A component model for dynamic correlations*

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Abstract

The idea of component models for volatility is extended to dynamic correlations. We propose a model of dynamic correlations with a short- and long-run component specification. We call this class of models DCC-MIDAS as the key ingredients are a combination of the Engle (2002) DCC model, the Engle and Lee (1999) component GARCH model to replace the original DCC dynamics with a component specification and the Engle, Ghysels, and Sohn (2006) GARCH-MIDAS component specification that allows us to extract a long-run correlation component via mixed data sampling. We provide a comprehensive econometric analysis of the new class of models, including conditions for positive semi-definiteness, and provide extensive empirical evidence that supports the model specification.

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1 Introduction

Component models have been widely used for volatility dynamics.[1] The motivation is typically based on either one of the following two arguments. First, the component structure allows for a parsimonious representation of complex dependence structures. Second, the components are sometimes linked to economic principles, namely the idea that there are different short- and long-run sources that affect volatility. The purpose of this paper is to propose a component model of dynamic correlations with a short- and long-run component specification.[2] We call this class of models DCC-MIDAS as the key ingredients are a combination of the Engle (2002) DCC model, the Engle and Lee (1999) component GARCH model to replace the original DCC dynamics with a component specification and the Engle, Ghysels, and Sohn (2006) GARCH-MIDAS component specification that allows us to extract a long-run correlation component via mixed data sampling.

We address the specification, estimation and interpretation of correlation models that distinguish short and long run components. We show that the changes in correlations are indeed very different. Dynamic correlations are a natural extension of the GARCH-MIDAS model to Engle (2002) DCC model. The idea captured by the DCC-MIDAS model is similar to that underlying GARCH-MIDAS. In the latter case, two components of volatility are extracted, one pertaining to short term fluctuations, the other pertaining to a secular component. In the GARCH-MIDAS the short run component is a GARCH component, based on daily (squared) returns, that moves around a long-run component driven by realized volatilities computed over a monthly, quarterly or bi-annual basis. The MIDAS weighting scheme helps us extract the slowly moving secular component around which daily volatility moves. Engle, Ghysels, and Sohn (2006) explicitly link the extracted MIDAS component to macroeconomic

[1]Engle and Lee (1999) introduced a GARCH model with a long and short run component. Several others have proposed related two-factor volatility models, see e.g. Ding and Granger (1996), Gallant, Hsu, and Tauchen (1999), Alizadeh, Brandt, and Diebold (2002), Chernov, Gallant, Ghysels, and Tauchen (2003), Adrian and Rosenberg (2004), Christoffersen, Jacobs, Ornthanalai, and Wang (2008), among many others. Chernov, Gallant, Ghysels, and Tauchen (2003) examine quite an exhaustive set of diffusion models for the stock price dynamics and conclude quite convincingly that at least two components are necessary to adequately capture the dynamics of volatility.

[2]It should be noted that there have been several prior attempts to think of component models for correlations, see inter alia Karolyi and Stulz (1996). Our approach focuses on autoregressive conditional correlation models.

sources. It is the same logic that is applied here to correlations. Namely, the daily dynamics obey a DCC scheme, with the correlations moving around a long run component. Short-lived effects to correlations will be captured by the autoregressive dynamic structure of DCC, with the intercept of the latter being a slowly moving process that reflects the fundamental or long-run causes of time variation in correlation.[3]

To estimate the parameters of the DCC-MIDAS model we follow the two-step procedure of Engle (2002). We start by estimating the parameters of the univariate conditional volatility models. The second step consists of estimating the DCC-MIDAS parameters with the standardized residuals. We also discuss the regularity conditions we need to impose on the *MIDAS-filtered* long run correlation component as models of correlations are required to yield positive definite matrices.

The paper concludes with an empirical illustration, showing the benefits of the component specification. Empirical specification tests reveal the superior empirical fit, both in- and out-of-sample of the new class of DCC-MIDAS correlation models.

The remainder of the paper is organized as follows. Section 2 introduces the correlation component model and compares the DCC-MIDAS class of models with original DCC models. Sections 3 and 4 cover regularity conditions and estimation, while section 5 contains the empirical applications. Section 6 concludes the paper.

2 A new class of component correlation models

The purpose of this section is to introduce the class of DCC/MIDAS dynamic correlation models. In a first subsection we provide some preliminaries. The second subsection introduces the structure of DCC/MIDAS.

2.1 Notation and Preliminaries

Consider a set of n assets and let the vector of returns be denoted as $\mathbf{r}_t = [r_{1,t}, \ldots, r_{n,t}]'$. The novelty of our approach consists of describing the dynamics of conditional vari-

[3]In principle we can link the secular correlation component to macroeconomic sources, very much like Engle, Ghysels, and Sohn (2006) and Schwert (1989), who study long historical time series and link volatility directly to various key macroeconomic time series.

ances and correlations, where we take into account both short and long run components. The long run component at time t will be a judiciously chosen weighted average of historical correlations. The assumption is that the long run component can be filtered from empirical correlations. Of course, what is critical is the choice of weights, which will be one of the key ingredients of the model specification. To proceed let us assume that the vector of returns $\mathbf{r}_t = [r_{1,t}, \ldots, r_{n,t}]'$ follows the process:

$$\begin{aligned} \mathbf{r}_t &\sim_{i.i.d.} N(\boldsymbol{\mu}, \boldsymbol{H}_t) \\ \boldsymbol{H}_t &= \boldsymbol{D}_t \boldsymbol{R}_t \boldsymbol{D}_t \end{aligned} \tag{2.1}$$

$\boldsymbol{\mu}$ is where the vector of unconditional means, $\boldsymbol{H}_t$ is the conditional covariance matrix and $\boldsymbol{D}_t$ is a diagonal matrix with standard deviations on the diagonal, and:

$$\begin{aligned} \boldsymbol{R}_t &= E_{t-1}[\boldsymbol{\xi}_t \boldsymbol{\xi}_t'] \\ \boldsymbol{\xi}_t &= D_t^{-1}(\mathbf{r}_t - \boldsymbol{\mu}) \end{aligned} \tag{2.2}$$

Therefore $\mathbf{r}_t = \boldsymbol{\mu} + \boldsymbol{H}_t^{\frac{1}{2}} \boldsymbol{\xi}_t$ with $\boldsymbol{\xi}_t \sim_{i.i.d.} N(\mathbf{0}, \boldsymbol{I}_n)$. At the outset, it should be noted that component models for correlations also prompt us to think about component models for volatility which feed into the correlation specification. Indeed the decomposition of the conditional covariance matrix $\boldsymbol{H}_t = \boldsymbol{D}_t \boldsymbol{R}_t \boldsymbol{D}_t$ appearing in equation (2.1) with $\boldsymbol{D}_t$ a diagonal matrix of standard deviations and $\boldsymbol{R}_t$ the conditional correlation matrix suggests a two-step model specification (and estimation) strategy. Consequently, we will first specify $\boldsymbol{D}_t$ followed by $\boldsymbol{R}_t$.

The univariate volatility models build on recent work by Engle and Rangel (2005) and in particular Engle, Ghysels, and Sohn (2006). Both proposed component models for volatility, where long an short run volatility dynamics are separated. Engle and Rangel (2005) introduce a Spline-GARCH model where the daily equity volatility is a product of a slowly varying deterministic component and a mean reverting unit GARCH. Unlike conventional GARCH or stochastic volatility models, this model permits low frequency volatility to change over time. Engle and Rangel (2005) use an exponential spline as a convenient non-negative parameterization. The recent work of Engle, Ghysels, and Sohn (2006) combines insights from Engle and Rangel (2005) with a framework that is suited to combine data that are sampled at different frequencies. The new approach is inspired by the recent work on mixed data sampling, or MIDAS discussed in Ghysels, Santa-Clara, and Valkanov (2005), Ghysels, Santa-Clara, and Valkanov (2006), Forsberg and Ghysels (2004), among others.[4] Engle, Ghysels, and Sohn (2006) replace the spline specification with a MIDAS polynomial.

The new class of models is called GARCH-MIDAS, since it uses a mean reverting unit *daily* GARCH process, similar to Engle and Rangel (2005), and a MIDAS polynomial which applies to *monthly, quarterly,* or *bi-annual* macroeconomic or financial variables. In what follows we will refer to g_i and m_i as the short and long run variance components respectively for asset i. Engle, Ghysels, and Sohn (2006) consider various specifications for g_i and we select only a specific one where the long run component is held constant across the days of the month, quarter or half-year. Alternatively, one can specify m_i based on rolling samples that change from day to day. The findings in Engle, Ghysels, and Sohn (2006) show that they yield very similar empirical fits - so we opted for the simplest to implement which involves locally constant long run components. We will denote by N_v^i the number of days that m_i is held fixed. The superscript i indicates that this may be asset-specific. The subscript v differentiates it from a similar scheme that will be introduced later for correlations. It will be convenient to introduce two time scales t and τ. In particular, while $g_{i,t}$ moves daily, $m_{i,\tau}$ changes only once every N_v^i days.

More specifically we assume that for each asset $i = 1, \ldots, n$, univariate returns follow the GARCH-MIDAS process:

$$r_{i,t} \quad = \quad \mu_i + \sqrt{m_{i,\tau} \cdot g_{i,t}}\xi_{i,t}, \quad \forall t = \tau N_v^i, \ldots, (\tau+1)N_v^i \tag{2.3}$$

where $g_{i,t}$ follows a GARCH(1,1) process:

$$g_{i,t} \quad = \quad (1 - \alpha_i - \beta_i) + \alpha_i \frac{(r_{i,t-1} - \mu_i)^2}{m_{i,\tau}} + \beta_i g_{i,t-1} \tag{2.4}$$

while the MIDAS component $m_{i,\tau}$ is a weighted sum of K_v^i lags of realized variances

[4] In the context of volatility, Ghysels, Santa-Clara, and Valkanov (2005) studied the traditional risk-return trade-off and used monthly data to proxy expected returns while the variance was estimated using daily squared returns. The idea was carried a step further in Ghysels, Santa-Clara, and Valkanov (2006) and Forsberg and Ghysels (2004), both papers focusing on predicting volatility at various horizons with high frequency financial data using MIDAS.

(RV) over a long horizon:

$$m_{i,\tau} \quad = \quad \overline{m}_i + \theta_i \sum_{l=1}^{K_v^i} \varphi_l(\omega_v^i) RV_{i,\tau-l} \tag{2.5}$$

where the realized variances involve N_v^i daily squared returns, namely:

$$RV_{i,\tau} \quad = \sum_{j=(\tau-1)N_v^i+1}^{\tau N_v^i} (r_{i,j})^2.$$

Note that N_v^i could for example be a quarter or a month. The above specification corresponds to the block sampling scheme as defined in Engle, Ghysels, and Sohn (2006), involving so called Beta weights defined as:

$$\varphi_l(\omega_v^i) \quad = \quad \frac{(1 - \frac{l}{K_v^i})^{\omega_v^i - 1}}{\sum_{j=1}^{K_v^i} (1 - \frac{j}{K_v^i})^{\omega_v^i - 1}} \tag{2.6}$$

In practice we will consider cases where the parameters N_v^i and K_v^i are independent of i, i.e. the same across all series. Similarly, we can also allow for different decay patterns ω_v^i across various series, but once again we will focus on cases with common ω_v (see the next subsection for further discussion). Obviously, despite the common parameter specification, we expect that the $m_{i,\tau}$ substantially differ across series, as they are data-driven.

2.2 The class of DCC/MIDAS dynamic correlation models

Dynamic correlations are a natural extension of the GARCH-MIDAS model to the Engle (2002) DCC model. More specifically, we will introduce two components, a long-run and short-run one. In the case of volatility we noted that $m_{i,\tau}$ can be formulated either via keeping it locally constant, or else based on a local moving window. Engle, Ghysels, and Sohn (2006) find for volatility that the difference between the two appears to be negligible. For correlations, we have potentially the same choice. Since the trailing local specification is more general, we adopt this for our formulation. Namely, using the standardized residuals $\xi_{i,t}$ it is possible to obtain a matrix $\boldsymbol{Q}_t$

whose elements are:

$$\begin{aligned} q_{i,j,t} &= \overline{\rho}_{i,j,t}\left(1-a-b\right)+a\xi_{i,t-1}\xi_{j,t-1}+bq_{i,j,t-1} \\ \overline{\rho}_{i,j,t} &= \sum_{l=1}^{K_c^{ij}}\varphi_l\left(\omega_r^{ij}\right)c_{i,j,t-l} \\ c_{i,j,t} &= \frac{\sum_{k=t-N_c^{ij}}^{t}\xi_{i,k}\xi_{j,k}}{\sqrt{\sum_{k=t-N_c^{ij}}^{t}\xi_{i,k}^2}\sqrt{\sum_{k=t-N_c^{ij}}^{t}\xi_{j,k}^2}} \end{aligned} \tag{2.7}$$

where the weighting scheme is similar to that appearing in (2.5). Note that in the above formulation of $c_{i,j,t}$ we could have used simple cross-products of $\xi_{i,t}$. The normalization will allow us later to discuss regularity conditions in terms of correlation matrices. Correlations can then be computed as:

$$\rho_{i,j,t} = \frac{q_{i,j,t}}{\sqrt{q_{i,i,t}}\sqrt{q_{j,j,t}}}$$

We regard $q_{i,j,t}$ as the short run correlation between assets i and j, whereas $\overline{\rho}_{i,j,t}$ is a slowly moving long run correlation. Rewriting the first equation of system (2.7) as

$$q_{i,j,t}-\overline{\rho}_{i,j,t} = a\left(\xi_{i,t-1}\xi_{j,t-1}-\overline{\rho}_{i,j,t}\right)+b\left(q_{i,j,t-1}-\overline{\rho}_{i,j,t}\right) \tag{2.8}$$

conveys the idea of short run fluctuations around a time varying long run relationship. The idea captured by the DCC-MIDAS model is similar to that underlying GARCH-MIDAS. In the latter case, two components of volatility are extracted, one pertaining to short term fluctuations, the other pertaining to a secular component. In the GARCH-MIDAS the short run component is a GARCH component, based on daily (squared) returns, that moves around a long-run component driven by realized volatilities computed over a monthly, quarterly or bi-annual basis. The MIDAS weighting scheme helps us extracting the slowly moving secular component around which daily volatility moves. Engle, Ghysels, and Sohn (2006) explicitly link the extracted MIDAS component to macroeconomic sources. It is the same logic that is applied here to correlations. Namely, the daily dynamics obey a DCC scheme, with the correlations moving around a long run component. Short-lived effects on correlations will be captured by the autoregressive dynamic structure of DCC, with the intercept of the latter being a slowly moving process that reflects the fundamental or secular causes of time variation in correlation. In principle we can link the long run correlation component to macroeconomic sources, very much like Engle, Ghysels, and Sohn (2006) study long historical time series, similar to Schwert (1989) and link volatility directly to various key macroeconomic time series. Note that in equation (2.5) we can allow for different weighting schemes across series. Likewise, the specification in (2.7) can potentially accommodate weights ω_r^{ij}, lag lengths N_c^{ij} and span lengths of historical correlations K_c^{ij} to differ across any pair of series.[5] Typically we will use a single setting common to all pairs of series, similar to the choice of a common MIDAS filter in the univariate models. We will discuss in the next subsection the implications of a single versus multiple parameter choices for the DCC-MIDAS filtering scheme.

It is also worth noting that our DCC-MIDAS model shares features with a local dynamic conditional correlation (LDCC) model introduced in Feng (2007), where variances are decomposed into a conditional and a local (unconditional) parts. The correlation structure is modeled by a multivariate nonparametric ARCH-type approach that accommodates the presence of regressors.

On the subject of alternative specifications, we should also note that Engle and Lee (1999) shows one can build a simple component GARCH model from a GARCH(2,2) model. One could therefore think of a simple component DCC model built from a DCC(2,2) model. Such a model could be a natural benchmark as well. It has, however, a much more challenging structure. In particular, based on the analogy with higher order GARCH models we expect that the higher order dynamics for correlation matrices is quite involved. It is the reason why we opted for the component structure of DCC-MIDAS. Nevertheless, the analogy with Engle and Lee (1999) is certainly a worthwhile project for future research.

To conclude this subsection we fix some notation that will allow us to discuss the general model specification. First, we will collect all the elements ω_r^{ij} into a vector $\underline{\omega}_r$, keeping in mind that it may only contain a single element if all weights are equal and we denote $N_c = \max_{i,j} N_c^{ij}$. We then can write and collect the set of correlations

[5]Note that $(\omega_r^{ij}, N_c^{ij}, K_c^{ij}) = (\omega_r^{ji}, N_c^{ji}, K_c^{ji})$ are identical for all i and j.

appearing in equation (2.7) yielding generically in matrix form:

$$\boldsymbol{R}_t = (\boldsymbol{Q}_t^*)^{-1/2}\boldsymbol{Q}_t(\boldsymbol{Q}_t^*)^{-1/2} \tag{2.9}$$

$$\boldsymbol{Q}_t^* = diag\boldsymbol{Q}_t, \tag{2.10}$$

$$\boldsymbol{Q}_t = (1-a-b)\overline{\boldsymbol{R}}_t(\underline{\omega}_r) + a\boldsymbol{\xi}_t\boldsymbol{\xi}_t' + b\boldsymbol{Q}_{t-1} \tag{2.11}$$

where

$$\overline{\boldsymbol{R}}_t(\underline{\omega}_r) = \sum_{l=1}^{K_c} \boldsymbol{\Phi}_l(\underline{\omega}_r) \odot \boldsymbol{C}_{t-l}$$

$$\boldsymbol{C}_t = \begin{pmatrix} v_{1,t} & 0 & 0 \\ \vdots & \ddots & 0 \\ 0 & \cdots & v_{n,t} \end{pmatrix}^{-\frac{1}{2}} \left(\sum_{k=t-N_c}^{t} \boldsymbol{\xi}_k\boldsymbol{\xi}_k' \right) \begin{pmatrix} v_{1,t} & 0 & 0 \\ \vdots & \ddots & 0 \\ 0 & \cdots & v_{n,t} \end{pmatrix}^{-\frac{1}{2}}$$

$$v_{i,t} = \sum_{k=t-N_c}^{t} \boldsymbol{\xi}_{i,k}^2, \quad \forall i = 1,\ldots,n \tag{2.12}$$

where $\boldsymbol{\Phi}_l(\underline{\omega}_r) = \varphi_l(\underline{\omega}_r)\iota\iota'$ and $\odot$ stands for the Hadamard product.[6]

3 Estimation

To estimate the parameters of the DCC-MIDAS model we follow the two-step procedure of Engle (2002). We start by collecting the parameters of the univariate conditional volatility models into a vector $\Psi \equiv [(\alpha_i, \beta_i, \omega_i, m_i, \theta_i), i = 1,\ldots,n]$. and the parameters of the conditional correlation model into $\Xi \equiv (a, b, \underline{\omega}_r)$. Then the quasi-likelihood function QL can be written as:

$$\begin{aligned} QL(\Psi,\Xi) &= QL_1(\Psi) + QL_2(\Psi,\Xi) \\ &\equiv -\sum_{t=1}^{T}\left(n\log(2\pi) + 2\log|\boldsymbol{D}_t| + \mathbf{r}_t'\boldsymbol{D}_t^{-2}\mathbf{r}_t\right) - \sum_{t=1}^{T}\left(\log|\boldsymbol{R}_t| + \boldsymbol{\xi}_t'\boldsymbol{R}_t^{-1}\boldsymbol{\xi}_t + \boldsymbol{\xi}_t'\boldsymbol{\xi}_t\right) \end{aligned} \tag{3.1}$$

[6]In a later section we will generalize this setup to allow for multiple MIDAS filters in the long-run dynamics of correlations.

Given the structure of the log likelihood function, namely the separation of $QL(\Psi, \Xi)$ into $QL_1(\Psi)$ and $QL_2(\Psi, \Xi)$, we can first estimate the parameters of the univariate GARCH-MIDAS processes, i.e. the parameters in Ψ, using $QL_1(\Psi)$ and therefore each single series separately - yielding $\hat{\Psi}$. The second step consists of estimating the DCC-MIDAS parameters with the standardized residuals $\hat{\boldsymbol{\xi}}_t = \hat{\boldsymbol{D}}_t^{-1}(\mathbf{r}_t - \hat{\boldsymbol{\mu}})$ using $QL_2(\hat{\Psi}, \Xi)$. The estimation of the MIDAS polynomial parameters in the dynamic correlations require some further discussion. The approach we adopt is inspired by the estimation of MIDAS polynomial parameters in the GARCH-MIDAS model.

So far we were not very explicit about the choice of the polynomial characteristics K_v^i and N_v^i in equation (2.5) and the choice of K_c and N_c in equation (2.7). In the former case, i.e. the univariate GARCH-MIDAS models, K_v^i determines the number of lags spanned in each MIDAS polynomial specifications for τ_t. The other is how to compute RV, weighted by the MIDAS polynomials. As pointed out by Engle, Ghysels, and Sohn (2006), this amounts to model selection with a fixed parameter space, and therefore is achieved via profiling the likelihood function for various combinations of K_v^i and N_v^i. To determine the long run component of conditional correlations, $\overline{\boldsymbol{R}}_t$ we proceed in exactly the same way, namely we select the number of lags K_c for historical correlations and the time span over which to compute the historical correlations N_c in equation (2.7). The similarity between the two procedures is not surprising, given the fact that DCC models build extensively on the ideas of GARCH and in both cases we have a MIDAS filter extracting a component which behaves like a time-varying intercept. We will provide an explicit discussion of the procedure in the empirical applications, given the similarity with Engle, Ghysels, and Sohn (2006).

The asymptotic properties of the two-step estimator are discussed in Engle and Sheppard (2001), Comte and Lieberman (2003), Ling and McAleer (2003) and McAleer, Chan, Hoti, and Lieberman (2006). These papers deal with fixed parameter DCC models. It is beyond the scope of the current paper to establish the asymptotic properties of the MLE estimator when the MIDAS stochastic intercept is present. Recent work by Dahlhaus and Subba Rao (2006) discussed general time-varying coefficient ARCH(∞) processes and regularity conditions for (local) MLE estimation. The GARCH-MIDAS and DCC-MIDAS processes are to a certain degree special cases of their setup. Namely, Dahlhaus and Subba Rao (2006) allow all parameters to vary and assume a nonparametric setting to capture the time-varying coefficients. This leads them, like Feng (2007), to consider kernel-based estimators. Our setting is parametric, as the MIDAS filter is a parametric specification, and therefore presumably simpler. We leave the regularity conditions that guarantee standard asymptotic results for the two-step estimation of DCC-MIDAS as an open question for future research. Ghysels and Wang (2007) provide a rigorous analysis of the ML estimation of the GARCH-MIDAS model. Their paper shows that the asymptotic theory is quite involved. One can easily extrapolate and infer from this that the DCC-MIDAS asymptotics are quite involved too. We therefore refrained from a detailed development of the asymptotics of the DCC-MIDAS model. However, we do cover in this paper the regularity conditions we need to impose on the *MIDAS-filtered* long run correlation component to obtain positive definite matrices. This is the topic of the next section.

4 Regularity Conditions

In this section, we turn our attention to the long run component and the choice of weights ω_r^{ij}, keeping the lag lengths N_c^{ij} and span lengths of historical correlations K_c^{ij} fixed across all pairs of series. Hence, we focus on the memory decay in the long run correlations.

4.1 Long-Run dynamics

The first case to consider is the one of a common decay parameter ω_r independent of the pair of returns series selected. The covariance matrices can be shown to be positive definite under a relatively mild set of assumptions. When considering equation (2.11) it is apparent that the matrix $\boldsymbol{Q}_t$ is a weighted average of three matrices. The matrix $\overline{\boldsymbol{R}}_t$ is positive semi-definite because it is a weighted average of correlation matrices. The matrix $\boldsymbol{\xi}_t \boldsymbol{\xi}_t'$ is always positive semi-definite by construction. Therefore, if the matrix $\boldsymbol{Q}_0$ is initialized to a positive semi-definite matrix, it follows that $\boldsymbol{Q}_t$ must be positive semi-definite at each point in time.

The positive semi-definiteness of the covariance matrix can be guaranteed without putting any restriction on the structure of the conditional variance estimators for the individual return series. This means, for example, that it is possible to assume a different number of GARCH-MIDAS lags for each return.

The case of two or more weighting schemes is more involved and therefore becomes more interesting. Indeed it is not always the case that the matrix $\overline{\boldsymbol{R}}_t\left(\underline{\omega}_r\right)$ is positive semi-definite for any choice of MIDAS parameters and specific restrictions on the parameter space ought to be imposed. The goal of this section is to provide sufficient conditions under which the sequence of matrices $\{\boldsymbol{\Phi}_l\}_{l=1}^{K_c}$ defined in (2.11) is positive semi-definite. Since the sequence of matrices $\{\boldsymbol{C}_{t-i}\}_{i=1}^{L_c}$ is positive semi-definite by construction, we can invoke the Schur product theorem to state that any matrix $\overline{\boldsymbol{R}}_t\left(\underline{\omega}_r\right) = \sum_{l=1}^{K_c} \boldsymbol{\Phi}_l\left(\underline{\omega}_r\right) \odot \boldsymbol{C}_{t-l}$ is positive semi-definite as well.[7]

In this section we examine the case of block matrices, whose dynamics can be accounted for by three parameters: the first two for block of correlations and the third one for the off-diagonal correlations. The following three definitions set the stage for the kind of matrices that we deal with in this section.

Definition 1 (Diagonal MIDAS Block)**.** *Let $\boldsymbol{\Phi}_l^D\left(n_a, \omega_r^a\right)$ be a symmetric, square matrix of size n_a such that all elements on the off-diagonal are equal to $\varphi_l(\omega_r^a)$ and all elements on the main diagonal are ones.*

Definition 2 (Off-Diagonal MIDAS Block)**.** *Let $\boldsymbol{\Phi}_l^F\left(n_a, n_b, \omega_r^c\right)$ be a matrix of size n_a, n_b such that all elements are equal to $\varphi_l(\omega_r^c)$.*

Definition 3 (Block MIDAS matrix)**.** *Let*

$$\boldsymbol{\Phi}_l\left(\omega_r^a, \omega_r^b, \omega_r^c, n_a, n_b\right) = \begin{pmatrix} \boldsymbol{\Phi}_l^D\left(n_a, \omega_r^a\right) & \boldsymbol{\Phi}_l^F\left(n_a, n_b, \omega_r^c\right) \\ \boldsymbol{\Phi}_l^F\left(n_a, n_b, \omega_r^c\right)' & \boldsymbol{\Phi}_l^D\left(n_b, \omega_r^b\right) \end{pmatrix}$$

be a block matrix with $\boldsymbol{\Phi}_l^D\left(n_a, \omega_r^a\right)$ and $\boldsymbol{\Phi}_l^D\left(n_b, \omega_r^b\right)$ defined as in 1 and $\boldsymbol{\Phi}_l^F\left(n_a, n_b, \omega_r^c\right)$ defined as in 2.

The following lemmas lead up to the main proposition of this section.

Lemma 1. *The determinant of the matrix $\boldsymbol{\Phi}_l$ in Definition 3 is equal to*

$$det\left(\boldsymbol{\Phi}_l\right) = det_A \cdot det_{BCAC}$$

[7]For a proof of the Schur product theorem see Horn and Johnson (1985), Theorem 7.5.3.

where

$$
\begin{aligned}
det_A &= \det\left(\mathbf{\Phi}_l^D\left(n_a, \omega_r^a\right)\right) &(4.2)\\
det_{BCAC} &= det\left(\mathbf{\Phi}_l^D\left(n_b, \omega_r^b\right) - \mathbf{\Phi}_l^F\left(n_a, n_b, \omega_r^c\right)' \mathbf{\Phi}_l^D\left(n_a, \omega_r^a\right)^{-1} \mathbf{\Phi}_l^F\left(n_a, n_b, \omega_r^c\right)\right) &(4.3)
\end{aligned}
$$

Proof. See Appendix. □

This lemma suggests that in order to ensure the positive definiteness of each weighting matrix we can focus separately on the conditions that make the determinant of the first block matrix positive and on those that make the determinant of the function of matrices defined in (4.3) positive. We shall start by focusing on the first diagonal matrix.

Lemma 2. *If* $\varphi_l(\omega_r^a) \leq 1$ *then all leading principal minors of* $\mathbf{\Phi}_l^D\left(n_a, \omega_r^a\right)$ *are non-negative.*

Proof. See Appendix. □

According to the previous lemma, the only condition that needs to be verified for the leading principal minors of $\mathbf{\Phi}_l$ up to the determinant of the first block matrix to be positive is that $\varphi_l(\omega_r^a)$ is less than one. This condition is always satisfied when using MIDAS filters.

The next two lemmas deal with the determinant of the function of sub-matrices defined in (4.3).

Lemma 3. *If* $\varphi_l(\omega_r^a) \geq 0$ *and* $\varphi_l(\omega_r^b) \leq 1$*, the scalar*

$$
\zeta\left(\varphi_l(\omega_r^b), \varphi_l(\omega_r^c), n_b\right) = \left(1 - \varphi_l(\omega_r^b)\right)^{n_b - 1}\left(1 + (n_b - 1)\varphi_l(\omega_r^b)\right) - n_a n_b \left[\varphi_l(\omega_r^c)\right]^2 \quad (4.4)
$$

is always smaller than det_{BCAC} *defined in (4.3).*

Proof. See Appendix. □

Lemma 4. *If* $\varphi_l(\omega_r^b) \leq 1$ *the function*

$$
\zeta\left(\varphi_l(\omega_r^b), \varphi_l(\omega_r^c), n_b\right) = \left(1 - \varphi_l(\omega_r^b)\right)^{n_b - 1}\left(1 + (n_b - 1)\varphi_l(\omega_r^b)\right) - n_a n_b \left[\varphi_l(\omega_r^c)\right]^2
$$

is always non-increasing in n_b.

Proof. See Appendix. □

We are now ready to state the first of the two propositions of this sub-section.

Proposition 1. *Let* $\varphi_l(\omega_r^a) < 1$*,* $\varphi_l(\omega_r^b) < 1$*, and* $\varphi_l(\omega_r^c) < 1$ *and*

$$\left(1 - \varphi_l(\omega_r^b)\right)^{n_b - 1} \left(1 + (n_b - 1)\varphi_l(\omega_r^b)\right) - n_a n_b \left[\varphi_l(\omega_r^c)\right]^2 \quad > \quad 0 \tag{4.5}$$

the matrix $\mathbf{\Phi}_l\left(\omega_r^a, \omega_r^b, \omega_r^c, n_a, n_b\right)$ *is positive definite.*

Proof. Follows directly from lemmas 1, 2, and 4. □

The assumption that $\varphi_l(\omega_r^a) < 1$, $\varphi_l(\omega_r^b) < 1$, and $\varphi_l(\omega_r^c) < 1$ is always verified when using MIDAS filters, since they are all positive and are forced to sum up to one. Therefore, it amounts to checking that equation (4.5) is satisfied $\forall l$ to ensure that the weighting matrices are positive definite. This can amount to checking a non-negligible amount of conditions in the case of lengthy MIDAS polynomial. For example, in the empirical applications, we show that the likelihood is maximized for 144 lags. A more useful theorem can be stated that amounts to checking only the first one of the conditions above. Its proof is a direct consequence of the following lemma and of proposition 1.

Lemma 5. *Let* $\varphi_l(\omega_r^b) > \varphi_{l+k}(\omega_r^b)$ *and* $\varphi_l(\omega_r^c) > \varphi_{l+k}(\omega_r^c)$ *for some positive scalar* k*. Then*

$$\zeta\left(\varphi_l(\omega_r^b), \varphi_l(\omega_r^c), n_b\right) \leq \zeta\left(\varphi_{l+k}(\omega_r^b), \varphi_{l+k}(\omega_r^c), n_b\right)$$

where the function $\zeta(\cdot,\cdot,\cdot)$ *is defined in (4.4).*

Proof. See Appendix. □

Proposition 2. *Let* ω_r^a*,* ω_r^b*, and* ω_r^c *be the characteristic parameters of the MIDAS filters* $\{\varphi_l(\omega_r^j)\}_{l=1}^{K_c}$*,* $\forall j = \{a, b, c\}$ *and let* $\left\{\mathbf{\Phi}_l^D\left(n_a, \omega_r^a\right), \mathbf{\Phi}_l^D\left(n_b, \omega_r^b\right), \mathbf{\Phi}_l^F\left(n_a, n_b, \omega_r^c\right)\right\}_{l=1}^{K_c}$ *be the associated sequences of block matrices according to definitions 1 and 2. If*

$$\left(1 - \varphi_1(\omega_r^b)\right)^{n_b - 1} \left[1 + (n_b - 1)\varphi_1(\omega_r^b)\right] - n_a n_b \left[\varphi_1(\omega_r^c)\right]^2 \quad > \quad 0$$

then each matrix in the sequence $\left\{\mathbf{\Phi}_l\left(\omega_r^a, \omega_r^b, \omega_r^c, n_a, n_b\right)\right\}_{l=1}^{K_c}$ *is positive definite.*

Proof. Follows directly from proposition 1, from lemma 5 and from the fact that each element of a MIDAS filter is bounded above by one. □

This proposition conveniently states that in order to ensure the positive-definiteness of the long-run correlation matrix, one only needs to check one simple condition involving the first terms of the MIDAS polynomial. This condition is quite easily satisfied. For example, assume to have two sets of 10 series each (i.e. $n_a = n_b = 10$): one that is better described by a long memory dynamics (say that $\omega_r^a = 2$) and one that can be characterized as a short memory process (say that $\omega_r^b = 15$). Also assume that the cross-correlations can be described as a MIDAS-average of the two process (i.e. say that $\omega_r^c = \frac{2+15}{2}$). The length of the MIDAS polynomial is K_c = 144, a parameter that is shown to be optimal in the empirical application. Then, the left hand side of the condition of Proposition 2 is equal to 0.4047. This simple example documents that even in a 20 by 20 system, this parameterizations is quite flexible to ensure the positive definiteness of the resulting long-run correlation matrix.

The multiple-MIDAS filters cases that we analyze in the empirical section are all based on weighting matrices of the type:

$$\mathbf{\Phi}_l\left(\omega_r^a, \omega_r^b, \omega_r^c, n_a, n_b\right) = \begin{bmatrix} 1 & \varphi_l(\omega_r^a) & \varphi_l(\omega_r^c) \\ \varphi_l(\omega_r^a) & 1 & \varphi_l(\omega_r^c) \\ \varphi_l(\omega_r^c) & \varphi_l(\omega_r^c) & 1 \end{bmatrix}$$

where $n_a = 2$, $n_b = 1$, and K_c = 144. The condition of proposition 2 boils down to $1 - 2\varphi_1(\omega_r^c)^2 > 0$, which is always satisfied for $\omega_r^c < 175$. Since a decay factor larger than 20 or 30 is hardly ever reached, we can state that this kind of 3 by 3 systems is always positive definite.

4.2 Short-Run dynamics

In general it will also prove convenient to allow for multiple sets of parameters to describe the DCC part of the correlation dynamics. To do so, we consider a richer structure for the short-run dynamics. In particular, let us reconsider equation (2.11), and without imposing the common parameters a and b across all asset combinations. In particular, let there be matrices of parameters G, A, and B such that the short-run dynamics are written as:

$$\boldsymbol{Q}_t \quad = \quad G \odot \overline{\boldsymbol{R}}_t(\underline{\omega}_r) + A \odot \boldsymbol{\xi}_{t-1}\boldsymbol{\xi}'_{t-1} + B \odot \boldsymbol{Q}_{t-1}$$

The Hadamard products imply that we relax the common parameter restriction and potentially apply different parameters to the asset combinations. In this subsection we study the positive semi-definiteness of the DCC part of the system, by imposing restrictions on the matrices of parameters G, A, and B. We start with the case of three blocks of matrices.

Definition 4 (DCC Block matrices)**.** *Let* $\{a_i\}_{i=1}^3$*, and* $\{b_i\}_{i=1}^3$ *be positive scalars. The DCC block matrices,* A*,* B *and* G *are*

$$A = \begin{bmatrix} a_1 \cdot \iota'_{N_j}\iota_{N_j} & a_3 \cdot \iota'_{N_j}\iota_{N_k} \\ a_3 \cdot \iota'_{N_k}\iota_{N_j} & a_2 \cdot \iota'_{N_k}\iota_{N_k} \end{bmatrix}, B = \begin{bmatrix} b_1 \cdot \iota'_{N_j}\iota_{N_j} & b_3 \cdot \iota'_{N_j}\iota_{N_k} \\ b_3 \cdot \iota'_{N_k}\iota_{N_j} & b_2 \cdot \iota'_{N_k}\iota_{N_k} \end{bmatrix}, G = \iota'_{N_j+N_k}\iota_{N_j+N_k} - A - B$$

where $\iota_{N_l} = [\underbrace{\begin{matrix} 1 & \dots & 1 \end{matrix}}_{N_l}]$, $\forall l \in \{j, k\}$.

The following three assumptions are needed in order to prove the positive semi-definiteness of the correlations matrices arising from the system above.

Assumption 1. *Let* $\{a_i\}_{i=1}^3$*, and* $\{b_i\}_{i=1}^3$ *be non-negative scalars and let* $1 - a_i - b_i \geq 0$*,* $\forall i = \{1, 2, 3\}$.

Assumption 2. *Let* $a_1 a_2 - a_3^2 \geq 0$ *and* $b_1 b_2 - b_3^2 \geq 0$.

Assumption 3. *The matrices* $G \odot \overline{\boldsymbol{R}}_t\left(\underline{\omega}_r\right)$ *are positive semi-definite.*

Note that the first assumption is equivalent to the one needed to prove the existence of well-defined correlation matrices in the standard DCC model.

We are now ready to state the main theorem of this sub-section.

Proposition 3. *Let the conditions of Assumptions 1-3 be satisfied. Then the DCC block matrices,* A*,* B*, and* G *are positive semi-definite.*

Proof. Any principal minor that is the determinant of a sub-matrix of order equal or larger than three is zero, since it has at least two identical columns or rows. Any principal minor that is the determinant of a sub-matrix of order equal or larger than two is non-negative, because of Assumptions 2 and 3. The fact that all entries of the matrices are non-negative (by Assumption 1) concludes the proof. □

A special case of this specification is the Generalized-DCC model of Cappiello, Engle, and Sheppard (2006).

Definition 5 (Generalized DCC matrices)**.** *Let there be* 2 *pairs of DCC parameters* $\{a_j, b_j\}_{j=1}^{2}$. *The generalized DCC matrices,* A^g*,* B^g*, and* G^g *are*

$$A^g = \boldsymbol{a}\boldsymbol{a}', B^g = \boldsymbol{b}\boldsymbol{b}', G^g = (\boldsymbol{\iota}\boldsymbol{\iota}' - \boldsymbol{a}\boldsymbol{a}' - \boldsymbol{b}\boldsymbol{b}')$$

where

$$\boldsymbol{a}' = \left[\underbrace{\sqrt{a_1} \quad \ldots,}_{N_k} \quad \underbrace{\sqrt{a_2} \quad \ldots}_{N_j} \right] \quad \textit{and} \quad \boldsymbol{b}' = \left[\underbrace{\sqrt{b_1} \quad \ldots,}_{N_k} \quad \underbrace{\sqrt{b_2} \quad \ldots}_{N_j} \right]$$

This specification satisfies Assumption 2 above with equality, but there is no specific reason why that should be the case and, in any event, this is a testable restriction. Therefore the DCC block structure provides a more flexible specification, that under mild assumptions delivers well-defined correlation matrices.

5 Empirical Applications

We start in a first subsection with some small scale empirical examples that explore the various issues that are typically encountered in empirical applications: (1) model specification for the long and short run dynamics of correlations, and (2) testing of models. We consider examples involving stocks and bonds. The equity portfolios are formed based on an industry classification.[8] The second example focus on the eco-

[8]Data were downloaded from Kenneth French's website and correspond to the 10 industry portfolio classification. Accordingly, each NYSE, AMEX, and NASDAQ stock is assigned to an industry portfolio based on its four-digit SIC code. We report the details on the specific portfolios that we employed in our empirical analysis in the Appendix.

nomic significance of the new models we propose in the paper and uses the methodology proposed by Engle and Colacito (2006) pertaining to minimum variance portfolio management. The example focuses on asset allocation with multiple international equities (five international stock markets) and a single MIDAS filter. The section is divided in subsections which cover the various examples.

5.1 Correlation Dynamics of Industry Portfolios and a 10 Year Bond

In this subsection we turn our attention to a smaller set of assets. The purpose of this section is more illustrative in nature - as we address various issues that are typically encountered in empirical applications. We investigate the short and long run correlation dynamics of industry portfolios and a 10 year bond.[9] The sample starts on 1971-07-15 and ends on 2006-06-30. We first address the issue of selecting the number of MIDAS lags. We follow a procedure suggested in Engle, Ghysels, and Sohn (2006), since the GARCH-MIDAS and DCC-MIDAS class of models share similar model selection issues. A convenient property of both models, is that the lag selection of the MIDAS filter involves a fixed number of parameters. In particular, Engle, Ghysels, and Sohn (2006) compare various GARCH-MIDAS models with different time spans via profiling of the likelihood function. We skip the details here as the procedure is described in their paper. The procedure can be summarized as selecting the smallest number of MIDAS lags after which the log-likelihoods of the volatilities seem to reach their plateau.[10]

Since we consider more than two assets we have the possibility that several long run MIDAS filters as well as multiple DCC parameters apply.[11] We provide two examples involving three asset returns, one where a single MIDAS filter suffices, and another where there is clearly a need for two filters. The former involves two industries and a bond, namely Energy and Hi-Tech portfolios vs. 10 year bond. The results appear in Figures 1 and Table 1.

[9] As noted before, the Appendix reports the specifics of the industry definitions.

[10] The details for the choice of MIDAS lags in this example are available upon request to the authors.

[11] The appendix reports the specifics and the names of the models that are being estimated in this subsection.

In Table 3 we report likelihood ratio tests for various nested model specifications involving separate parameters for the DCC dynamics and/or MIDAS filters. Each entry in the table represents the p-value for testing that the likelihood of the model of the column is significantly higher than the likelihood of the model on the corresponding row. The first row of the table documents that the baseline one DCC - one MIDAS model may not be enough to account for the dynamics of the system. The specifications with two sets of DCC parameters seem to yield significant lower likelihoods in all pairwise comparisons. It is also the case that adding an additional MIDAS parameter does not improve the model performance when a second pair of DCC parameters has already been added. This is always true with the only possible exception of the comparison between one and two MIDAS with two sets of non-generalized DCC parameters. The model with three distinct sets of DCC parameters does not appear to significantly improve the likelihood.

These results convey that one MIDAS parameter is enough to account for the long-run dynamics of the system. The short-run dynamics is the one needing a more flexible specification in this example. A model with two sets of DCC parameters and one MIDAS parameter seems to suffice to accurately describe the joint dynamics of the three assets. However, we cannot draw any conclusion as to whether we should employ the generalized or non-generalized DCC structure using the likelihood ratio tests, as the two models are non-nested. The two left-most columns of Table 2 show that the generalized specification with two sets of DCC parameters and one MIDAS appears to perform better than its non-generalized counterpart, according to the AIC and BIC criteria.

The next and final example shows that this is not always the case. In Table 6 we report likelihood ratio tests for Ten year Bonds combined with Manufacturing and Retail industries.[12] The parameter estimates, for the single MIDAS filter appear in Table 4 whereas the multiple filter case appears in Table 5. The bottom part of the table shows that when the MIDAS parameters are estimated in all possible permutations of bivariate systems, a value close to 7 appears to do the job for bond vs. manufacturing and for bond versus retail. A decisively shorter memory achieves the maximum likelihood when it comes to accounting for the long run dynamics of the

[12] We use the term Retail, although the industry is more broadly defined. As noted in the Technical Appendix that 'Retail' is defined as: Wholesale, Retail, and Some Services (Laundries, Repair Shops) SIC codes: 5000-5999, 7200-7299, 7600-7699.

correlation between retail and manufacturing. The top part of Table 5 shows that it can indeed be quite restrictive to force one MIDAS parameter to describe the long-run dynamics of all pairs of correlations. The introduction of an additional MIDAS parameter not only brings the outcome of the estimation closer to what suggested by the analysis of the bivariate systems, but it also sizeably increases the log-likelihood. Table 6 shows that this increase is significant at a 1% confidence level.

The variances and correlations appear respectively in Figures 2 and 3. The former shows the single filter patterns and the latter shows the patterns with two distinct filters. We observe that the second filter clearly changes the long run component correlation across the two industries.

When we employ the Generalized DCC-MIDAS model, we obtain the parameters' estimates reported in Table 5. These estimates seem to confirm the need for a second MIDAS filter to be applied to the correlation between the manufacturing and the retail portfolios. Figure 4 shows that the long-run correlations filtered using this specification appear to be a little smoother when the 10-year bond is one of the assets compared to the results obtained under the previous specification. The low-frequency correlation of the two portfolios is instead a little noisier. Aside for these small differences, we take the results as confirming the need for multiple set of correlation parameters.

5.2 Portfolio Choice and Correlation Dynamics

We perform a comparison of covariance matrix estimators, by employing the methodology proposed by Engle and Colacito (2006). For convenience, we briefly summarize their approach. An investor chooses optimal portfolio weights for N securities, in order to minimize expected portfolio variance subject to the constraint that portfolio weights must add up to some scalar $\overline{w}$:

$$\begin{aligned} &\min_{w_t} w_t' \boldsymbol{H}_t w_t \\ &\text{s.t. } w_t' \iota = \overline{w} \end{aligned} \tag{5.1}$$

where $\boldsymbol{H}_t$ is the estimated one-period ahead conditional covariance matrix and ι is an $N \times 1$ vector of ones. Let the true conditional covariance matrix be denoted as $\boldsymbol{\Omega}_t$. An investor choosing optimal portfolio weights according to the estimated $\boldsymbol{H}_t$ would end up with the following amount of volatility:

$$\frac{\sigma_t}{\overline{w}} = \frac{\sqrt{\iota' \boldsymbol{H}_t^{-1} \, \boldsymbol{\Omega}_t \, \boldsymbol{H}_t^{-1} \iota}}{\iota' \boldsymbol{H}_t^{-1} \iota} \tag{5.2}$$

An investor choosing portfolio weights with the knowledge of the true covariance matrix $\boldsymbol{\Omega}_t$ would achieve the following portfolio volatility for each unit of investment:

$$\frac{\sigma_t^*}{\overline{w}} = \frac{1}{\sqrt{\iota' \boldsymbol{\Omega}_t^{-1} \, \iota}} \tag{5.3}$$

Engle and Colacito (2006) show that $\sigma_t^* \leq \sigma_t$ for any suboptimal estimator of the conditional covariance matrix. In order to quantify the gains from superior covariance information, we can look at the ratio $\frac{\sigma_t - \sigma_t^*}{\sigma_t^*}$. This ratio is always larger than zero and it can be interpreted as the percentage reduction in portfolio investment that could have been achieved by knowing the true covariance matrix. For example, a number like 1% means that an investor wanting to allocate \$100 million and choosing portfolio weights according $\boldsymbol{H}_t$ could have saved \$1 million to achieve the minimum variance portfolio.

Suppose to have two alternative time series of conditional covariance matrices, one produced by a DCC estimator, $\boldsymbol{H}_t^{DCC}$, and one produced by a DCC-MIDAS estimator, $\boldsymbol{H}_t^{DCC-M}$. In each period, a set of minimum variance portfolio weights is constructed based on each covariance matrix. Let portfolio returns attained according to each estimator be denoted as

$$\pi_t^j = \left(w_t^j\right)' (r_t) \,, \quad \forall j \in \{DCC, DCC - M\}$$

where r_t stands for the demeaned vector of asset returns. Let the difference of the squared returns on the two portfolios be denoted as

$$u_t = \left(\pi_t^{DCC}\right)^2 - \left(\pi_t^{DCC-M}\right)^2$$

The null hypothesis is that the portfolio variances of the two estimators are equal. This can be tested using the Diebold and Mariano (1995) procedure. By regressing $\boldsymbol{u}$ on a constant and correcting the covariance matrix for heteroskedasticity, the null of equal variance is simply a test that the mean of u is zero.

We assume that both DCC and DCC-MIDAS employ a GARCH(1,1) as their measure of the forecasted variances. This should enable our tests to be more revealing about specific differences in correlation estimators. We perform both an in-sample and an out-of-sample comparison of estimators, as opposed to Engle and Colacito (2006), that only test in-sample differences. We think that there is even more scope for an out-of-sample forecasting comparison, in light of the recent events that have affected the financial industry. Specifically, we are interested in two forecasting exercises. The first one uses the sample up to January 2008 to estimate models' parameters. The second one uses data up to September 2008 as the pre-sample. The two sample choices reflect roughly the information set of an investor at the onset of the Bear Stern collapse and Lehmann Brothers' bankruptcy, respectively.

In our empirical investigation, we focus on International stock market returns in five countries: United States, United Kingdom, Japan, France, and Germany. Data are collected at daily frequency over the January 1988- October 2009 sample period. All returns are expressed in US Dollars.[13] We construct portfolios of different sizes, by selecting all possible combinations of two, three, and four returns' series.

Table 7 reports the results of our analysis. At least three key findings seem to emerge. First, the DCC-MIDAS correlation outperforms the DCC estimator in a large majority of the cases. This is suggestive of the potential efficiency gains that can be obtained even in the context of a myopic, short-horizon asset allocation exercise, by modeling the low frequency correlation dynamics. Second, efficiency gains are as large as 360 basis points in sample and 260 basis points out of sample and the gains from better correlation information are consistently higher in sample than they are out-of-sample. These numbers are per se non negligible and even more so when compared to those reported in Engle and Colacito (2006). In their application, the largest benefits are obtained by comparing the DCC model with a constant unconditional estimate of volatilities and correlations. For the case of minimum variance portfolios, the efficiency gain is typically in a ballpark of 100 basis points or less. The DCC-MIDAS appears to substantially improve their findings. Last, but not least, the gains are

[13]Data source is MSCI-Barra. Series names are MSDUFR, MSDUGR, MSDUJN, MSDUUK, MSDUUS. With the exception of table 8, parameters' estimates are not reported in the paper for space constraints, but they are available upon request to the authors.

increasing in the size of the portfolio. In sample, there appears to be an efficiency improvement of about 100 basis points for each asset that is being added to the analysis. Modeling correlation's slowly moving components is therefore extremely important for large dimensional systems.

The case of the correlation between UK, Germany, and France is revealing of the potential source of the benefits from employing the DCC-MIDAS estimator. Figure 5 reports the forecasted correlations for the two alternative estimators. As suggested by Engle and Colacito (2006) this case is of particular interest in the context of a portfolio choice exercise, because the correlations are extremely large. Modeling the low-frequency dynamics of correlations enables the DCC-MIDAS to outperform the DCC estimator also at higher frequencies. This is mostly due to the fact that the MIDAS polynomial allows the long-run correlation to move frequently over time, therefore reducing the persistence of the short-run correlation dynamics (see table 8 for parameters' estimates).

We can push the result of the potential benefits from using the DCC-MIDAS estimator one step forward. Figure 6 shows the efficiency gains for portfolios of increasing size obtained from all possible permutations of G-7 countries' stock market returns. It is quite apparent that at least in sample, the gain is increasing in the size of the portfolio and can be as high as 450 basis points, when comparing the DCC-MIDAS estimator with the standard DCC model. The benefits are roughly 20%-40% larger when the competing estimator employs constant unconditional correlations.

6 Concluding remarks

We introduced a class of DCC-MIDAS component models of dynamic correlations with a short- and long-run component specification. The key ingredients are a combination of the Engle (2002) DCC model, the Engle and Lee (1999) component GARCH model to replace the original DCC dynamics with a component specification and the Engle, Ghysels, and Sohn (2006) GARCH-MIDAS component specification that allows us to extract a long-run correlation component via mixed data sampling. We addressed the specification, estimation and interpretation of correlation models that distinguish short and long run components. We show that the changes in correlations are indeed very different. An empirical illustration shows the benefits of the component specification. Empirical specification tests are introduced and applied. They reveal the superior empirical fit of the new class of DCC-MIDAS correlation models. While we left the regularity conditions that guarantee standard asymptotic results for the two-step estimation of DCC-MIDAS as an open question for future research we did cover one important part of the regularity conditions dealing with the positive definiteness of the MIDAS-filtered long run correlation component.

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Technical Appendix

Proofs of Lemmas

Proof of Lemma 1. Follows directly from decomposing the matrix as

$$\boldsymbol{\Phi}_l = \begin{pmatrix} \boldsymbol{\Phi}_l^D(n_a,\omega_r^a) & \mathbf{0} \\ \boldsymbol{\Phi}_l^F(n_a,n_b,\omega_r^c)' & \boldsymbol{I}_{n_b} \end{pmatrix} \cdot \begin{pmatrix} \boldsymbol{I}_{n_a} & \boldsymbol{\Phi}_l^D(n_a,\omega_r^a)^{-1}\boldsymbol{\Phi}_l^F(n_a,n_b,\omega_r^c) \\ \mathbf{0} & \boldsymbol{\Phi}_l^D\left(n_b,\omega_r^b\right) - \boldsymbol{\Phi}_l^F(n_a,n_b,\omega_r^c)'\boldsymbol{\Phi}_l^D(n_a,\omega_r^a)^{-1}\boldsymbol{\Phi}_l^F(n_a,n_b,\omega_r^c) \end{pmatrix}$$

□

Proof of Lemma 2. It follows directly from observing that any leading principal minor of $\boldsymbol{\Phi}_l^D(n_a,\omega_r^a)$ can be written as

$$det_k = (1-\varphi_l(\omega_r^a))^{k-1}(1+(k-1)\varphi_l(\omega_r^a)), \quad \forall k \geq 1$$

□

Proof of Lemma 3. Since

$$det_{BCAC} = -n_a n_b \frac{\left(1-\varphi_l(\omega_r^b)\right)^{n_b-1}}{1+(1+n_a)\varphi_l(\omega_r^a)}[\varphi_l(\omega_r^c)]^2 + \left(1-\varphi_l(\omega_r^b)\right)^{n_b-1}\left(1+(n_b-1)\varphi_l(\omega_r^b)\right)$$

it amounts to showing that

$$\frac{\left(1-\varphi_l(\omega_r^b)\right)^{n_b-1}}{1+(1+n_a)\varphi_l(\omega_r^a)} \leq 1$$

This is always the case since the numerator is always smaller than unity (because $\varphi_l(\omega_r^b) \leq 1$ by assumption) and the denominator is always larger than one (because $\varphi_l(\omega_r^a) \geq 0$ by assumption).

□

Proof of Lemma 4. To simplify the proof we slightly modify the notation for $\zeta\left(\varphi_l(\omega_r^b),\varphi_l(\omega_r^c),n_b\right)$

to $\zeta(n_b)$, suppressing the other arguments. Now, denote $\zeta(n_b)$ as the difference

$$\zeta(n_b) = p(n_b) - q(n_b)$$

where

$$\begin{aligned} p(n_b) &= \left(1-\varphi_l(\omega_r^b)\right)^{n_b-1}\left(1+(n_b-1)\varphi_l(\omega_r^b)\right) \\ q(n_b) &= n_a n_b \left[\varphi_l(\omega_r^c)\right]^2 \end{aligned}$$

The term $-q(n_b)$ is trivially always decreasing in n_b. The term $p(n_b)$ can be written as

$$p(n_b) = 1 + \left[\varphi_l(\omega_r^b)\right]^2 \sum_{j=1}^{n_b-1} (-1)^j \cdot j \cdot (\varphi_l(\omega_r^b) - 1)^{j-1}$$

with $p(n_b = 1) = 1$. The increments:

$$p(n_b) - p(n_b - 1) = \left[\varphi_l(\omega_r^b)\right]^2 (-1)^{n_b-1} \cdot (n_b - 1) \cdot (\varphi_l(\omega_r^b) - 1)^{n_b-2}$$

are always negative, because if n_b is odd (even), the term $(-1)^{n_b-1}$ is positive (negative), while the term $(\varphi_l(\omega_r^b) - 1)^{n_b-2}$ is negative (positive), since $\varphi_l(\omega_r^b) \leq 1$, by assumption.

□

Proof of Lemma 5. Similar to the previous proof, we slightly modify the notation for $\zeta\left(\varphi_l(\omega_r^b), \varphi_l(\omega_r^c), n_b\right)$ to $\zeta\left(\varphi_l(\omega_r^b), \varphi_l(\omega_r^c)\right)$, suppressing this time the n_b argument. Now, decompose $\zeta\left(\varphi_l(\omega_r^b), \varphi_l(\omega_r^c)\right)$ as

$$\zeta\left(\varphi_l(\omega_r^b), \varphi_l(\omega_r^c)\right) = p\left(\varphi_l(\omega_r^b)\right) - q\left(\varphi_l(\omega_r^c)\right)$$

where

$$\begin{aligned} p\left(\varphi_l(\omega_r^b)\right) &= 1 + \sum_{j=1}^{n_b-1} \left[\varphi_l(\omega_r^b)\right]^2 (-1)^j \cdot j \cdot \left[\varphi_l(\omega_r^b)\right]^{j-1} \\ &= 1 + \sum_{j=1}^{n_b-1} p_j\left(\varphi_l(\omega_r^b)\right) \\ q\left(\varphi_l(\omega_r^c)\right) &= n_a n_b \left[\varphi_l(\omega_r^c)\right]^2 \end{aligned}$$

The term $-q\left(\varphi_l(\omega_r^c)\right)$ is trivially decreasing in $\varphi_l(\omega_r^c)$. For the other term:

$$\begin{aligned} p_j\left(\varphi_l(\omega_r^b)\right) &= -\left[\varphi_l(\omega_r^b)\right]^2 (-1)^j \cdot j \cdot \left|\varphi_l(\omega_r^b)\right|^{j-1} \\ &\leq -\left[\varphi_{i+k}(\omega_r^b)\right]^2 (-1)^j \cdot j \cdot \left|\varphi_{i+k}(\omega_r^b)\right|^{j-1} = p_j\left(\varphi_{i+k}(\omega_r^b)\right), \quad \forall j = \{1, \cdots, n_b - 1\} \end{aligned}$$

□

Details on industry portfolios classification

Data are download from Kenneth French web-site. The Energy, Manufacturing, Hi-Tech, and Retail portfolios that we use in the empirical section correspond to the collection of the following SIC codes:

1. Energy: Oil, Gas, and Coal Extraction and Products
   SIC codes: 1200-1399, 2900-2999

2. Manufacturing: Machinery, Trucks, Planes, Chemicals, Off Furn, Paper, Com Printing
   SIC codes: 2520-2589, 2600-2699, 2750-2769, 2800-2829, 2840-2899, 3000-3099, 3200-3569, 3580-3621, 3623-3629, 3700-3709, 3712-3713, 3715-3715, 3717-3749, 3752-3791, 3793-3799, 3860-3899

3. Hi-Tech: Computers, Software, and Electronic Equipment, Industrial controls, computer programming and data processing, Computer integrated systems design, computer processing, data prep, information retrieval services, computer facilities management service, computer rental and leasing, computer maintenance and repair, computer related services, R&D labs, research, development, testing labs
   SIC codes: 3570-3579, 3622-3622, 3660-3692, 3694-3699, 3810-3839, 7370-7372, 7373-7373, 7374-7374, 7375-7375, 7376-7376, 7377-7377, 7378-7378, 7379-7379, 7391-7391, 8730-8734

4. Retail: Wholesale, Retail, and Some Services (Laundries, Repair Shops)
   SIC codes: 5000-5999, 7200-7299, 7600-7699

Summary of specifications

In the empirical section, we will analyze the performance of several combinations of short- and long-run specifications for a number of 3 by 3 systems. To simplify the reading of the results, we summarize and label the models in this sub-section.

1. MIDAS=1: the typical MIDAS correlation weighting matrix is

$$\mathbf{\Phi}_l\left(\omega_1\right) = \begin{bmatrix} 1 & \varphi_l(\omega_1) & \varphi_l(\omega_1) \\ \varphi_l(\omega_1) & 1 & \varphi_l(\omega_1) \\ \varphi_l(\omega_1) & \varphi_l(\omega_1) & 1 \end{bmatrix}$$

2. MIDAS=2: the typical MIDAS correlation weighting matrix is

$$\mathbf{\Phi}_l\left(\omega_1, \omega_2\right) = \begin{bmatrix} 1 & \varphi_l(\omega_2) & \varphi_l(\omega_1) \\ \varphi_l(\omega_2) & 1 & \varphi_l(\omega_1) \\ \varphi_l(\omega_1) & \varphi_l(\omega_1) & 1 \end{bmatrix}$$

   Hence, the second MIDAS polynomial insists on the correlation of the first two assets with the third asset.

3. DCC=1: the short-run dynamics are governed by the scalars a_1 and b_1.

4. DCC=2: the short-run dynamics are described by the following matrices:

$$A = \begin{bmatrix} a_2 & a_2 & a_1 \\ a_2 & a_2 & a_1 \\ a_1 & a_1 & a_1 \end{bmatrix} \qquad B = \begin{bmatrix} b_2 & b_2 & b_1 \\ b_2 & b_2 & b_1 \\ b_1 & b_1 & b_1 \end{bmatrix}$$

5. DCC=3: the short-run dynamics are described by the following matrices:

$$A = \begin{bmatrix} a_2 & a_2 & a_3 \\ a_2 & a_2 & a_3 \\ a_3 & a_3 & a_1 \end{bmatrix} \qquad B = \begin{bmatrix} b_2 & b_2 & b_3 \\ b_2 & b_2 & b_3 \\ b_3 & b_3 & b_1 \end{bmatrix}$$

6. DCC=2 (G): the short-run dynamics are described by the following matrices:

$$A = \begin{bmatrix} a_2 & a_2 & \sqrt{a_1 a_2} \\ a_2 & a_2 & \sqrt{a_1 a_2} \\ \sqrt{a_1 a_2} & \sqrt{a_1 a_2} & a_1 \end{bmatrix} \qquad B = \begin{bmatrix} b_1 & b_1 & \sqrt{b_1 b_2} \\ b_1 & b_1 & \sqrt{b_1 b_2} \\ \sqrt{b_1 b_2} & \sqrt{b_1 b_2} & b_2 \end{bmatrix}$$

TABLE 1
ENERGY, HI-TECH AND 10 YEAR BOND

	μ	α	β	θ	ω	m
Energy	0.070	0.087	0.804	0.199	12.602	0.545
	(0.000)	(0.000)	(0.016)	(0.065)	(0.000)	(0.123)
Hi-Tech	0.063	0.087	0.837	0.186	9.997	0.726
	(0.000)	(0.064)	(0.001)	(0.000)	(0.000)	(0.332)
Bond	0.022	0.059	0.915	0.204	3.090	0.284
	(0.002)	(0.000)	(0.000)	(0.002)	(0.000)	(0.000)

	a	b	ω
DCC-MIDAS	0.018	0.977	1.683
	(0.004)	(0.000)	(0.000)
DCC	0.016	0.981	–
	(0.001)	(0.001)	–

Notes - The top panel reports the estimates of the GARCH-MIDAS coefficients for the Energy portfolio, Hi-Tech portfolio and 10 year Bond. The bottom panel reports the estimates of the DCC-MIDAS and original DCC parameters. The number of MIDAS lags is 36 for the GARCH processes and 144 for the DCC process. The sample covers 1971-07-15 until 2006-06-30.

TABLE 2
ENERGY, HI-TECH, AND 10 YEAR BOND: MULTIPLE MIDAS AND DCC

	a_1	a_2	a_3	b_1	b_2	b_3	ω_1	ω_2	$Log - Likelihood$	AIC	BIC
DCC=1	0.018	–	–	0.977	–	–	1.683	–	−11990.18	23,986.36	24,007.62
MIDAS=1	(0.004)	(–)	(–)	(0.000)	(–)	(–)	(0.000)	(–)			
DCC=1	0.018	–	–	0.977	–	–	1.368	3.607	−11989.05	23,986.09	24,014.44
MIDAS=2	(0.000)	(–)	(–)	(0.001)	(–)	(–)	(0.031)	(0.022)			
DCC=2	0.021	0.017	–	0.976	0.976	–	1.708	–	−11986.53	23,983.06	24,018.49
MIDAS=1	(0.000)	(0.000)	(–)	(0.000)	(0.000)	(–)	(0.000)	(–)			
DCC=2	0.021	0.016	–	0.976	0.976	–	1.527	3.738	−11984.18	23,980.35	24,022.87
MIDAS=2	(0.000)	(0.000)	(–)	(0.000)	(0.000)	(–)	(0.000)	(0.000)			
DCC=2 (G)	0.023	0.017	–	0.973	0.982	–	2.595	–	−11985.16	23,980.31	24,015.74
MIDAS=1	(0.031)	(0.028)	(–)	(0.028)	(0.003)	(–)	(0.046)	(–)			
DCC=2 (G)	0.023	0.011	–	0.973	0.981	–	1.516	4.157	−11983.59	23,979.19	24,021.70
MIDAS=2	(0.023)	(0.031)	(–)	(0.027)	(0.018)	(–)	(0.102)	(0.022)			
DCC=3	0.021	0.013	0.016	0.977	0.976	0.977	1.942	–	−11986.11	23,986.20	24,035.80
MIDAS=1	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(–)			
DCC=3	0.021	0.014	0.016	0.976	0.977	0.976	1.396	3.684	−11984.01	23,984.00	24,040.69
MIDAS=2	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)			

Notes - Each row reports the estimated coefficients, the log-likelihood, and the Akaike and Schwartz information criteria for the Energy, Hi-Tech, and 10 year Bond Portfolio for an increasing number of DCC and MIDAS parameters. When multiple sets of parameters are used, the second DCC and/or MIDAS coefficients are applied only to the correlation of Energy vs Hi-Tech, except for the case of the generalized DCC, in which the product of the two parameters affects also all other correlations. For the case of 3 DCC sets of parameters, the third coefficient is applied to the correlations of Energy and Hi-Tech vs Bond. The number of MIDAS lags is 36 for the GARCH processes and 144 for the DCC process. The sample starts on 1971-07-15 and ends on 2006-06-30.

TABLE 3

ENERGY, HI-TECH AND 10 YRS BOND: LIKELIHOOD RATIO TESTS

		DCC=1 MIDAS=1	DCC=1 MIDAS=2	DCC=2 MIDAS=1	DCC=2 MIDAS=2	DCC=2 (G) MIDAS=1	DCC=2 (G) MIDAS=2	DCC=3 MIDAS=1	DCC=3 MIDAS=2
DCC=1	MIDAS=1	–	0.132	0.026*	0.007**	0.007**	0.004**	0.086	0.030*
DCC=1	MIDAS=2	–	–	–	0.008**	–	0.004**	–	0.039*
DCC=2	MIDAS=1	–	–	–	0.030*	–	–	0.651	0.167
DCC=2	MIDAS=2	–	–	–	–	–	–	–	0.839
DCC=2 (G)	MIDAS=1	–	–	–	–	–	0.077	0.388	0.511
DCC=2 (G)	MIDAS=2	–	–	–	–	–	–	–	0.666
DCC=3	MIDAS=1	–	–	–	–	–	–	–	0.040*

Notes - Each entry represents the p-value for testing that the likelihood of the model on the column is significantly higher than the likelihood of the model on the corresponding row. When multiple sets of parameters are used, the second DCC and/or MIDAS coefficients are applied only to the correlation of Energy vs Hi-Tech, except for the case of the generalized DCC, in which the product of the two parameters affects also all other correlations. For the case of 3 DCC sets of parameters, the third coefficient is applied to the correlations of Energy and Hi-Tech vs Bond. The number of MIDAS lags is 36 for the GARCH processes and 144 for the DCC process. The sample starts on 1971-07-15 and ends on 2006-06-30.

TABLE 4
MANUFACTURING, SHOPS, AND 10 YEAR BOND

	μ	α	β	θ	ω	m
Bond	0.022	0.059	0.914	0.204	3.090	0.284
	(0.001)	(0.000)	(0.000)	(0.002)	(0.000)	(0.000)
Manufacturing	0.072	0.103	0.801	0.175	10.925	0.564
	(0.000)	(0.000)	(0.078)	(0.019)	(0.005)	(0.082)
Shops	0.069	0.101	0.816	0.171	11.529	0.632
	(0.001)	(0.002)	(0.000)	(0.003)	(0.001)	(0.013)

	a	b	ω
DCC-MIDAS	0.029	0.954	11.680
	(0.000)	(0.000)	(1.215)
DCC	0.217	0.975	–
	(0.000)	(0.000)	–

Notes - The top panel reports the estimates of the GARCH-MIDAS coefficients for the 10 year Bond, Manufacturing portfolio, and the Shops portfolio. The bottom panel reports the estimates of the DCC-MIDAS and of the original DCC parameters. The number of MIDAS lags is 36 for the GARCH processes and 144 for the DCC process. The sample covers 1971-07-15 until 2006-06-30.

TABLE 5
MANUFACTURING, SHOPS, AND 10 YEAR BOND: MULTIPLE MIDAS AND DCC

	a_1	a_2	a_3	b_1	b_2	b_3	ω_1	ω_2	$Log - Likelihood$	AIC	BIC
DCC=1	0.029	–	–	0.954	–	–	11.68	–	−9242.53	18,491.06	18,512.32
MIDAS=1	(0.000)	(–)	(–)	(0.000)	(–)	(–)	(1.216)	(–)			
DCC=1	0.030	–	–	0.950	–	–	7.585	26.300	−9237.99	18484	18512.32
MIDAS=2	(0.000)	(–)	(–)	(0.000)	(–)	(–)	(0.000)	(0.000)			
DCC=2	0.0357	0.034	–	0.943	0.943	–	14.297	–	−9238.863	18487.73	18526.15
MIDAS=1	(0.022)	(0.025)	(–)	(0.002)	(0.003)	(–)	(0.001)	(–)			
DCC=2	0.036	0.034	–	0.943	0.942	–	7.768	27.029	−9233.92	18,479.85	18,522.37
MIDAS=2	(0.000)	(0.000)	(–)	(0.000)	(0.001)	(–)	(0.000)	(0.000)			
DCC=2 (G)	0.034	0.031	–	0.932	0.955	–	16.818	–	−9240.41	18,490.82	18,526.25
MIDAS=1	(0.000)	(0.000)	(–)	(0.000)	(.000)	(–)	(0.000)	(–)			
DCC=2 (G)	0.033	0.029	–	0.939	0.956	–	9.681	26.949	−9237.38	18,486.76	18,529.27
MIDAS=2	(0.062)	(0.132)	(–)	(0.011)	(0.028)	(–)	(0.113)	(0.111)			
DCC=3	0.034	0.0341	0.033	0.943	0.955	0.948	12.371	–	−9233.81	18,481.63	18,531.23
MIDAS=1	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(–)			
DCC=3	0.035	0.034	0.033	0.941	0.952	0.945	8.253	24.663	−9230.61	18,477.20	18,533.89
MIDAS=2	(0.000)	(0.013)	(0.015)	(0.019)	(0.008)	(0.020)	(0.022)	(0.022)			

Bivariate Models

	a	b	ω	Log-Likelihood
Bond vs Manufacturing	0.038	0.946	7.037	−9733.577
	(0.000)	(0.000)	(0.000)	
Bond vs Shops	0.038	0.936	7.608	−9818.094
	(0.001)	(0.003)	(0.000)	
Manufacturing vs Shops	0.031	0.952	24.587	−6939.103
	(0.001)	(0.000)	(0.000)	

Notes - Each row of the top panel reports the estimated coefficients, the log-likelihood, and the Akaike and Schwartz information criteria for the Manufacturing, Shops, and 10 year Bond Portfolio for an increasing number of DCC and MIDAS parameters. When multiple sets of parameters are used, the second DCC and/or MIDAS coefficients are applied only to the correlation of Manufacturing vs Shops, except for the case of the generalized DCC, in which the product of the two parameters affects also all other correlations. For the case of 3 DCC sets of parameters, the third coefficient is applied to the correlations of Manufacturing and Shops vs Bond. The bottom panel reports the estimates of the DCC-MIDAS model for the three bivariate systems obtained from all possible permutations of Bond, Manufacturing, and Shops. The number of MIDAS lags is 36 for the GARCH processes and 144 for the DCC process. The sample starts on 1971-07-15 and ends on 2006-06-30.

TABLE 6

10 YRS BOND, MANUFACTURING AND SHOPS: LIKELIHOOD RATIO TESTS

		DCC=1 MIDAS=1	DCC=1 MIDAS=2	DCC=2 MIDAS=1	DCC=2 MIDAS=2	DCC=2 (G) MIDAS=1	DCC=2 (G) MIDAS=2	DCC=3 MIDAS=1	DCC=3 MIDAS=2
DCC=1	MIDAS=1	—	0.003**	0.026*	0.000**	0.12	0.016*	0.000**	0.000**
DCC=1	MIDAS=2	—	—	—	0.017*	—	0.538	—	0.005**
DCC=2	MIDAS=1	—	—	—	0.002**	—	—	0.006**	0.000**
DCC=2	MIDAS=2	—	—	—	—	—	—	—	0.036*
DCC=2 (G)	MIDAS=1	—	—	—	—	—	0.014*	0.001**	0.000**
DCC=2 (G)	MIDAS=2	—	—	—	—	—	—	—	0.001**
DCC=3	MIDAS=1	—	—	—	—	—	—	—	0.011*

Notes - Each entry represents the p-value for testing that the likelihood of the model on the column is significantly higher than the likelihood of the model on the corresponding row. When multiple sets of parameters are used, the second DCC and/or MIDAS coefficients are applied only to the correlation of Manufacturing vs Shops, except for the case of the generalized DCC, in which the product of the two parameters affects also all other correlations. For the case of 3 DCC sets of parameters, the third coefficient is applied to the correlations of Manufacturing and Shops vs Bond. The number of MIDAS lags is 36 for the GARCH processes and 144 for the DCC process. The sample starts on 1971-07-15 and ends on 2006-06-30.

TABLE 7
DIEBOLD AND MARIANO TESTS ON INTERNATIONAL PORTFOLIOS

Countries	In Sample	9/1/2008-onward	1/1/2008-onward
US, UK	0.395	0.097**	0.131***
	(-1.407)	(-2.047)	(-2.631)
US, GER	0.13	-0.043	-0.083**
	(-0.564)	(0.827)	(2.274)
US, JPN	1.2***	0.331***	0.219***
	(-2.377)	(-2.647)	(-2.914)
US, FRA	0.371	-0.159**	-0.104***
	(-1.273)	(2.409)	(3.762)
UK, GER	1.04	-0.153	0.079
	(-1.002)	(0.809)	(-0.349)
UK, JPN	0.955***	0.148	0.152*
	(-3.106)	(-1.568)	(-1.792)
UK, FRA	-0.286	-0.095	0.198*
	(1.204)	(1.283)	(-1.778)
GER, JPN	1.167***	0.302**	0.223*
	(-3.070)	(-2.002)	(-1.638)
GER, FRA	2.603	2.326	2.149
	(-1.437)	(-1.144)	(-1.123)
JPN, FRA	1.166***	0.214	0.228*
	(-3.109)	(-1.419)	(-1.638)
US, UK, GER	0.735	-0.64**	-0.465**
	(-1.231)	(2.214)	(2.422)
US, UK, JPN	2.023***	0.514***	0.409***
	(-3.326)	(-3.204)	(-3.069)
US, UK, FRA	0.307	-0.372***	-0.55
	(-0.745)	(2.629)	(0.355)
US, GER, JPN	1.719***	0.295	0.225
	(-2.999)	(-1.608)	(-1.411)
US, GER, FRA	2.638	1.484	1.383
	(-1.316)	(-0.958)	(-0.915)
US, JPN, FRA	2.036***	0.699***	0.477***
	(-3.444)	(-2.874)	(-2.539)
UK, GER, JPN	1.527*	-0.152	0.168
	(-1.877)	(0.402)	(-0.646)
UK, GER, FRA	2.915	1.871	2.01
	(-1.232)	(-1.292)	(-1.310)
UK, JPN, FRA	0.811**	-0.081	0.041
	(-2.086)	(0.623)	(-0.377)
GER, JPN, FRA	3.528**	2.626	2.387
	(-2.010)	(-1.175)	(-1.247)
US, UK, GER, JPN	2.246***	-0.114	-0.03
	(-2.890)	(0.505)	(0.165)
US, UK, GER, FRA	2.824	0.771	1.138
	(-1.373)	(-0.731)	(-0.967)
UK, GER, JPN, FRA	3.613**	0.958	0.812
	(-2.258)	(-0.864)	(-0.784)
% of DCC-MIDAS lower variance	95.65	60.87	78.26
Average DCC-MIDAS Gain	1.55	0.42	0.49
Bivariate DCC-MIDAS gain	0.87	0.30	0.32
Tri-variate DCC-MIDAS Gain	1.82	0.63	0.61
Four-variate DCC-MIDAS Gain	2.89	0.54	0.64

Notes - Each column reports the percentage efficiency gains (losses) from using the DCC-MIDAS estimator, computed as the ratio of (5.2) and (5.3). The numbers in parenthesis are t-stats for the associated Diebold and Mariano test. One, two and three stars denote significance at 10%, 5%, and 1% confidence levels. The last four lines summarize the percentage gains, by breaking them into portfolio sizes.

TABLE 8
GERMANY VS. FRANCE

	a	b	ω
DCC-MIDAS	0.042	0.921	4.619
	(0.000)	(0.000)	(0.000)
DCC	0.026	0.973	–
	(0.005)	(0.005)	–

Notes - Estimates of the DCC-MIDAS and original DCC parameters for the system including Germany and France. The sample is 1988:1 to 2009:10.

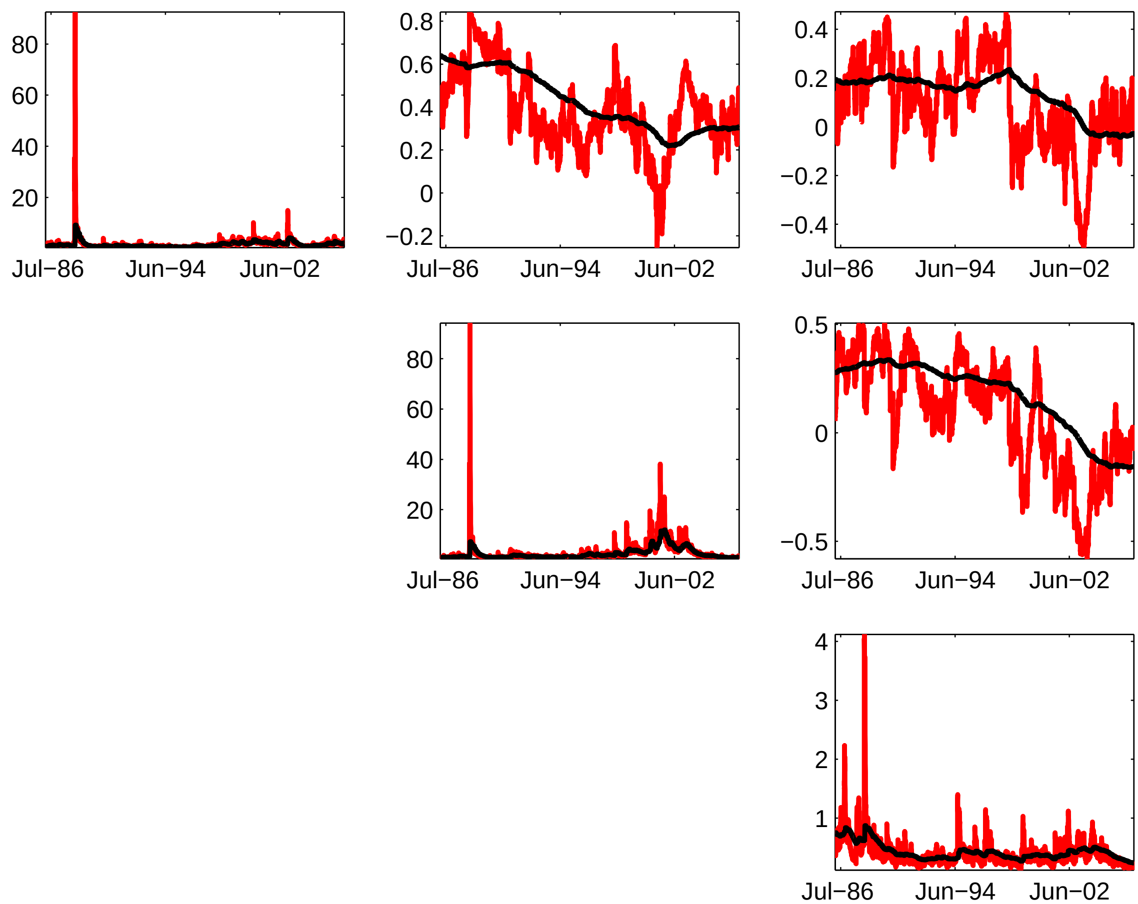

FIG. 1 - Long and short run volatilities and correlations for the energy and hi-tech portfolios and the 10 year bond. The pictures on the main diagonal refer to conditional variances of energy and hi-tech portfolios and of 10 year bond and those on the off diagonal report conditional correlations among the same group of asset returns. In each diagonal panel the dark line refers to the long-run volatility and the light line represents the short-run volatility. In each off-diagonal panel the dark line is the long-run correlation and the light line is the total correlation.

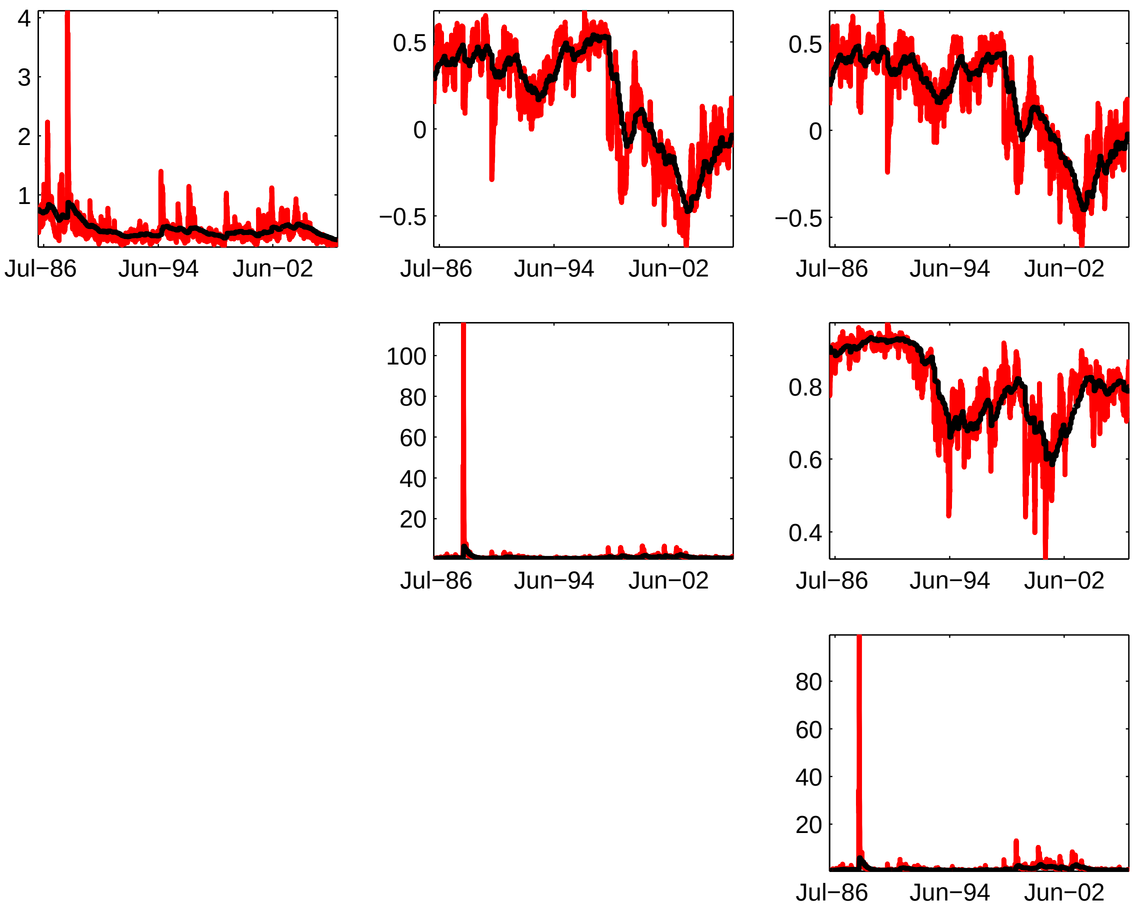

FIG. 2 - Long and short run volatilities and correlations for the 10 year bond and Manufacturing and Shops portfolios. The pictures on the main diagonal refer to conditional variances of bond, manufacturing and shops and those on the off diagonal report conditional correlations among the same group of asset returns. In each diagonal panel the dark line refers to the long-run volatility and the light line represents the short-run volatility. In each off-diagonal panel the dark line is the long-run correlation and the light line is the total correlation.

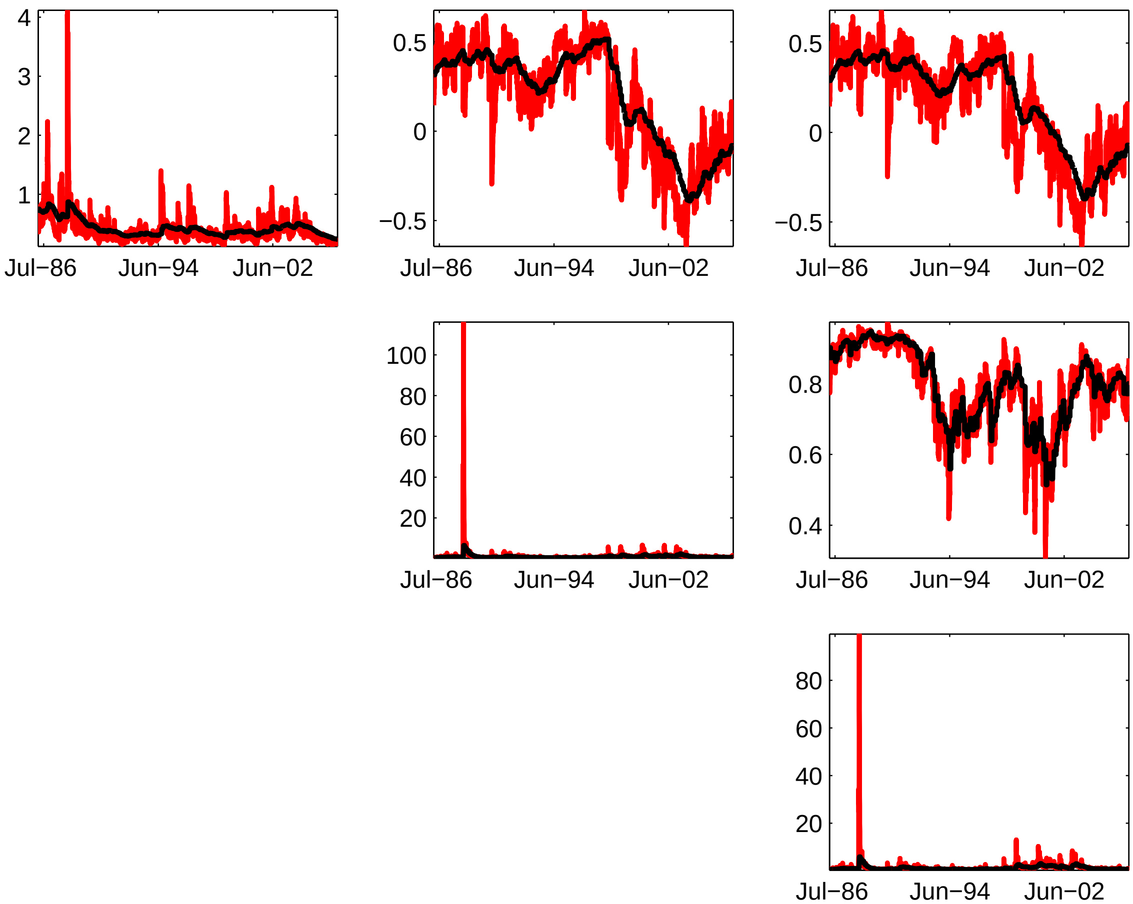

FIG. 3 - Long and short run volatilities and correlations for the 10 year bond and Manufacturing and Shops portfolios with 2 MIDAS filters. The second MIDAS filter is applied to the correlation between the manufacturing and the shops portfolios. The pictures on the main diagonal refer to conditional variances of bond, manufacturing and shops and those on the off diagonal report conditional correlations among the same group of asset returns. In each diagonal panel the dark line refers to the long-run volatility and the light line represents the short-run volatility. In each off-diagonal panel the dark line is the long-run correlation and the light line is the total correlation.

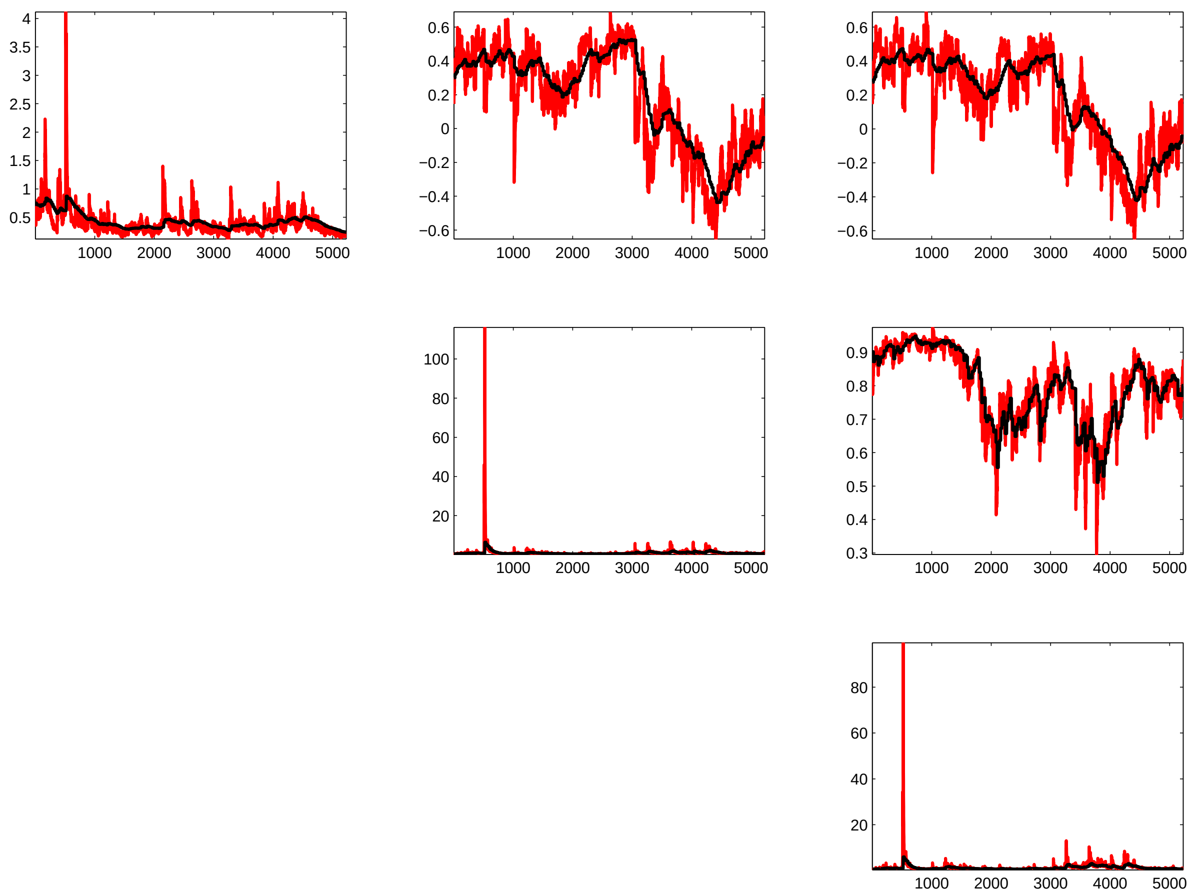

FIG. 4 - Long and short run volatilities and correlations for the 10 year bond and Manufacturing and Shops portfolios using the Generalized DCC-MIDAS with 2 DCC set of parameters and 2 MIDAS filters. The second set of parameters is applied to the correlation between the manufacturing and the shops portfolios. The pictures on the main diagonal refer to conditional variances of bond, manufacturing and shops and those on the off diagonal report conditional correlations among the same group of asset returns. In each diagonal panel the dark line refers to the long-run volatility and the light line represents the short-run volatility. In each off-diagonal panel the dark line is the long-run correlation and the light line is the total correlation.

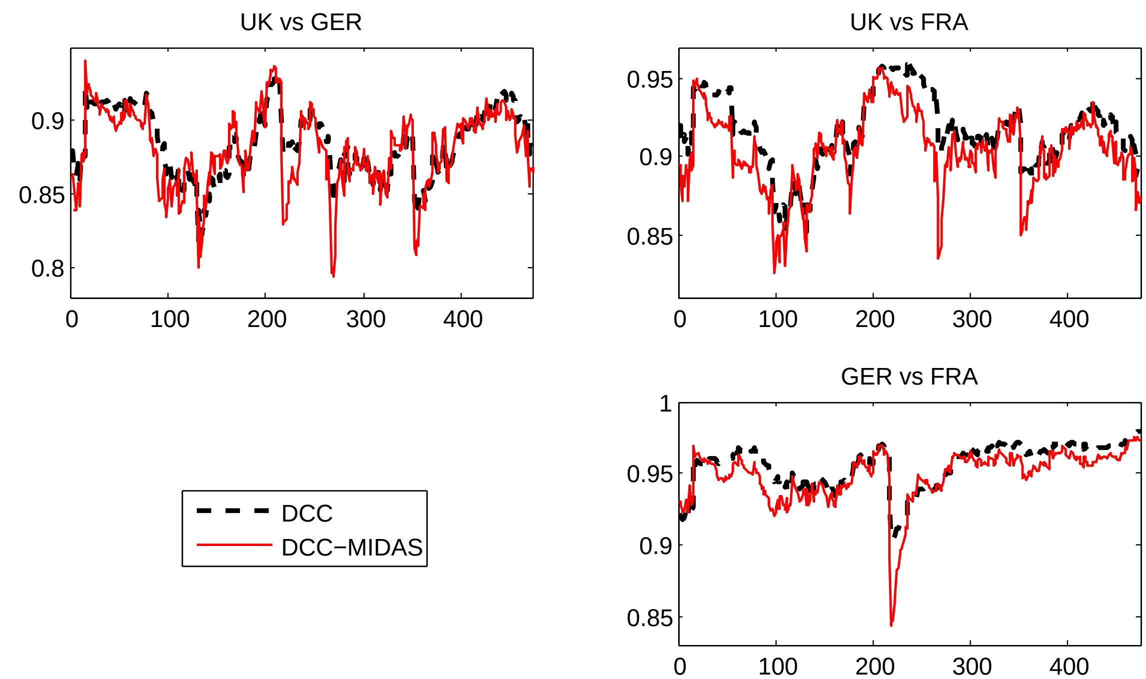

FIG. 5 - Out of sample forecasts of correlations for the DCC and the DCC-MIDAS model. The system consists of the asset returns for UK, Germany, and France. The parameters of the model are estimated on the pre-sample ending on 12/31/2007. The estimates of the correlation parameters are reported in table 8.

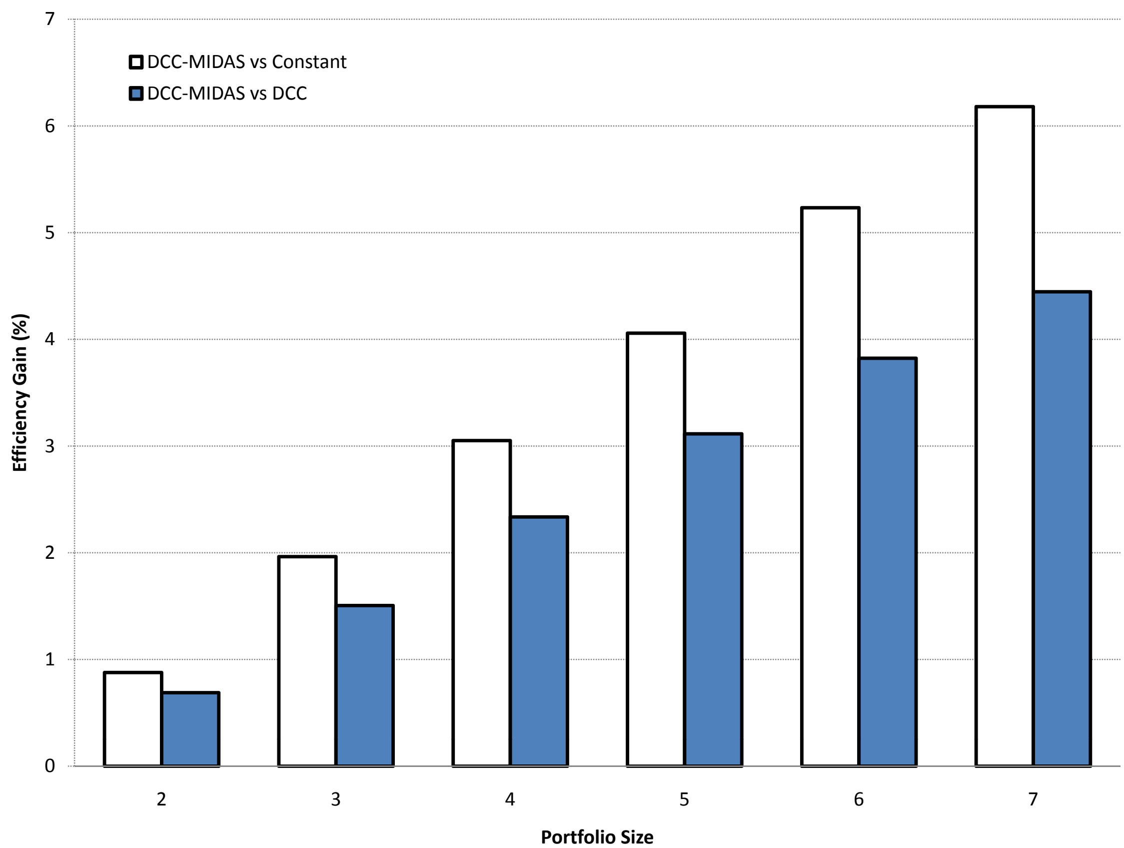

FIG. 6 - Efficiency gains by portfolio size. Each bar reports the average efficiency gain for all the permutations of G-7 countries' stock market returns in a portfolio of the size displayed on the horizontal axis. The white bars refer to the efficiency gain from using the DCC-MIDAS model instead of a constant unconditional correlation measure. The dark bars are efficiency gains for the DCC-MIDAS over the standard DCC model.