota = ME\left(R_{t+1}\right) + E\left(R_{t+1} - \mathrm{ER}_{t+1}\right)\left(R_{t+1} - \mathrm{ER}_{t+1}\right)\beta_M$$

$$\Rightarrow \beta_M = \Omega^{-1}\left(\iota - \mathrm{ME}(\mathrm{R}_{t+1})\right)$$

$$\mathrm{Var}\left(\mathrm{m}_{\mathrm{t+1}}(M)\right) = \beta_M^{'}\Omega\beta_M$$

$$= \left[\Omega^{-1}\left(\iota\text{-ME}(\mathrm{R}_{\mathrm{t+1}})\right)\right]^{'}\Omega\left[\Omega^{-1}\left(\iota\text{-ME}(\mathrm{R}_{\mathrm{t+1}})\right)\right]$$

$$\mathrm{Var}\left(\mathrm{m}_{\mathrm{t+1}}(M)\right) = \left(\iota\text{-ME}(\mathrm{R}_{\mathrm{t+1}})\right)^{'}\Omega^{-1}\left(\iota\text{-ME}(\mathrm{R}_{\mathrm{t+1}})\right)$$