The canonical model: an economy à la Lucas

- One representative agent with standard preferences

$$U(C) = E_0 \left[\sum_{t=0}^{T} \beta^t u(C_t) \right]$$

- One single consumption good which is not storable

- One firm and one share: its dividends (output) $\overline{Y}_t$ and its price p_t^m

- All other assets in zero-net supply

Equilibrium

- FOC for optimality

$$E_t\left[\frac{u'\left(C_{t+1}^*\right)}{u'\left(C_t^*\right)}R_{t+1}^i\right]=1 \qquad \forall t,i$$

- Market-clearing:

$$\begin{aligned} d\left(w_t^*\right) &= \overline{Y}_t + p_t^m, \qquad \forall t \\ C_t^* &= \overline{Y}_t, \qquad \forall t \end{aligned}$$

- Optimality and market-clearing

$$E_t\left[\frac{u'\left(\overline{Y}_{t+1}\right)}{u'\left(\overline{Y}_t\right)}R_{t+1}^i\right]=1 \qquad \forall t,i$$

Main problems of theoretical models

- The risk-premium they predict is too low for reasonable values of relative risk aversion: **the equity premium puzzle**

- The risk-free rate they predict is too high for reasonable values of the subjective discount factor and the elasticity of intertemporal substitution: **the risk-free rate puzzle**

A first pass at the equity premium

- In equilibrium with power utility we have that

$$E_t\left[\beta\left(\frac{\overline{Y}_{t+1}}{\overline{Y}_t}\right)^{-\gamma}R^m_{t+1}\right] = 1$$

$$E_t\left[\beta\left(\frac{\overline{Y}_{t+1}}{\overline{Y}_t}\right)^{-\gamma}R^f_{t+1}\right] = 1$$

and any other return of course!

- Assuming joint lognormality of consumption and and returns and homoskedasticity

$$E\left(r^m_{t+1}-r^f_{t+1}\right)+\frac{\sigma^2_m}{2}=\gamma\sigma_{mc}$$

- Use data available to obtain $\widehat{\gamma}=240$!!!!!

A first pass at the equity premium

- The equity premium is too high to be razionalized with the standard model and reasonable (experimental evidence) γ

- What went wrong?

- – Is it consumption too smooth?

- – Is it the low correlation between consumption and stock returns?

Epstein-Zin-Weil

- Preferences that disentangle risk aversion and the elasticitity of IS

$$\frac{\Lambda_{t+1}}{\Lambda_t} = \left[\left(\frac{C^*_{t+1}}{C^*_t} \right)^{-\frac{1}{\psi}} \right]^{\theta} \left(R^m_{t+1} \right)^{\theta - 1}$$

where

$$\theta = \frac{1-\gamma}{1 - 1/\psi}$$

- With the same distributional assumptions

$$r^f_{t+1} = -\log \beta + \frac{1}{\psi} E_t \left(\triangle c_{t+1} \right) + \frac{\theta - 1}{2} \sigma^2_m - \frac{\theta}{2\psi^2} \sigma^2_c$$

Epstein-Zin-Weil

- High risk aversion does not need to imply a high rate: solve the risk-free rate puzzle
  - Unfortunately ψ is found to be small in experimental evidence

- Equity premium for risky assets including the market portfolio

$$E\left(r_{t+1}^{i} - r_{t+1}^{f}\right) + \frac{\sigma_i^2}{2} = \theta \frac{\sigma_{ic}}{\psi} + (1-\theta)\,\sigma_{im}$$

  the covariance σ_{im} may be high: solve the equity premium puzzle

  - However, the market-portfolio and consumption are linked through the budget constraint

Internal habit persistence

- Repeated exposure to a stimulus diminishes the response to it

- Common problem: high volatility of the risk-free rate

- Constantinides (1990) and Campbell and Cochrane:

$$U(C) = E_0 \left[\sum_{t=0}^{T} \beta \frac{(C_t - X_t)^{1-\gamma}}{1-\gamma} \right]$$

- Internal habit: X_t is a function of past own consumption

- Nests HARA utility with $\delta > 0$ and $\gamma > 0$ with relative risk aversion

$$\frac{\gamma}{1 - \delta / C}$$

Internal habit persistence

- It can solve the risk-free rate puzzle

$$r^f_{t+1} \simeq -\log\beta + \gamma E_t(\triangle c_{t+1}) - (ERA_t)^2 \frac{\sigma_c^2}{2}$$

- Skip the volatility problem by:
  - Constantinides assumes consumption is not $i.i.d.$
  - Campbell and Cochrane impose a highly nonlinear evolution of ERA_t which makes r^f_{t+1} constant

- Caveats:
  - It can not solve the equity premium puzzle
  - The way they get around the volatility problem is potentially counterfactual

External habit persistence

- Catching up with the Joneses

- Abel (1990):

$$U(C) = E_0\left[\sum_{t=0}^{T}\beta\frac{(C_t/X_t)^{1-\gamma}}{1-\gamma}\right]$$

and

$$X_t = \left[C_{t-1}^{D}\overline{Y}_{t-1}^{1-D}\right]^{\alpha}$$

- It nexts both internal and external habit

Abel (1990)

TABLE 1—UNCONDITIONAL EXPECTED RETURNS
$\beta = 0.99$; $E\{x\} = 1.018$; $\text{VAR}\{x\} = (0.036)^2$

α	Stocks	Bills	Consols
A. Time-separable preferences ($\gamma = 0$)			
0.5	1.93	1.87	1.87
	[1.93]	[1.87]	[1.87]
1.0	2.83	2.70	2.70
	[2.83]	[2.70]	[2.70]
6.0	10.34	9.52	9.52
	[10.33]	[9.51]	[9.51]
10.0	14.22	12.85	12.85
	[14.13]	[12.72]	[12.72]
B. Relative consumption ($\gamma = 1$; $D = 0$)			
0.5	2.80	2.76	2.73
	[2.80]	[2.76]	[2.73]
1.0	2.83	2.70	2.70
	[2.83]	[2.70]	[2.70]
6.0	6.70	2.07	5.84
	[6.72]	[2.06]	[5.86]
10.0	14.73	1.59	13.16
	[14.95]	[1.55]	[13.32]
C. Habit formation ($\gamma = 1$; $D = 1$)			
0.86	33.56	4.53	35.25
0.94	6.83	3.48	7.44
1.00	2.83	2.70	2.70
1.06	8.43	1.93	7.40
1.14	38.28	0.93	35.16