HYBRID GARCH Models and Intra-Daily Return Periodicity*

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Abstract

We use the HYBRID GARCH model of Chen, Ghysels, and Wang (2009) to predict future volatility at daily horizons using intra-daily returns. The latter requires us to address intra-daily periodic patterns. We propose two approaches and compare their relative merits. The first approach uses raw intra-daily data - with the HYBRID process capturing the intra-daily periodic patterns - whereas the second approach involves pre-adjusted intra-daily returns. We find that the former approach dominates both in-sample and out-of-sample, although for different HYBRID GARCH model specifications.

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1 Introduction

Writing a paper on seasonality and GARCH honors Svend Hylleberg in more than one way. Obviously, his work on seasonality is well known and at the frontier of the field. Indeed, much has been written on the topic of seasonality in economic time series by Svend Hylleberg and others (see e.g. two monographs devoted on the subject - Hylleberg (1992) and Ghysels and Osborn (2001)). His connection with GARCH is more indirect, as a close friend of Rob Engle and the advisor of Tim Bollerslev.

Financial return series feature a specific type of pattern, known as intra-daily periodicity or diurnal effects. Wood, McInish, and Ord (1985), one of the earliest studies employing intra-daily data, documents the well known U-shaped pattern. Although the inverse U-shaped pattern of volatility and trading during the day has been documented across different markets, the reasons for this phenomenon are not entirely understood. Admati and Pfleiderer (1988) consider a partial equilibrium setting where uninformed liquidity traders have a discretion over the timing of their trades. They show that periods of concentrated trading and volatility can arise, with heavy trading by both informed and uninformed traders. In their analysis, trade clustering appears when the private signals of the informed traders are highly correlated. A few papers have studied price discovery and the opening mechanism of markets and their impact on volatility, see e.g. Biais, Hillion, and Spatt (1999), Cao, Ghysels, and Hatheway (2000), Madhavan and Panchapagesan (2000), among others. Other sources of diurnal effects are driven by institutional arrangements. Macroeconomic news is typically announced prior to the opening of markets - causing high volatility at the beginning of trading. End-of-day increased volatility have been, relatively speaking, less studied - see however inter alia Forster and George (1996).

The authors introduced recently a class of **H**igh Frequenc**Y** Data-**B**ased P**R**oject**I**on-**D**riven GARCH, or HYBRID GARCH models that allows a mixture of frequencies in terms of prediction horizons and conditioning information (i.e. past returns). Loosely speaking, this is a 'GARCH-version' of Ghysels, Santa-Clara, and Valkanov (2005), without ARCH-in-Mean and allowing for intra-daily periodic patterns. Broadly speaking there are two approaches to seasonality: (1) pre-filter series and construct non-periodic models subsequently, or (2) build periodic features of the data into the model specification. Intra-daily seasonality has been tackled also along the two aforementioned approaches.[1] In the current paper we apply this

[1]See e.g. Andersen and Bollerslev (1997), Andersen and Bollerslev (1998), Bollen and Inder (2002),

class of models to intra-daily data and compare the two approaches to intra-daily periodicity, namely accommodate for it within the model or pre-filter the returns prior to their use. The class of HYBRID GARCH models is versatile and flexible to allow for both within the same framework, which makes the comparison appealing. The paper is exclusively focused on empirical implementation - exploiting the versatility of the HYBRID GARCH setting.

The remainder of the paper is organized as follows. Section 2 describes the model specification. Section 3 reports the empirical results. The last section provides a summary and concluding remarks.

2 Model Specifications

In this section we briefly review the HYBRID GARCH model introduced in Chen, Ghysels, and Wang (2009) and then focus on the specification of intra-daily periodicity - the main topic of the paper. A subsection is devoted to each topic.

2.1 HYBRID GARCH models

A generic HYBRID GARCH model has the following dynamics for volatility:

$$V_{\tau+1|\tau} = \tilde{a} + \tilde{b} V_{\tau|\tau-1} + \tilde{c} H_\tau \tag{2.1}$$

We call H_τ a HYBRID process. When H_τ is simply a daily squared return we have the volatility dynamics of a standard daily GARCH(1,1), or H_τ a weekly squared return those of a standard weekly GARCH(1,1). However, what would happen if we want to attribute an individual weight to each of the five days in a week. Or what would happen if we weight intra-daily data differently. This is an example of a parameter-driven HYBRID process $H_\tau \equiv H(\theta, \vec{r}_\tau)$ where $\vec{r}_\tau = (r_{1,\tau}, r_{2,\tau}, \ldots, r_{m-1,\tau}, r_{m,\tau})^T$ is $\mathbb{R}^m-$valued random vector. We will focus on cases involving intra-daily returns. The general setting of mixed frequency data is discussed in detail by Chen, Ghysels, and Wang (2009). They distinguish three cases: (1) data-driven HYBRID processes, (2) structural HYBRID processes and (3) HYBRID filtering processes. In case (1) the HYBRID process H_τ does not depend on parameters. The obvious

Dacorogna, Gençay, Müller, Olsen, and Pictet (2001), Martens, Chang, and Taylor (2002), Taylor and Xu (1997), among others.

case would be a simple return process such that $V_{\tau+1|\tau}$ is the conditional volatility of the next period. More recently, however, other purely data-driven examples of what we call generic HYBRID processes have been suggested. For example Engle and Gallo (2006), de Vilder and Visser (2008), Visser (2008), Shephard and Sheppard (2009) suggest the use of (daily) realized volatilities, high-low range or realized kernels or generic realized measures as they are called by Shephard and Sheppard (2009). The cases (2) and (3) are of direct interest to the present paper. Structural HYBRID processes appear in the context of temporal aggregation - a topic discussed extensively in the (weak) GARCH literature, see e.g. Drost and Nijman (1993), Drost and Werker (1996), Meddahi and Renault (2004), among others. Finally, the HYBRID process $H(\theta, \vec{r}_\tau)$ can involve parameters that are *not* explicitly related to $\tilde{a}$, $\tilde{b}$ and $\tilde{c}$ appearing in (2.1). There is no underlying high frequency data DGP that is being assumed, unlike in the structural HYBRID case. One can view this as a GARCH model driven by a filtered high frequency process - where the filter weights - (hyper-)parameterized by θ are estimated jointly with the volatility dynamics parameters.

To be more precise, we will consider the following specification for equation (2.1):

$$\begin{aligned} H(\theta, \vec{r}_\tau) &= \sum_{j=1}^{m} \omega_j r_{j,\tau}^2 \\ &= \left[\sum_{j=1}^{m} \exp\left(\sum_{i=1}^{j-1} (\theta_0 + \theta_1 i + \theta_2 i^2)\right) r_{m-j+1,\tau}^2\right] \end{aligned} \tag{2.2}$$

where the use of the (quadratic) Exponential Almon MIDAS filter, with $\theta = (\theta_0, \theta_1, \theta_2)$, is inspired by a similar approach proposed in Ghysels, Santa-Clara, and Valkanov (2005).[2] In its unconstrained form the unknown parameters $(\tilde{a}, \tilde{b}, \tilde{c}, \theta_0, \theta_1, \theta_2)$ are estimated without restrictions and the resulting HYBRID process is not directly related to any underlying high frequency data (i.e. intra-daily) return DGP. However, there will be many cases where the parameters $(\tilde{a}, \tilde{b}, \tilde{c})$ and $(\theta_0, \theta_1, \theta_2)$ are tied - yielding a structural HYBRID process that is related to a high frequency data DGP. In the subsequent subsections we will provide several examples.

To conclude it should be noted that various specifications of H_τ have profound implications for how one estimates HYBRID GARCH models and what type of underlying DGP is assumed. We do not elaborate on these topics here, as they are covered in the aforementioned

[2] We opted for the Exponential Almon because it yields a convenient approach to testing restrictions, as opposed to the Beta polynomial proposed by Chen and Ghysels (2009) to handle intra-daily periodicity in the context of a MIDAS regression. Note also that the Exponential Almon lags in equation (2.2) are constructed using the convention that $\sum_1^0 \cdot \equiv 0$, and they are not normalized to add up to one.

theoretical companion paper.

2.2 Intra-daily returns and HYBRID GARCH

We will start with a motivating example that is based on the periodic GARCH(1,1) model of Bollerslev and Ghysels (1996). In particular, consider m periods and the following volatility structure:

$$r_{i,\tau} = \sqrt{v_{i,\tau|i-1,\tau}}\varepsilon_{i,\tau}$$

and where

$$v_{i,\tau|i-1,\tau} = a_i + b_i v_{i-1,\tau|i-2,\tau} + c_i r^2_{i-1,\tau}$$

for $i = 2, \ldots, m$ (assuming the time point $(0, \tau)$ is equivalent to $(m, \tau - 1)$, e.g., $v_{1,\tau|0,\tau} \equiv v_{1,\tau|m,\tau-1}$) and for $i = 1$ we have:

$$r_{1,\tau} = \sqrt{v_{1,\tau|m,\tau-1}}\varepsilon_{1,\tau}$$

and where

$$v_{1,\tau|m,\tau-1} = a_1 + b_1 v_{m,\tau-1|m-1,\tau-1} + c_1 r^2_{m,\tau-1}$$

and $0 < b_i + c_i < 1 \; \forall \; i$. Therefore, for $0 \leq j < i \leq m$,

$$v_{i,\tau|j,\tau} = \sum_{h=j+2}^{i} \left(a_h \prod_{l=h+1}^{i} (b_l + c_l) \right) + \prod_{l=j+2}^{i} (b_l + c_l) v_{j+1,\tau|j,\tau}$$

with the conventions $\sum_{i=a}^{b} A_i = 0$ and $\prod_{i=a}^{b} A_i = 1$ if $a > b$ for arbitrary A_i. Let the accumulated volatility over the period τ be denoted as $V_{\tau|\tau-1} \equiv \sum_{i=1}^{m} v_{i,\tau|m,\tau-1}$. This periodic structure yields the following HYBRID GARCH dynamics:

$$V_{\tau+1|\tau} = \tilde{a} + \tilde{b} V_{\tau|\tau-1} + \tilde{c} H(\theta, \vec{r}_\tau) \tag{2.3}$$

where $\tilde{a} = \tilde{a}_p(m)$, $\tilde{b} = \left(\prod_{i=1}^{m} b_i\right)$, $\tilde{c} = \tilde{c}_p(m)$ with $a_{m+1} \equiv a_1$, $b_{m+1} \equiv b_1$ and $c_{m+1} \equiv c_1$. Moreover, $H(\theta, \vec{r}_\tau)$, $\tilde{a}_p(m)$ and $\tilde{c}_p(m)$ are as follows:

$$H(\theta, \vec{r}_\tau) = \sum_{j=1}^{m} \left(\prod_{i=2+j}^{m+1} b_i \right) c_{j+1} r^2_{j,\tau}$$

$$\tilde{a}_p(m) = \left[\sum_{j=1}^{m}\sum_{i=2}^{j} a_i \prod_{l=i+1}^{j}(b_l + c_l)\right]\left[1 - \left(\prod_{i=1}^{m} b_i\right)\right] + \left[\sum_{j=1}^{m}\prod_{i=2}^{j}(b_i + c_i)\right]\left[\sum_{j=1}^{m} a_j \prod_{i=j+1}^{m+1} b_i\right]$$

$$\tilde{c}_p(m) = \sum_{j=1}^{m}\prod_{i=2}^{j}(b_i + c_i)$$

While the above is an example of structural HYBRID process, it would be difficult to estimate $m \times 3$ parameters in the periodic GARCH case, i.e. all a_i, b_i and c_i, for all i. There is a convenient shortcut, inspired by the Exponential Almon MIDAS filter proposed in Ghysels, Santa-Clara, and Valkanov (2005) and appearing in the previous subsection. Namely, if we ignore the periodicity of a_i and c_i in equation (2.3) we have a convenient representation:

$$V_{\tau+1|\tau} = \tilde{a} + \left(\prod_i b_i(\theta)\right) V_{\tau|\tau-1} + \tilde{c}\left[\sum_{j=1}^{m}(\prod_{i=1}^{j-1} b_i(\theta)) r^2_{m-j+1,\tau}\right] \tag{2.4}$$

where we estimate only five parameters, i.e. daily parameters $\tilde{a}$ and $\tilde{c}$ as well as the periodic intra-daily pattern via $b_i(\theta)$ with only three parameters $\theta = (\theta_0, \theta_1, \theta_2)$. Hence, the specification amounts to setting $\tilde{b} = \prod_i b_i(\theta)$ in equation (2.2).

Note that the Exponential Almon specification makes the restriction imposed on the lagged volatility easy to test as it links the MIDAS polynomial weights with the lagged coefficient in the HYBRID GARCH model. More specifically, the slope coefficient for $V_{\tau|\tau-1}$ should be equal to:

$$\prod_{i=1}^{m} b_i(\theta) = \prod_{i=1}^{m} \exp\{\theta_0 + \theta_1 i + \theta_2 i^2\} = \exp\sum_{i=1}^{m}\{\theta_0 + \theta_1 i + \theta_2 i^2\} \tag{2.5}$$

which implies that the slope has to be equal to the exponential of the sum of the intra-daily MIDAS weight. An easy test would therefore be to estimate equation (2.4) unrestricted:

$$V_{\tau+1|\tau} = \tilde{a} + \tilde{b}V_{\tau|\tau-1} + \tilde{c}\left[\sum_{j=1}^{m}(\prod_{i=1}^{j-1} b_i(\theta)) r^2_{m-j+1,\tau}\right] \tag{2.6}$$

and impose the restriction appearing in (2.5) that $\ln(\tilde{b})$ equals the sum of the MIDAS weights. While we will call this a periodic specification - it is clear that its main advantage is to save one parameter.

We noted that purely data-driven examples of generic HYBRID processes have been suggested in the literature. Consider for example the following RV-driven GARCH:

$$V_{\tau+1|\tau} = a + bV_{\tau|\tau-1} + cRV_{\tau} \tag{2.7}$$

where $RV_{\tau} = \sum_{i=1}^{m} r_{i,\tau}^2$. Comparing with equation (2.10) implies that we can write:

$$V_{\tau+1|\tau} = a + bV_{\tau|\tau-1} + cRV_{\tau} + c\left[\sum_{j=1}^{m}(\omega_j - 1)r_{j,\tau}^2\right] \tag{2.8}$$

where $\omega_{m-j+1} \equiv \prod_{i=1}^{j-1} b_i(\theta)$. This means that the RV-driven and HYBRID GARCH models differ by the last term in the above equation - called a MIDAS correction by Chen, Ghysels, and Wang (2009). We will use the RV-driven model as a benchmark - as discussed later.

To streamline the presentation, we list all models defined in this and the following sections in Table 1. In particular, so far we have defined the following models (removing the tildes from the above equations):

- HYBRID GARCH

$$V_{\tau+1|\tau} = a + bV_{\tau|\tau-1} + c\sum_{j=1}^{m}\exp\left(\sum_{i=1}^{j-1}(\theta_0 + \theta_1 i + \theta_2 i^2)\right)r_{m-j+1,\tau}^2 \tag{2.9}$$

- Periodic HYBRID GARCH

$$V_{\tau+1|\tau} = a + \exp\left(\sum_{i=1}^{m}(\theta_0 + \theta_1 i + \theta_2 i^2)\right)V_{\tau|\tau-1} + c\sum_{j=1}^{m}\exp\left(\sum_{i=1}^{j-1}(\theta_0 + \theta_1 i + \theta_2 i^2)\right)r_{m-j+1,\tau}^2 \tag{2.10}$$

  where we have the hypothesis that $b = \exp\left(\sum_{i=1}^{m}(\theta_0 + \theta_1 i + \theta_2 i^2)\right)$.

- RV GARCH

$$V_{\tau+1|\tau} = a + bV_{\tau|\tau-1} + cRV_{\tau} \tag{2.11}$$

2.3 Parameter estimation

In this subsection we summarize results on parameter estimation, skipping the technical details, which are covered in Chen, Ghysels, and Wang (2009). We start from the generic HYBRID GARCH model appearing in equation (2.1), where the HYBRID filtering process $H_\tau = H(\theta, \vec{r}_\tau)$ is a measurable function of intraday returns, and it satisfies the following conditions. Let Θ collect all the parameters in H,

Assumption 2.1 *H is a mapping from $\Theta \times \overbrace{L^2 \times \ldots \times L^2}^{m}$ to L^2. It is nonnegative almost surely. Fix $\vec{x}$, $H(., \vec{x}) \in C^3$. H, $\partial_\theta H$, $\partial_\theta^2 H$ and $\partial_\theta^3 H$ are measurable; $E \sup_{\theta \in \overline{\Theta^0}} H(\theta, \vec{x})^2$, $E \sup_{\theta \in \overline{\Theta^0}} (\partial_\theta H(\theta, \vec{x}))^2$, $E \sup_{\theta \in \overline{\Theta^0}} (\partial_\theta^2 H(\theta, \vec{x}))^2$ are finite. Moreover, 1, H_τ, and each component of $\partial_\theta(H_\tau)$ are linearly independent if $\tilde{c}$ is not a constant; Otherwise, 1, and each component of $\partial_\theta(H_\tau)$ are linearly independent.*

Suppose that the solution to (2.1) is $V_{\tau+1|\tau}(\Phi)$ where $\Phi = (\tilde{a}, \tilde{b}, \tilde{c}, \theta)$, and there exists Φ_0 such that $V_{\tau+1|\tau}(\Phi_0) = \sigma^2_{\tau+1|\tau}$ where, letting $\mathcal{F}_\tau$ represent all the information up to day τ, $\sigma^2_{\tau+1|\tau} \equiv E(RV_{\tau+1}|\mathcal{F}_\tau) = E(R^2_{\tau+1}|\mathcal{F}_\tau)$ and $R_{\tau+1} \equiv \sum_{j=1}^m r_{j,\tau+1}$ the daily return of day $\tau + 1$. Define

$$\hat{\Phi}_T^{lhr2} = \arg\min_{\Phi \in \mathcal{C}} \frac{1}{T} \sum_{\tau=1}^{T} \left(\log \hat{V}_\tau(\Phi) + \frac{R_\tau^2}{\hat{V}_\tau(\Phi)} \right) \tag{2.12}$$

where $\mathcal{C}$ is a well-chosen convex compact subset satisfying $\Phi_0 \in \mathcal{C}^0 \subset \mathcal{C}$. $\hat{V}_\tau$ is defined recursively by

$$\hat{V}_\tau(\Phi) = \tilde{a} + \tilde{b}\hat{V}_{\tau-1}(\Phi) + \tilde{c}H_{\tau-1}(\theta), \tau \geq 1 \quad \text{and} \quad V_0 = v \tag{2.13}$$

and v is any arbitrary deterministic value.[3] While we do not formally discuss stability conditions, it is worth noting the following observation: the financial crisis featured many large return shocks, if the HYBRID models were unstable - then their forecasting performance would be dismal. It turns out, as we will discuss in the empirical result section - that their performance during the financial crisis is quite the opposite, namely they outperform RV-driven and other daily models.

[3] Chen, Ghysels, and Wang (2009) constructs four estimators for Model (2.1): two based on minimum-distance, two based on likelihood function. Only likelihood-based estimation is adopted here because it requires less restrictive moment conditions. Moreover, we selected this particular procedure as it is the closet to the estimation of daily GARCH models discussed later.

The distributional properties of $\hat{\Phi}_T^{lhr2}$ are summarized in the following theorem,

Theorem 2.1 *Under the assumption that the intraday return process is square integrable, and it is strictly periodically stationary and periodically ergodic, $\hat{\Phi}_T^{lhr2}$ is a strongly consistent estimator of Φ_0.*[4] *If it is further assumed that $E(r^4) < \infty$,*

$$\sqrt{T}(\hat{\Phi}_T^{lhr2} - \Phi_0) \Longrightarrow N\left(0, (\Sigma^{lh})^{-1}\Omega^{lhr2}(\Sigma^{lh})^{-1}\right)$$

where

$$0 < \Sigma^{lh} = E\left(V_{\tau|\tau-1}^{-2}(\Phi_0)\nabla V_{\tau|\tau-1}(\Phi_0)\nabla V_{\tau|\tau-1}(\Phi_0)'\right) < \infty, \tag{2.14}$$

$$0 < \Omega^{lhr2} = E\left(V_{\tau|\tau-1}^{-4}(\Phi_0)(R_\tau^2 - V_{\tau|\tau-1}(\Phi_0))^2\nabla V_{\tau|\tau-1}(\Phi_0)\nabla V_{\tau|\tau-1}(\Phi_0)'\right) < \infty \tag{2.15}$$

It should also be noted that an estimator of the asymptotic variance-covariance matrix, $(\hat{\Sigma}^{lh})^{-1}\hat{\Omega}^{lhr2}(\hat{\Sigma}^{lh})^{-1}$, is obtained as follows:

$$\hat{\Omega}^{lhr2} = 1/T\sum_{\tau=1}^{T}\left(\hat{V}_{\tau|\tau-1}^{-4}(\hat{\Phi}_T^{lhr2})(R_\tau^2 - \hat{V}_{\tau|\tau-1}(\hat{\Phi}_T^{lhr2}))^2\nabla\hat{V}_{\tau|\tau-1}(\hat{\Phi}_T^{lhr2})\nabla\hat{V}_{\tau|\tau-1}(\hat{\Phi}_T^{lhr2})'\right),$$

and $\hat{\Sigma}^{lh}(\Phi) = 1/T\sum_{\tau=1}^{T}\left(\hat{V}_{\tau|\tau-1}^{-2}(\Phi)\nabla\hat{V}_{\tau|\tau-1}(\Phi)\nabla\hat{V}_{\tau|\tau-1}(\Phi)'\right)$, where Φ is replaced by $\hat{\Phi}_T^{lhr2}$.

A rigorous proof of Theorem 2.1 is available in Section 4.2 of Chen, Ghysels, and Wang (2009).

2.4 HYBRID GARCH with pre-filtered returns

A special case of the derivations in the previous subsection is the non-periodic GARCH(1,1). Consider in equation (2.2) a constant case, $\theta_1 = \theta_2 = 0$. More specifically, in the constant polynomial case we can write the model explicitly as:

$$V_{\tau+1|\tau} = \tilde{a} + \exp(m\theta_0)V_{\tau|\tau-1} + cd(m)\sum_{j=1}^{m}\exp((j-1)\theta_0)r_{m-j+1,\tau}^2 \tag{2.16}$$

where $d(m) = (1 - (\exp(\theta_0) + c)^m)/(1 - (\exp(\theta_0) + c))$ and $\tilde{a} = [a[1 - \exp(m\theta_0)][1 - \exp(\theta_0)]][m(1 - \exp(\theta_0)) - cd(m)][1 - (\exp(\theta_0) + c)]$. Obviously, this HYBRID process is

[4] See Aknouche and Bibi (2009) for the definition of periodic ergodicity.

not well designed to address intra-daily seasonal fluctuations. A standard answer to this is to consider seasonal adjustment or pre-filtering of the data and build a model that does not feature seasonality or periodicity (see Hylleberg (1992) and Ghysels and Osborn (2001) for the many issues related to this). In the case of returns, the filtering would be one pertaining to volatility.

The filtering approach is inspired by Engle, Sokalska, and Chanda (2005). Namely, the HYBRID GARCH models with pre-filtering involve the returns $\widetilde{r}_{i,\tau} \equiv r_{i,\tau}/\sqrt{s_i}, s_i = T^{-1}\sum_{\tau=1}^{T} r_{i,\tau}^2/V_{\tau|\tau-1}^D, i = 1, ..., m$, and $V_{\tau|\tau-1}^D$ estimated with a Daily ASYGARCH(1,1) model proposed by Glosten, Jagannathan, and Runkle (1993) model - defined in Table 1.

The scaled returns can be used to estimate the model in equation (2.16). Note, however, that when it comes to parameter counts (something that will be important for the BIC selection criterion for instance), we will ignore the m parameter estimates s_j. Namely, as is typically the case, the SA is handled prior to model estimation. Finally, it should be noted that we obtain equation (2.16) by setting $\theta_1 = \theta_2 = 0$, $\tilde{b} = \exp(m\theta_0)$, in equation (2.2).

It is also worth pointing out a connection with Areal and Taylor (2002) who studied the ex post measure of daily volatility using intra-day returns which exhibit intra-day periodicity structure. The daily variance in their paper is estimated by a weighted sum of intraday squared returns, $V_{\tau|\tau} = \sum_{j=1}^{m} \omega_j r_{j,\tau}^2$, and $r_{j,\tau}^2 = \lambda_j V_{\tau|\tau} e_{j,\tau}$ where $V_{\tau|\tau} = Var(R_\tau|\mathcal{F}_\tau)$ is the daily variance for day τ, λ_j represents the periodic pattern, and $\{e_{j,\tau}\}$ are i.i.d. random variables with mean 1. Moreover, the weights ω_j's are also related to the periodicity expressed via the λ_j's. When we reconsider the RV-driven GARCH model in (2.7) with the weighting scheme suggested by Areal and Taylor (2002) we obtain a GARCH model driven by a HYBRID process that is also data-driven with a weighting scheme designed to measure $V_{\tau|\tau}$, presumably more accurately, than RV. We can again link such a model via a MIDAS correction - as in equation (2.8) - with respect to the exponential Almon weighting schemes in the (Periodic) HYBRID optimized for the purpose of prediction as in equations (2.10) and (2.9).

We also consider *periodic* specifications of the HYBRID models using pre-filtered returns - that is models with constraints on the slope coefficients - as in equation (2.10). The fact that this restriction saves one parameter will turn out to be appealing as the empirical results will show.

To summarize, using pre-filtered returns we have the following models:

- HYBRID GARCH with pre-filtering:

$$V_{\tau+1|\tau} = a + bV_{\tau|\tau-1} + c\sum_{j=1}^{m} \exp\left(\sum_{i=1}^{j-1}(\theta_0 + \theta_1 i + \theta_2 i^2)\right)\tilde{r}^2_{m-j+1,\tau} \tag{2.17}$$

- Periodic HYBRID GARCH with pre-filtering:

$$V_{\tau+1|\tau} = a + \exp(m\theta_0)V_{\tau|\tau-1} + c\sum_{j=1}^{m} \exp((j-1)\theta_0)\tilde{r}^2_{m-j+1,\tau} \tag{2.18}$$

2.5 News Impact and Asymmetries

So far we presented only symmetric GARCH-type models. Chen and Ghysels (2009) examine whether the sign and magnitude of intra-daily returns have impact on expected volatility the next day or over longer future horizons. They use a MIDAS regression setting, starting with a semi-parametric approach which captures with minimal interference the mapping between intra-daily returns and future volatility. They revisit the concept of news impact curves introduced by Engle and Ng (1993). Overall, they find that moderately good (intra-daily) news reduces volatility (the next day), while both very good news (unusual high intra-daily positive returns) and bad news (negative returns) increase volatility, with the latter having a more severe impact. Inspired by the GJR model proposed by Glosten, Jagannathan, and Runkle (1993), we consider the ASYMGJR model with $NIC(r) = c(1 + d\mathbf{1}_{r<0})r^2$, where the indicator function $\mathbf{1}_A$ is 1 if A is true; otherwise, 0. Another possible way to allow for asymmetric effects is via a location shift, as in the Asymmetric GARCH model in Engle (1990), yielding the ASYMLS model with $NIC(r) = c(r - d)^2$.

This leads to the following model specifications:

- HYBRID ASYGARCH

$$V_{\tau+1|\tau} = a + bV_{\tau|\tau-1} + c\sum_{j=1}^{m} \exp\left(\sum_{i=1}^{j-1}(\theta_0 + \theta_1 i + \theta_2 i^2)\right)(1 + d\mathbf{1}_{r_{m-j+1,\tau}<0})r^2_{m-j+1,\tau} \tag{2.19}$$

- HYBRID QGARCH

$$V_{\tau+1|\tau} = a + bV_{\tau|\tau-1} + c\sum_{j=1}^{m} \exp\left(\sum_{i=1}^{j-1}(\theta_0 + \theta_1 i + \theta_2 i^2)\right)(r_{m-j+1,\tau} - d)^2 \qquad (2.20)$$

Note that periodic models can also involve news impact curves, namely:

- Periodic HYBRID ASYGARCH

$$\begin{aligned} V_{\tau+1|\tau} &= a + \exp\left(\textstyle\sum_{i=1}^{m}(\theta_0 + \theta_1 i + \theta_2 i^2)\right)V_{\tau|\tau-1} \\ &\quad + c\textstyle\sum_{j=1}^{m}\exp\left(\sum_{i=1}^{j-1}(\theta_0 + \theta_1 i + \theta_2 i^2)\right)(1 + d\mathbf{1}_{r_{m-j+1,\tau}<0})r_{m-j+1,\tau}^2 \end{aligned} \qquad (2.21)$$

- Periodic HYBRID QGARCH

$$\begin{aligned} V_{\tau+1|\tau} &= a + \exp\left(\textstyle\sum_{i=1}^{m}(\theta_0 + \theta_1 i + \theta_2 i^2)\right)V_{\tau|\tau-1} \\ &\quad + c\textstyle\sum_{j=1}^{m}\exp\left(\sum_{i=1}^{j-1}(\theta_0 + \theta_1 i + \theta_2 i^2)\right)(r_{m-j+1,\tau} - d)^2 \end{aligned} \qquad (2.22)$$

For the purpose of completeness we also can extend the notion of news impact to pre-filtered returns as well as periodic models with pre-filtering, namely:

- HYBRID ASYGARCH with pre-filtering

$$V_{\tau+1|\tau} = a + bV_{\tau|\tau-1} + c\sum_{j=1}^{m} \exp\left(\sum_{i=1}^{j-1}(\theta_0 + \theta_1 i + \theta_2 i^2)\right)(1 + d\mathbf{1}_{\widetilde{r}_{m-j+1,\tau}<0})\widetilde{r}_{m-j+1,\tau}^2 \qquad (2.23)$$

- HYBRID QGARCH with pre-filtering

$$V_{\tau+1|\tau} = a + bV_{\tau|\tau-1} + c\sum_{j=1}^{m} \exp\left(\sum_{i=1}^{j-1}(\theta_0 + \theta_1 i + \theta_2 i^2)\right)(\widetilde{r}_{m-j+1,\tau} - d)^2 \qquad (2.24)$$

- Periodic HYBRID GARCH with pre-filtering

$$V_{\tau+1|\tau} = a + \exp(m\theta_0)V_{\tau|\tau-1} + c\sum_{j=1}^{m} \exp((j-1)\theta_0)\widetilde{r}_{m-j+1,\tau}^2 \qquad (2.25)$$

- Periodic HYBRID ASYGARCH with pre-filtering

$$V_{\tau+1|\tau} = a + \exp(m\theta_0)V_{\tau|\tau-1} + c\sum_{j=1}^{m} \exp((j-1)\theta_0)(1 + d\mathbf{1}_{\widetilde{r}_{m-j+1,\tau}<0})\widetilde{r}^2_{m-j+1,\tau} \quad (2.26)$$

- Periodic HYBRID QGARCH with pre-filtering

$$V_{\tau+1|\tau} = a + \exp(m\theta_0)V_{\tau|\tau-1} + c\sum_{j=1}^{m} \exp((j-1)\theta_0)(\widetilde{r}_{m-j+1,\tau} - d)^2 \quad (2.27)$$

Finally, we noted earlier that we will consider the RV-driven GARCH models as a benchmark. We add three more benchmarks, including two featuring news impact curves involving daily returns, the other the standard GARCH(1,1) model. Hence, we have:

- Daily ASYGARCH

$$V_{\tau+1|\tau} = a + bV_{\tau|\tau-1} + c(1 + d\mathbf{1}_{R_\tau<0})R^2_\tau \quad (2.28)$$

- Daily QGARCH

$$V_{\tau+1|\tau} = a + bV_{\tau|\tau-1} + c(R_\tau - d)^2 \quad (2.29)$$

- Daily GARCH

$$V_{\tau+1|\tau} = a + bV_{\tau|\tau-1} + cR^2_\tau \quad (2.30)$$

3 Empirical Results

We analyze four datasets which consist of five-minute intra-day returns of respectively Dow Jones and S&P500 cash and futures markets. We will denote the four series as DJC and SPC for the two cash market series and DJF and SPF for the futures market series. The data are described in Table 2. The samples start in 1993 or 1997 and hence do not include the 1987 crash, and end in December 2008. For each dataset, there are two in-sample periods ending in 2004 and in 2006 with the corresponding two-year out-of-sample periods 2005-2006 and 2007-2008, respectively. The choice of these prediction samples is on purpose. The 2005-2006 sample is one of low volatility, whereas of course the 2007-2008 episode covers the recent financial crisis. The distinction will be important when we turn our attention to out-of-sample forecasting. Finally, for each day there are m =78 five-minute returns. We do not pre-filter the data for the purpose of eliminating micro-structure noise. In Chen, Ghysels, and Wang (2009) we document the fact that RV-driven GARCH models and models driven by realized measures, such as those advocated by Shephard and Sheppard (2009), have either similar out-of-sample forecast performance or sometimes worse performance - see also Patton and Sheppard (2009) for similar findings. Moreover, at the five minute frequency we do not expect that micro-structure noise plays a great role since we focus exclusively on market-wide indices.

Since the focus of our analysis is the intra-daily return pattern, we start with a look at Figure 1. The figure displays intra-daily return pattern as measured by $s_i, i = 1, ..., m$, where $s_i = 1/T \sum_{\tau=1}^{T} r_{i,\tau}^2 / V_{\tau|\tau-1}$, where $V_{\tau|\tau-1}$ is estimated from a daily ASYGARCH(1,1) model defined in equation (2.28). We note a much more pronounced intra-daily pattern with a spike of volatility at the market open for the cash market series (left panels DJC and SPC). The two right panels pertaining to the futures markets show more modest intra-daily periodicity.

For the purpose of model appraisal and selection we rely on several criteria, namely (1) In-sample Log-likelihood - appearing in Table 3, (2) the Bayes Information Criterion: $BIC = -2ln(L) + kln(n)$ where L stands for log-likelihood; k for number of parameters; n for sample size and appearing in Table 4, (3) the out-of-sample log-likelihood appearing in Table 5 for the sample configurations appearing in Table 2, (4) the out-of-sample Mean Squared Forecast Error (MSFE) appearing in Table 6, and finally the Giacomini and White (2006) (GW) test appearing in Table 7. The GW tests involve loss functions - either out-of-sample log-likelihood or MSFE - and compare two models 'A' and 'B'. We consider eighty five rolling samples using S&P 500 Cash Market data with a longer span than what is described in Table 2. The tests involve both an estimation sample and prediction sample. The ten-year in-sample estimation periods $k = 1, \ldots, 85$ start from January 1990 plus $(k-1)$ months, and ends in December 1999 plus $(k-1)$ months; the two-year out-of-sample period is from January 2000 plus $(k-1)$ months to December 2001 plus $(k-1)$ months. In calculating the Giacomini-White statistics, three instruments are used: constant, the last available (i.e. 24 lagged) difference of loss function, and the difference of BIC. The column 'Ratio of $A > B$' contains the ratio of selecting model A rather than B in the last 61 samples ($61 = 85 - 24$, discarding the 24 initialization samples) based on the decision rule for forecast selection discussed below. One other advantage of using the GW tests is that they allow for non-nested models or forecasting 'methods' as Giacomini and White (2006) call it. Our comparisons of daily GARCH, RV-driven GARCH with HYBRID GARCH processes that feature asymmetries are non-nested.

It is well known that it is difficult to measure the fit of GARCH-type models as the true conditional variance is unobserved (see e.g. Patton (2009)). Our forecast evaluations are based on loss functions - out-of-sample log-likelihood and MSFE - that according to Patton (2009) preserve ranking of forecasts despite the unobserved conditional variance. It should be noted that our ex post RV measures are computed as 'plain vanilla' estimates based on five-minute returns. We noted before that Patton and Sheppard (2009) and Chen, Ghysels, and Wang (2009) document the fact that RV-driven GARCH models and models driven by realized measures, have either similar out-of-sample forecast performance or sometimes worse performance.

We analyze exclusively on one-day forecast horizons. The empirical efficiency of the forecasts is assessed by comparing the respective loss functions and testing the following null hypothesis:

$$H_0 : E[\left((L^A_{t,t+1}) - (L^B_{t,t+1})\right) |I_t] = 0 \tag{3.1}$$

where I_t are the instruments with information known at at time t. The basic idea is that rejection occurs because the instruments can predict the loss differences $\Delta L^{AB}_{t,t+1} \equiv (L^A_{t,t+1}) - (L^B_{t,t+1})$ out of sample, which suggests using instruments to predict which method will yield lower loss. Giacomini and White (2006) propose the following two-step procedure:

- STEP 1: Regress $\Delta L^{AB}_{t,t+1}$ on the instruments, over the out-of-sample period $t = 25, ..., 85$, and let δ denote the regression coefficient.
- STEP 2: Use the decision rule: select Model A if the regression prediction is above (below) zero for the out-of-sample log-likelihood (MSFE) loss function. Select Model B otherwise.

We start with the in-sample log-likelihood appearing in Table 3. The bold-faced entries are the best models according to the chosen criterion - the in-sample log-likelihood - for each series appraised for the in-sample periods ending respectively in 2004 and 2006. For the S&P 500 series SPC and SPF, we find that the best model in both samples is the HYBRID

ASYGARCH. For the cash market series, the models are the same across the two samples, but they differ across series. For the DJC series the best model is HYBRID QGARCH with pre-filtering, whereas for the SPC series it is the HYBRID ASYGARCH model, as already noted. There is a common thread, however, all models involve asymmetric HYBRID processes. On that note we also should point out the vast improvement of asymmetric models when compared to RV-driven models. These results confirm those obtained by Chen and Ghysels (2009).

Turning our attention to the in-sample BIC criterion we find a more uniform picture across samples and series. The same model, Periodic HYBRID QGARCH with pre-filtering, is almost unanimously the best - and that is due to the fact that it saves on parameters, while keeping the importance of asymmetries as part of the HYBRID process specification. The latter underlines how important asymmetries are, as indeed the daily GARCH models, involving either daily returns or RV, have far less parameters than any of the parametric HYBRID specifications. Yet, they do not fit well and the differences in terms of BIC are large.

What matters more is out-of-sample performance, so we turn our attention to Tables 5 and 6. Now, each series is appraised for the two-year out-of-sample periods 2005-2006 and 2007-2008 - the latter covering the financial crisis. We put more emphasis on the latter, and in that respect it is remarkable to find that (1) in terms of out-of-sample log-likelihood there is one model that dominates, the Periodic HYBRID QGARCH with pre-filtering - with the exception for the DJC where the HYBRID GARCH with pre-filtering is best with only a slim margin better than the Periodic HYBRID QGARCH with pre-filtering model. While for the MSFE in Table 6 we find a similar result, although it is the Periodic HYBRID ASYGARCH with pre-filtering model that dominates, we find the remarkable result that the Daily ASYGARCH model is the best for the SPF series during the financial crisis. This is remarkable because that means that a model with daily returns outperforms models based on high frequency data. The margin is again very slim compared to the Periodic HYBRID GARCH with pre-filtering, Periodic HYBRID ASYGARCH, Periodic HYBRID ASYGARCH with pre-filtering and Periodic HYBRID QGARCH with pre-filtering models - yet the margin is *not* slim with respect to the RV-driven models for instance. Chen and Ghysels (2009) did not report such findings - i.e. in all cases they found with the same data and sample configurations that HF-data based regression models always dominated GARCH models with daily returns. Most of their appraisals and findings were based on the GW test, which in a sense is more rigorous. We therefore turn to the GW test next.

Because there are potentially many pairwise combinations to investigate, we do not report all of them. Instead, we only focus on four models - using the numbers in across all the tables: (2) Daily ASYGARCH, (4) RV GARCH, (9) Periodic HYBRID ASYGARCH, and (15) Periodic HYBRID ASYGARCH with pre-filtering. This selection allows us to investigate whether daily GARCH models with asymmetries are dominated by RV-driven models. The answer is negative, namely the latter perform significantly better according to the GW test statistic and the selection procedure described above in STEP1 and STEP2 - the latter indicating that 100 % of the time one would prefer the RV-driven model. Turning next to comparison of the HYBRID GARCH models with or without pre-filtering, we see from Table 7, that they dominate RV-driven GARCH models. All these results coincide with the findings of Chen and Ghysels (2009) - who use different models which share similar features regarding intra-daily NIC's - when compared with (symmetric) RV-driven models. Finally, the last row in Table 7 tells us that according to the GW procedure the models *not* involving pre-filtering do perform better that models with pre-filtering. Note that the results pertain to SPC series - which features according to Figure 1 strong intra-daily periodic patterns in particular with the market open volatility spike.

To further elaborate on the GW test findings, we turn our attention to Figure 2 which displays the loss differences $\Delta L_{t,t+1}^{AB}$, for various combinations of models. In particular, Figure 2 covers the differences of Out-of-sample log-likelihoods between models. In each panel, two models are compared. The differences are (a) RV GARCH minus Daily ASYGARCH; (b) Periodic HYBRID ASYGARCH minus Daily ASYGARCH; (c) Periodic HYBRID ASYGARCH with pre-filtering minus Daily ASYGARCH; (d) Periodic HYBRID ASYGARCH minus RV GARCH; (e) Periodic HYBRID ASYGARCH with pre-filtering minus RV GARCH; and (f) Periodic HYBRID ASYGARCH minus Periodic HYBRID ASYGARCH with pre-filtering. Hence, we cover a slightly smaller set of model combinations than in Table 7. For the log-likelihood comparisons, panel (b) shows clear dominance of the Periodic HYBRID ASYGARCH over the Daily ASYGARCH since the difference is always positive. In panels (a), (c), (d) through (e) we see an occasional drop below zero for the out-of-sample log-likelihood, but clearly one can conclude that in panel (a) RV GARCH dominates Daily ASYGARCH; in panel (c) Periodic HYBRID ASYGARCH with pre-filtering dominates Daily ASYGARCH; in panel (d) Periodic HYBRID ASYGARCH dominates RV GARCH; and in panel (e) Periodic HYBRID ASYGARCH with pre-filtering dominates RV GARCH. Panel (f) is interesting as it shows that Periodic HYBRID ASYGARCH has a higher out-of-sample log-likelihood than Periodic HYBRID ASYGARCH with pre-filtering, *except* during the financial crisis. Hence, while pre-filtering was not necessary, somehow during the financial crisis it seems to have had a marginal impact. Overall, though pre-filtering seems less appealing.

4 Conclusion

We used the HYBRID GARCH model of Chen, Ghysels, and Wang (2009) to predict volatility at daily horizons using intra-daily returns. The use of such returns forces one to think about how to treat intra-daily periodicity. We considered an exhaustive selection of model specifications, ranging from models based on daily returns or RV, models with HYBRID processes that are symmetric or instead featuring intra-daily news impact curves, or HYBRID processes involving pre-filtered returns. While the out-of-sample log-likelihood and MSFE seem to prefer pre-filtering, when we turn to the GW test based on repeated out-of-sample appraisals we find that the use of pre-filtered returns is inferior. This means that we have essentially a relatively simple class of volatility models based on high-frequency data that handle intra-day periodicity well.

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Table 1: Summary of Model Specifications

All models have the same mean part: $R_\tau = \sqrt{V_{\tau|\tau-1}}\varepsilon_\tau$, $\tau = 1, ..., T$, where R_τ is the daily return, the sum of high frequency returns $\sum_{i=1}^{m} r_{i,\tau}$. In the asymmetric model specifications we use the indicator function $\mathbf{1}_A$ is 1 if A is true; otherwise, 0. The HYBRID GARCH models with pre-filtering involve the returns $\widetilde{r}_{i,\tau} \equiv r_{i,\tau}/\sqrt{s_i}$, $s_i = T^{-1}\sum_{\tau=1}^{T} r_{i,\tau}^2/V_{\tau|\tau-1}$, $i = 1, ..., m$, and $V_{\tau|\tau-1}$ estimated with a Daily ASYGARCH(1,1) model.

Daily GARCH	$V_{\tau+1\|\tau} = a + bV_{\tau\|\tau-1} + cR_\tau^2$
Daily ASYGARCH	$V_{\tau+1\|\tau} = a + bV_{\tau\|\tau-1} + c(1 + d\mathbf{1}_{R_\tau<0})R_\tau^2$
Daily QGARCH	$V_{\tau+1\|\tau} = a + bV_{\tau\|\tau-1} + c(R_\tau - d)^2$
Daily RV GARCH	$V_{\tau+1\|\tau} = a + bV_{\tau\|\tau-1} + cRV_\tau$
HYBRID GARCH	$V_{\tau+1\|\tau} = a + bV_{\tau\|\tau-1} + c\sum_{j=1}^{m}\exp\left(\sum_{i=1}^{j-1}(\theta_0 + \theta_1 i + \theta_2 i^2)\right)r_{m-j+1,\tau}^2$
HYBRID ASYGARCH	$V_{\tau+1\|\tau} = a + bV_{\tau\|\tau-1} + c\sum_{j=1}^{m}\exp\left(\sum_{i=1}^{j-1}(\theta_0 + \theta_1 i + \theta_2 i^2)\right)(1 + d\mathbf{1}_{r_{m-j+1,\tau}<0})r_{m-j+1,\tau}^2$
HYBRID QGARCH	$V_{\tau+1\|\tau} = a + bV_{\tau\|\tau-1} + c\sum_{j=1}^{m}\exp\left(\sum_{i=1}^{j-1}(\theta_0 + \theta_1 i + \theta_2 i^2)\right)(r_{m-j+1,\tau} - d)^2$
Periodic HYBRID GARCH	$V_{\tau+1\|\tau} = a + \exp\left(\sum_{i=1}^{m}(\theta_0 + \theta_1 i + \theta_2 i^2)\right)V_{\tau\|\tau-1} + c\sum_{j=1}^{m}\exp\left(\sum_{i=1}^{j-1}(\theta_0 + \theta_1 i + \theta_2 i^2)\right)r_{m-j+1,\tau}^2$
Periodic HYBRID ASYGARCH	$V_{\tau+1\|\tau} = a + \exp\left(\sum_{i=1}^{m}(\theta_0 + \theta_1 i + \theta_2 i^2)\right)V_{\tau\|\tau-1} + c\sum_{j=1}^{m}\exp\left(\sum_{i=1}^{j-1}(\theta_0 + \theta_1 i + \theta_2 i^2)\right)(1 + d\mathbf{1}_{r_{m-j+1,\tau}<0})r_{m-j+1,\tau}^2$
Periodic HYBRID QGARCH	$V_{\tau+1\|\tau} = a + \exp\left(\sum_{i=1}^{m}(\theta_0 + \theta_1 i + \theta_2 i^2)\right)V_{\tau\|\tau-1} + c\sum_{j=1}^{m}\exp\left(\sum_{i=1}^{j-1}(\theta_0 + \theta_1 i + \theta_2 i^2)\right)(r_{m-j+1,\tau} - d)^2$
HYBRID GARCH with pre-filtering	$V_{\tau+1\|\tau} = a + bV_{\tau\|\tau-1} + c\sum_{j=1}^{m}\exp\left(\sum_{i=1}^{j-1}(\theta_0 + \theta_1 i + \theta_2 i^2)\right)\widetilde{r}_{m-j+1,\tau}^2$
HYBRID ASYGARCH with pre-filtering	$V_{\tau+1\|\tau} = a + bV_{\tau\|\tau-1} + c\sum_{j=1}^{m}\exp\left(\sum_{i=1}^{j-1}(\theta_0 + \theta_1 i + \theta_2 i^2)\right)(1 + d\mathbf{1}_{\widetilde{r}_{m-j+1,\tau}<0})\widetilde{r}_{m-j+1,\tau}^2$
HYBRID QGARCH with pre-filtering	$V_{\tau+1\|\tau} = a + bV_{\tau\|\tau-1} + c\sum_{j=1}^{m}\exp\left(\sum_{i=1}^{j-1}(\theta_0 + \theta_1 i + \theta_2 i^2)\right)(\widetilde{r}_{m-j+1,\tau} - d)^2$
Periodic HYBRID GARCH with pre-filtering	$V_{\tau+1\|\tau} = a + \exp(m\theta_0)V_{\tau\|\tau-1} + c\sum_{j=1}^{m}\exp((j-1)\theta_0)\widetilde{r}_{m-j+1,\tau}^2$
Periodic HYBRID ASYGARCH with pre-filtering	$V_{\tau+1\|\tau} = a + \exp(m\theta_0)V_{\tau\|\tau-1} + c\sum_{j=1}^{m}\exp((j-1)\theta_0)(1 + d\mathbf{1}_{\widetilde{r}_{m-j+1,\tau}<0})\widetilde{r}_{m-j+1,\tau}^2$
Periodic HYBRID QGARCH with pre-filtering	$V_{\tau+1\|\tau} = a + \exp(m\theta_0)V_{\tau\|\tau-1} + c\sum_{j=1}^{m}\exp((j-1)\theta_0)(\widetilde{r}_{m-j+1,\tau} - d)^2$

Table 2: Datasets

Four datasets which consist of five-minute intra-day returns of respectively Dow Jones and S&P500 cash and futures markets are used. The entries to the table provide details of the estimation and prediction samples.

		Estimation Sample Period		Out-of-sample Prediction Period	
		Start	End	Start	End
Dow Jones Cash Market (DJC)					
	Sample 1	04/01/1993	12/31/2004	01/01/2005	12/31/2006
	Sample 2	04/01/1993	12/31/2006	01/01/2007	12/31/2008
Dow Jones Futures Market (DJF)					
	Sample 1	10/06/1997	12/31/2004	01/01/2005	12/31/2006
	Sample 2	10/06/1997	12/31/2006	01/01/2007	12/31/2008
S&P 500 Cash Market (SPC)					
	Sample 1	04/01/1993	12/31/2004	01/01/2005	12/31/2006
	Sample 2	04/01/1993	12/31/2006	01/01/2007	12/31/2008
S&P 500 Futures Market (SPF)					
	Sample 1	04/01/1993	12/31/2004	01/01/2005	12/31/2006
	Sample 2	04/01/1993	12/31/2006	01/01/2007	12/31/2008

Table 3: In-sample Log-likelihood

All models are estimated via a standard QMLE using the data and estimation sample configurations described in Table 2 we report the in-sample log-likelihood. The model numbers correspond to (1) Daily GARCH, (2) Daily ASYGARCH, (3) Daily QGARCH, (4) RV GARCH, (5) HYBRID GARCH, (6) HYBRID ASYGARCH, (7) HYBRID QGARCH, (8) Periodic GARCH, (9) Periodic HYBRID ASYGARCH, (10) Periodic HYBRID QGARCH, (11) HYBRID GARCH with pre-filtering, (12) HYBRID ASYGARCH with pre-filtering, (13) HYBRID QGARCH with pre-filtering, (14) Periodic HYBRID GARCH with pre-filtering, (15) Periodic HYBRID ASYGARCH with pre-filtering, (16) Periodic HYBRID QGARCH with pre-filtering. All models are defined in Table 1.

	DJC	DJF	SPC	SPF	DJC	DJF	SPC	SPF
	Estimation sample ending 2004				Estimation sample ending 2006			
(1)	−3934.514	−2523.883	−3713.874	−3195.824	−4410.215	−2949.738	−4176.245	−3643.337
(2)	−3902.016	−2484.311	−3670.196	−3149.120	−4369.119	−2905.016	−4123.119	−3591.925
(3)	−3898.410	−2483.698	−3678.665	−3170.044	−4364.519	−2908.652	−4132.051	−3615.085
(4)	−3851.669	−2475.879	−3620.292	−3103.853	−4321.263	−2892.746	−4078.204	−3547.648
(5)	−3832.156	−2466.932	−3610.373	−3095.125	−4302.278	−2884.155	−4067.626	−3526.832
(6)	−3818.370	−2457.279	**−3593.625**	**−3075.827**	−4281.176	−2870.463	**−4045.462**	**−3517.494**
(7)	−3817.673	**−2456.097**	−3596.078	−3080.236	−4279.003	**−2865.548**	−4047.426	−3523.271
(8)	−3842.725	−2471.927	−3617.480	−3097.255	−4312.332	−2890.144	−4076.239	−3541.796
(9)	−3822.685	−2459.139	−3595.258	−3075.893	−4285.462	−2872.872	−4048.047	−3518.581
(10)	−3822.535	−2456.705	−3597.125	−3080.218	−4283.496	−2866.287	−4048.235	−3518.737
(11)	−3833.029	−2470.889	−3611.244	−3080.601	−4302.828	−2884.104	−4067.379	−3523.996
(12)	−3819.591	−2457.966	−3595.782	−3080.537	−4283.273	−2871.583	−4047.869	−3517.905
(13)	**−3816.499**	−2456.751	−3594.678	−3080.244	**−4276.792**	−2867.479	−4045.575	−3523.167
(14)	−3838.603	−2472.904	−3615.255	−3100.041	−4306.882	−2889.868	−4075.321	−3544.127
(15)	−3824.531	−2462.770	−3600.575	−3083.679	−4289.896	−2876.582	−4057.464	−3525.479
(16)	−3823.531	−2460.534	−3601.761	−3083.569	−4287.016	−2871.263	−4058.516	−3522.546

Table 4: Bayesian Information Criterion (BIC)

The table shows the $BIC = -2ln(L) + kln(n)$ where L stands for log-likelihood; k for number of parameters; n for sample size. The model numbers correspond to (1) Daily GARCH, (2) Daily ASYGARCH, (3) Daily QGARCH, (4) RV GARCH, (5) HYBRID GARCH, (6) HYBRID ASYGARCH, (7) HYBRID QGARCH, (8) Periodic GARCH, (9) Periodic HYBRID ASYGARCH, (10) Periodic HYBRID QGARCH, (11) HYBRID GARCH with pre-filtering, (12) HYBRID ASYGARCH with pre-filtering, (13) HYBRID QGARCH with pre-filtering, (14) Periodic HYBRID GARCH with pre-filtering, (15) Periodic HYBRID ASYGARCH with pre-filtering, (16) Periodic HYBRID QGARCH with pre-filtering. All models are defined in Table 1.

	DJC	DJF	SPC	SPF	DJC	DJF	SPC	SPF
	Estimation sample ending 2004				Estimation sample ending 2006			
(1)	7893.0081	5070.2890	7451.6072	6415.6310	8844.8815	5922.7294	8376.8380	7311.1377
(2)	7836.0074	4998.6528	7372.2052	6330.2194	8770.8399	5841.0366	8278.7019	7216.4689
(3)	7828.7936	4997.4268	7389.1424	6372.0668	8761.6394	5848.3075	8296.5667	7262.7882
(4)	7727.3195	4974.2814	7264.4431	6231.6892	8666.9775	5808.7445	8180.7561	7119.7605
(5)	7712.2731	4978.9095	7268.4662	6238.2168	8653.4587	5814.8168	8183.9495	7102.5905
(6)	7692.6954	4967.1110	7242.9234	6207.6169	8619.4062	5795.1823	8147.7374	7092.0690
(7)	7691.3019	4964.7469	7247.8287	6216.4335	8615.0597	5785.3524	8151.6647	7103.6237
(8)	7725.4174	4981.3920	7274.7268	6234.4836	8665.4155	5819.0436	8193.0587	7124.3643
(9)	7693.3311	4963.3240	7238.2352	6199.7531	8619.8266	5792.2512	**8144.7898**	7086.0888
(10)	7693.0309	4958.4558	7241.9697	6208.4035	8615.8957	5779.0795	8145.1660	7086.4012
(11)	7714.0198	4986.8237	7270.2085	6209.1696	8654.5579	5814.7134	8183.4555	7096.9197
(12)	7695.1379	4968.4864	7247.2374	6217.0361	8623.5994	5797.4230	8152.5512	7092.8914
(13)	7688.9537	4966.0555	7245.0286	6216.4497	8610.6379	5789.2159	8147.9632	7103.4150
(14)	7701.1865	4968.3314	7254.3703	6224.0660	8638.2156	5802.9885	8174.9909	7112.7171
(15)	7681.0356	4955.5713	**7232.9627**	6199.3365	8612.3949	5784.1686	8147.3920	7083.5759
(16)	**7679.0358**	**4951.0993**	7235.3343	**6199.1173**	**8606.6342**	**5773.5311**	8149.4964	**7077.7097**

Table 5: Out-of-sample Log-likelihood

The table shows the out-of-sample log-likelihood for one-day forecasts. The model numbers correspond to (1) Daily GARCH, (2) Daily ASYGARCH, (3) Daily QGARCH, (4) RV GARCH, (5) HYBRID GARCH, (6) HYBRID ASYGARCH, (7) HYBRID QGARCH, (8) Periodic GARCH, (9) Periodic HYBRID ASYGARCH, (10) Periodic HYBRID QGARCH, (11) HYBRID GARCH with pre-filtering, (12) HYBRID ASYGARCH with pre-filtering, (13) HYBRID QGARCH with pre-filtering, (14) Periodic HYBRID GARCH with pre-filtering, (15) Periodic HYBRID ASYGARCH with pre-filtering, (16) Periodic HYBRID QGARCH with pre-filtering. All models are defined in Table 1.

	DJC	DJF	SPC	SPF	DJC	DJF	SPC	SPF
	2005 − 2006				2007 − 2008			
(1)	−478.5166	−430.7371	−464.6039	−447.7330	−846.3687	−767.6787	−859.8930	−821.0473
(2)	−470.4186	−425.8949	−455.1380	−443.3789	−836.2857	−758.9410	−847.0802	−809.8318
(3)	−469.5727	−432.3489	−455.6101	−445.3131	−835.5850	−757.1499	−850.4956	−813.6325
(4)	−470.8697	−419.1603	−457.8534	−439.9789	−828.9899	−753.7793	−830.7144	−801.3956
(5)	−471.9376	−418.4940	−460.1829	−440.7687	−822.6416	−748.8929	−830.9502	−808.4426
(6)	−463.2963	−413.8521	−451.8641	−440.7295	−827.2022	−749.7874	−830.1785	−797.4795
(7)	−461.8264	**−411.7907**	−452.0606	−437.7413	−824.1031	−746.0214	−825.9423	−802.2934
(8)	−471.3150	−421.1074	−459.5837	−440.8861	−825.1762	−749.8417	−830.5527	−796.9818
(9)	−463.2881	−414.3410	−452.5061	−440.9421	−825.0816	−753.2476	−829.3142	−796.9702
(10)	−460.9516	−413.3971	**−451.3064**	**−437.7022**	−822.1039	−745.6000	−825.6619	−794.7381
(11)	−470.9219	−421.1563	−457.8137	−442.6212	**−821.8293**	−748.0744	−830.6745	−802.1922
(12)	−464.1296	−414.1696	−452.3851	−442.7299	−828.8603	−750.1408	−830.0495	−796.3431
(13)	**−460.1023**	−461.0099	−451.4925	−442.3397	−823.7998	−743.2444	−825.8771	−801.2834
(14)	−469.2613	−417.9510	−460.7677	−440.3479	−824.9763	−749.7599	−826.8513	−798.4835
(15)	−466.4513	−414.5152	−457.9318	−439.0933	−824.3092	−746.0752	−826.3141	−795.3206
(16)	−463.9412	−413.9957	−458.1541	−439.5578	−821.9754	**−742.2613**	**−823.5624**	**−793.5362**

Table 6: Mean Suqared Forecast Error (MSFE)

The table shows the MSFE for one-day forecasts. The model numbers correspond to (1) Daily GARCH, (2) Daily ASYGARCH, (3) Daily QGARCH, (4) RV GARCH, (5) HYBRID GARCH, (6) HYBRID ASYGARCH, (7) HYBRID QGARCH, (8) Periodic GARCH, (9) Periodic HYBRID ASYGARCH, (10) Periodic HYBRID QGARCH, (11) HYBRID GARCH with pre-filtering, (12) HYBRID ASYGARCH with pre-filtering, (13) HYBRID QGARCH with pre-filtering, (14) Periodic HYBRID GARCH with pre-filtering, (15) Periodic HYBRID ASYGARCH with pre-filtering, (16) Periodic HYBRID QGARCH with pre-filtering. All models are defined in Table 1.

	DJC	DJF	SPC	SPF	DJC	DJF	SPC	SPF
	2005 − 2006				2007 − 2008			
(1)	0.3496373	0.3819408	0.3916434	0.3217418	0.4883134	0.4252149	0.4875571	0.4412682
(2)	0.3249447	0.4218610	0.3760505	0.3283594	0.4131336	0.3678976	0.3981585	**0.3598022**
(3)	0.3776585	0.7010381	0.4231451	0.3226905	0.4461069	0.4194921	0.4304087	0.4127470
(4)	0.2598253	0.2487079	0.3591731	0.2234641	0.4459638	0.3972100	0.4271039	0.3971859
(5)	0.3301951	0.2601137	0.5107203	0.2811419	0.5318360	0.6328567	0.6308825	1.2234297
(6)	0.2240469	0.2123076	0.3084060	**0.2094457**	0.4568601	0.4875556	0.4956158	0.3820410
(7)	0.2614387	0.2438564	0.3661816	0.2117682	0.4183632	0.3815845	0.4192872	1.0648373
(8)	0.2863394	0.2732625	0.4042519	0.2672200	0.4049986	0.4147423	0.4247792	0.4147010
(9)	**0.2133530**	**0.2004185**	**0.2993128**	0.2095444	0.4046249	0.3822105	0.4139628	0.3637571
(10)	0.2426965	0.2396703	0.3405776	0.2115929	0.4137957	0.3784217	0.4110383	0.3835225
(11)	0.3130007	0.2681813	0.4210301	0.2943771	0.5542737	0.6509864	0.6804989	1.0829484
(12)	0.2295677	0.2090054	0.3195341	0.2983730	0.4472496	0.5480094	0.4965319	0.4148762
(13)	0.2314873	0.2508790	0.3486352	0.2970197	0.4113720	0.3879778	0.4302643	1.0609531
(14)	0.2626609	0.2328856	0.4387025	0.2242252	0.4097991	0.3958003	0.4108952	0.3965638
(15)	0.2612014	0.2109654	0.4254017	0.2129917	**0.381584**8	**0.3652255**	**0.3772302**	0.3685401
(16)	0.2765535	0.2344174	0.4689588	0.2194251	0.3944380	0.3779578	0.3868140	0.3795703

Table 7: Giacomini-White Tests

The table shows the Giacomini-White Tests for comparing the forecast accuracy for one-day forecasts. Eighty five rolling samples using S&P 500 Cash Market data are considered: in-sample $k, k = 1, ..., 85$, the ten-year in-sample period starts from January 1990 plus $(k-1)$ months, and ends in December 1999 plus $(k-1)$ months; the two-year out-of-sample period is from January 2000 plus $(k-1)$ months to December 2001 plus $(k-1)$ months. Two types of loss functions are considered, namely out-of-sample log-likelihood and MSFE. In calculating the Giacomini-White statistics, three instruments are used: constant, the last available (i.e. 24 lagged) difference of loss function, and the difference of BIC. The column 'Ratio of A > B' contains the ratio of selecting model A rather than B in the last 61 samples ($61 = 85 - 24$, discarding the 24 initialization samples) based on the decision rule for forecast selection on page 1558 in Giacomini and White (2006), described via STEP1 and STEP2 in the paper. We only report the results for a selected choice of model combinations. As noted earlier, all the model specifications are summarized in Table 1. In Tables 3 through 6 we use model numbers which correspond to (1) Daily GARCH, (2) Daily ASYGARCH, (3) Daily QGARCH, (4) RV GARCH, (5) HYBRID GARCH, (6) HYBRID ASYGARCH, (7) HYBRID QGARCH, (8) Periodic GARCH, (9) Periodic HYBRID ASYGARCH, (10) Periodic HYBRID QGARCH, (11) HYBRID GARCH with pre-filtering, (12) HYBRID ASYGARCH with pre-filtering, (13) HYBRID QGARCH with pre-filtering, (14) Periodic HYBRID GARCH with pre-filtering, (15) Periodic HYBRID ASYGARCH with pre-filtering, (16) Periodic HYBRID QGARCH with pre-filtering.

		Out -of-sample Log-likelihood			MSFE		
Model A	Model B	Stat	P-value	Ratio of A>B	Stat	P-value	Ratio of A>B
(4)	(2)	43.1258	$2.3142E-09$	1.0000	13.7150	$3.3199E-03$	0.3279
(9)	(2)	56.7091	$2.9650E-12$	1.0000	23.9972	$2.5000E-05$	0.5738
(15)	(2)	44.5765	$1.1383E-09$	1.0000	4.8264	$1.8496E-01$	0.4754
(9)	(4)	50.5052	$6.2360E-11$	1.0000	34.5475	$1.5181E-07$	1.0000
(15)	(4)	29.7882	$1.5291E-06$	0.8689	8.5312	$3.6219E-02$	0.6393
(9)	(15)	30.6705	$9.9725E-07$	0.7869	28.1007	$3.4595E-06$	0.7377

Figure 1: Intra-daily Periodic Patterns

The figure shows the periodic pattern $s_i, i = 1, ..., m$, where $s_i = \frac{1}{T}\sum_{t=1}^{T}\frac{r_{i,\tau}^2}{V_{\tau|\tau-1}}$, $V_{\tau|\tau-1}$ is estimated from a Daily ASYGARCH(1,1) model defined in Table 1.

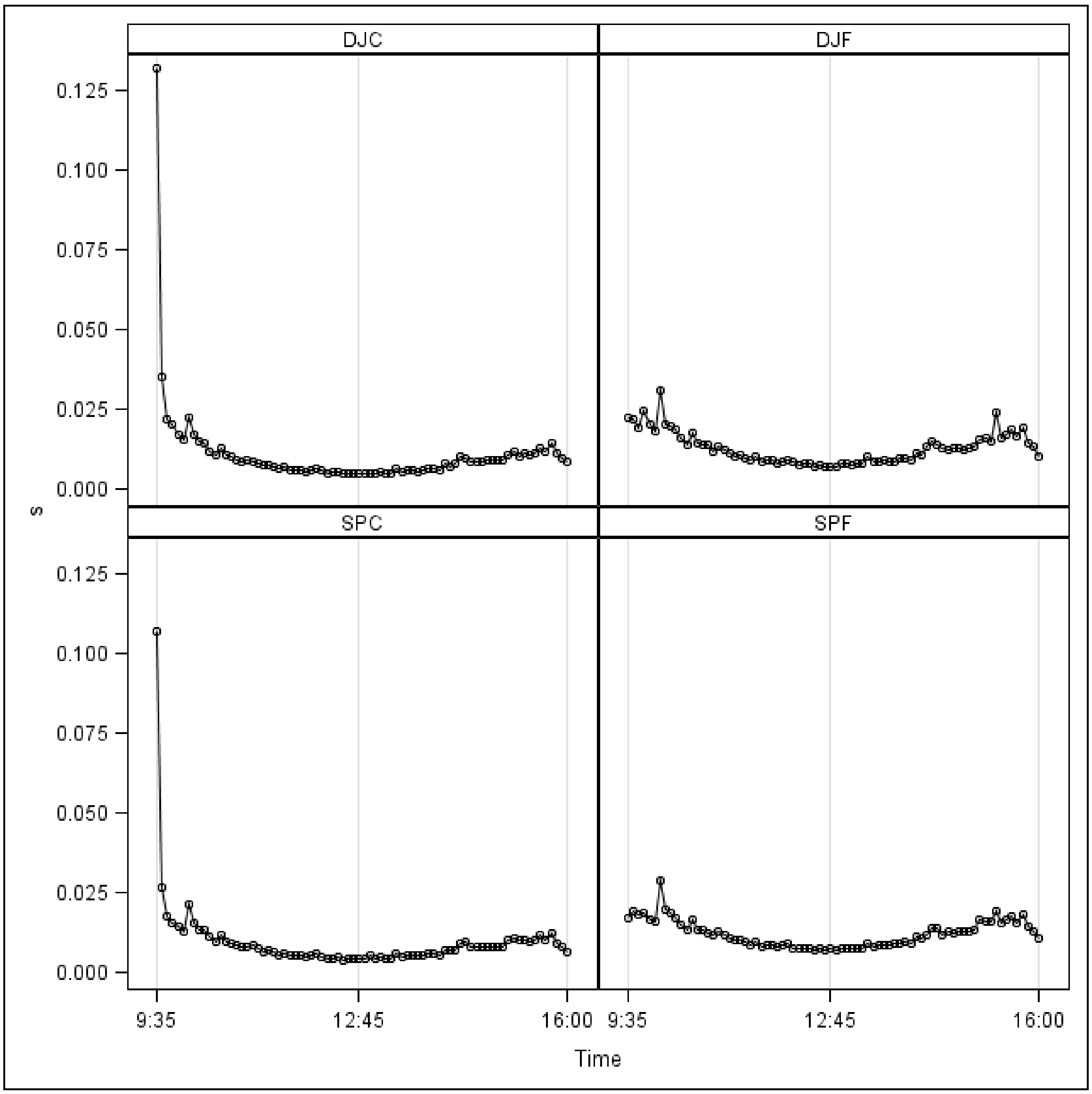

Figure 2: Differences of Out-of-sample Log-likelihood between Models

The figure shows the Differences of Out-of-sample Log-likelihood between Models. In each panel, two models are compared. The differences are (a) RV GARCH minus Daily ASYGARCH; (b) Periodic HYBRID ASYGARCH minus Daily ASYGARCH; (c) Periodic HYBRID ASYGARCH with pre-filtering minus Daily ASYGARCH; (d) Periodic HYBRID ASYGARCH minus RV GARCH; (e) Periodic HYBRID ASYGARCH with pre-filtering minus RV GARCH; and (f) Periodic HYBRID ASYGARCH minus Periodic HYBRID ASYGARCH with pre-filtering.