Conditional Betas: Asymmetric Responses to Good and Bad News*

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Abstract

Several parametric and non-parametric estimators of conditional betas have been suggested by the econometric literature. The conditional CAPM does not provide any structure on how betas should vary and it is not clear which estimator should be preferred. In this paper we fill this gap by investigating the economic structure imposed by a rational expectations equilibrium model on conditional betas. We show that uncertainty about the state of the economy combined with different cash-flow structures, like those implied by value and growth portfolios, result in very distinct dynamics of conditional betas. Furthermore, the same forces driving volatility asymmetry result in excess covariance during market downturns for cyclical assets and in asymmetric responses of conditional betas to news. In particular, the market betas of value portfolios increase more with negative news than with positive news.

Key words: Conditional Betas, Systematic Risk, Asset Pricing.

1 Introduction

It has long been recognized that the systematic risk of stocks, captured by the market beta, is time-varying. Indeed, in empirical applications as early as Fama and MacBeth (1973) betas were computed from rolling sample moments. The conditional capital asset pricing model (CAPM), however, does not impose any structure on how betas should vary. This has largely been tackled from an empirical perspective. Early parametrical approaches include the multivariate GARCH framework (Bollerslev, Engle, and Wooldridge, 1988) and the instrumental variables

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or "conditioned down" betas (Harvey, 1989). Recent parametric models suggest treating conditional betas as latent variables: Adrian and Franzoni (2009) suggest using the Kalman filter while Ang and Chen (2007) apply Markov chain Monte Carlo and Gibbs sampling to obtain time varying betas. Non-parametric approaches have been suggested by Andersen, Bollerslev, Diebold, and Wu (2006), who use high-frequency data to estimate betas and Ang, Chen, and Xing (2006) who point out how asymmetries in betas may be important.

As the econometric literature indicates, there is still an ongoing debate as to how conditional betas should be estimated. Ghysels (1998) points out that misspecified conditional betas can result in higher pricing errors than static betas. This is one of the reasons why many empirical works still use the rolling betas of Fama and MacBeth (1973) to avoid taking a stand on an econometric model (Lewellen and Nagel, 2006).

In this paper we contribute to this debate from an economic theoretical perspective. We investigate the dynamics of conditional betas implied by a rational expectations equilibrium. More specifically, we consider a multiple asset version of the rational expectations equilibrium model of Veronesi (1999) first suggested by Ribeiro and Veronesi (2002). In our model, the investor is uncertain about the true distribution of each asset's cash-flow stream. In particular, the investor does not observe the drift of the continuous process that characterizes cash-flows which can take on two values according to a Markov chain process. As a result of this uncertainty, investor decisions are affected by a learning process. The asset pricing formulas implied by the model bear many of the properties observed in real data. For instance, the asset return volatility implied by the model displays the clustering effect and the asymmetric response to news so pervasive in real data.

The results drawn from the model are the following: first, the asset pricing formulas are able to capture the cross-section variation in expected returns. We calibrate a five asset economy with the parameters implied by the log-dividend growth series of five book-to-market sorted portfolios. The theoretical values match the cross-sectional variation in expected returns and the standard deviation of the actual data reasonably well. The calibration exercise also reveals that cash-flows of the firms in the value portfolio are much more susceptible to changes in the economic conditions than those of firms in the growth portfolio.

Second, uncertainty regarding the state of the economy results in time-varying expected returns. A factor decomposition of expected excess returns also indicates that both the market beta and the price of risk are time-varying. The market beta and the price of risk are both important to the dynamics of the expected returns to assets with the value and growth characteristics. On the other hand, the dynamics of the expected returns to assets in the intermediate portfolios on the book-to-market spectrum are mostly determined by the price of risk.

Third, conditional betas can have opposite dynamics according to the asset's underlying cash-flow structure. The model implies that the conditional betas of assets whose profitability is highly sensitive to changes in the economic conditions, like the value portfolio, increase with uncertainty about the state of the economy. On the other hand, the conditional betas of assets that are less sensitive to shifts in the economic conditions, which includes anti-cyclical assets and also cyclical assets with a lower sensitivity, decrease with uncertainty.

Finally, the same forces that generate volatility asymmetry of asset returns, induce conditional betas to respond asymmetrically to positive and negative news. The model implies that the conditional betas of assets whose profitability is highly sensitive to changes in the economic conditions increase more after negative news than after positive news. On the other hand, for less sensitive assets, the conditional betas decrease more after negative news than after positive news.

The asymmetric implications to conditional betas of our model provide support to the empirical results found by Ang, Chen, and Xing (2006). The authors show that a downside beta, computed from rolling samples conditioned on the market returns being negative, is priced on the cross-section of stocks returns and command a significant risk premium. They link this risk premium to investors asymmetric preferences toward gains and losses. In our model, a different interpretation is possible: the downside betas actually capture the time-variation of the systematic risk of firms whose cash-flows are very sensitive to changes in the market conditions, like those in the value portfolios.

Our paper is related to Santos and Veronesi (2004), who derive implications to market betas within a general equilibrium model. The model assumes that the investor has habit persistent preferences and that the dividends in the economy are random shares of the total endowment of the economy. They find that betas can be decomposed into a cash-flow risk and a discount risk components and that the dynamic of conditional betas is determined by the component that is most important.

The paper proceeds as follows: in section (2), we solve the model and discuss the resulting asset pricing formulas. Then, in section (3) we calibrate the model with U.S. data. We simulate time series of asset returns using the theoretical formulas, compute ex-post moments and estimate univariate and multivariate GARCH models to assess the dynamics of covariance and market betas. We conclude the paper with a summary of the results and some final remarks.

2 The model

The model is a multiple asset version of the rational expectations equilibrium model of Veronesi (1999). This extended version was also considered by Ribeiro and Veronesi (2002) but for a different purpose. In that paper, the authors show how uncertainty about the state of the world economy can result in the observed excess covariation in international stock markets during downturns. Here, we will investigate the dynamics of the different components of the risk premia and, in particular, how good and bad news are incorporated into conditional betas.

The key assumption of the model is the uncertainty the investor faces about the true distribution of the assets' cash-flows. More specifically, the drifts of the continuous stochastic processes that describe the cash-flows can take two values according to an unobserved two state Markov switching process. It is further assumed that the investor optimally infers the true drifts from cash-flows realizations. This generates a learning process that results in asset prices that bear many of the empirical properties observed in real data. For instance, the asset return volatility implied by the model is time varying and displays the clustering effect observed in real data. It also reproduces the predictable asymmetry of stock returns. Finally, it provides a theoretical justification for the apparent puzzle that asset returns are more volatile than theirs underlying cash-flows.

Apart from the ability to replicate many of the stylized facts about stock returns, the model is appealing for it provides a tractable framework to incorporate a learning dynamic into pricing formulas. For instance, it allows us to assess how news about the economy can change the risk of assets. As we will see below, different cash-flow structures can result in opposite responses of market betas to news of the same sign.

2.1 The Economy

The economy has one representative investor that maximizes expected utility subject to a budget constraint. There are $n+1$ financial assets: a risk-free asset that is inelastically supplied with a known rate of return rdt and n risky assets that pay continuous stream of cash-flows given by:

$$dD_{it} = \theta_{it}dt + \sigma_i d\xi_t \quad i = 1, ..., n$$

where $d\xi_t$ is a $(n \times 1)$ vector of Brownian motions and σ_i a $(1 \times n)$ vector of diffusion coefficients. The n expressions presented above can be written in matrix notation as $dD_t = \theta_t dt + \Phi d\xi_t$, where θ_t is the $(n \times 1)$ vector of drift terms θ_{it}, and Φ is the $(n \times n)$ matrix that stacks the diffusion terms σ_i. Denote by $\Sigma = \Phi\Phi'$ the cash-flow covariance matrix. The market portfolio cash-flow is defined as the sum of all cash-flows times the available shares of each asset, $D_{mt} \equiv \sum_{i=1}^{n} \omega_i D_{it}$, where $\omega = [\omega_1, ..., \omega_n]'$ are the available shares.

The investor does not observe the random vector $\{\theta_t\}$ but knows it can take two values: $\theta_G = [\theta_{1G}, ..., \theta_{nG}]'$ in the good state and $\theta_B = [\theta_{1B}, ..., \theta_{nB}]'$ in the bad state. This random vector switches between the two states with conditional probabilities that follow a market-wide two-state Markov-chain process with parameters μ, the probability of going to a good state from a bad state, and λ, the probability of shifting from the good state to the bad state. We will label asset i cyclical if $\Delta\theta_i \equiv \theta_G - \theta_B > 0$ and anti-cyclical otherwise.

The investor optimally infers the true drifts of cash-flows from past observations. That is, he conditions his beliefs about the true drifts on the information set $\mathcal{F}_t = \sigma(D_\tau, \tau < t)$. As was shown by Veronesi (1999), the optimal prediction is conveniently described by a stochastic process. The following lemma is an extension of the univariate case for multiple assets.

Lemma 1. *The investor's belief that the economy is in the good state,* $\pi_t \equiv Prob(\theta_t = \theta_G|\mathcal{F}_t)$, *evolves according to the stochastic process:*

$$d\pi_t = (\lambda + \mu)(\pi_s - \pi_t)\,dt + \pi_t(1 - \pi_t)\Delta\theta'\Phi'^{-1}dv_t \tag{1}$$

where $\pi_s = \frac{\mu}{\lambda+\mu}$ *is the unconditional probability of* π_t, $\Delta\theta' = [\theta_{1G} - \theta_{1B}, ..., \theta_{nG} - \theta_{nB}]$, *and* $dv_t \equiv \Phi^{-1}(dD_t - E[dD_t|\mathcal{F}_t])$ *is a* $(n \times 1)$ *vector of standard Brownian motions with respect to the filtration* $\mathcal{F}_t$, *with* $E[dD_{it}|\mathcal{F}_t] = \theta_{iG}\pi_t + \theta_{iB}(1 - \pi_t)$ *for* $i = 1, ..., n$.

Proof. It follows from theorem 9.3 in Lipster and Shiryaev (2001). □

Note that π_t mean reverts towards its unconditional mean, π_s, at a rate of $\lambda + \mu$. Shocks to dv_t are weighted by a signal to noise ratio, $\Delta\theta'\Phi'^{-1}$, and by the uncertainty level about the state of the economy, $h(\pi_t) \equiv \pi_t(1 - \pi_t)$. The closer π_t is to 0.50, the more uncertain the investor is about the true state, and the larger the revisions to the conditional probability are. For ease of notation, let $\alpha_\pi \equiv (\lambda + \mu)(\pi_s - \pi_t)$ and $\sigma_\pi^2 \equiv \pi_t^2(1 - \pi_t)^2\Delta\theta'\Sigma^{-1}\Delta\theta$. We will also denote the $(1 \times n)$ vector by $\sigma_\pi \equiv \pi_t(1 - \pi_t)\Delta\theta'\Phi'^{-1}$.

As we will see below, the second moments of asset returns will be non-linear functions of uncertainty as captured by $h(\pi_t)$. In order to study the dynamics of these moments, it will be instructive to assess how uncertainty evolves by differentiating $h(\pi_t)$. We define the market at time t as good if $\pi_t \geq 0.5$ and as bad otherwise. The following corollary gives the conditional dynamics of uncertainty:

Corollary 2. *Define uncertainty as* $h(\pi_t) \equiv \pi_t(1 - \pi_t)$. *Then the following process describes the evolution of conditional uncertainty over time*

$$dh_t = \begin{cases} \left[\alpha_h - (\mu - \lambda)\sqrt{h^{max} - h_t}\right]dt - \sigma_h dv_t & \text{if the market is good, } \pi_t \geq 0.5 \\ \left[\alpha_h + (\mu - \lambda)\sqrt{h^{max} - h_t}\right]dt + \sigma_h dv_t & \text{if the market is bad, } \pi_t < 0.5 \end{cases} \tag{2}$$

where $\alpha_h \equiv 2(\lambda + \mu)(h^{max} - h_t) - h_t^2\Delta\theta'\Sigma^{-1}\Delta\theta$, $\sigma_h \equiv 2h_t\sqrt{h^{max} - h_t}\Delta\theta'\Phi'^{-1}$ *is a* $(1 \times n)$ *row vector and* $h^{max} = \frac{1}{4}$. dv_t *is the same* $(n \times 1)$ *vector of standard Brownian motions with respect to* $\mathcal{F}_t = \sigma(D_\tau, \tau < t)$ *defined in proposition (1).*

Proof. The result follows from the application of Ito's lemma to $h(\pi_t)$. □

Note that the sign on the term σ_h in equation (2) shows that positive news in a bad market and negative news in a good market increase uncertainty[1].

Whenever expansions last longer than recessions, $\lambda < \mu$, the unconditional mean π_s will be greater than 0.50, that is, the market will be good more often than not[2]. As a result, it follows from corollary (2) that increases in uncertainty are more likely to arise after bad news than after good news. We will see below that this asymmetric response of uncertainty to news will also induce asymmetries in sample moments of asset returns, volatility and covariances.

In this economy investor preferences are represented by a constant absolute risk aversion utility function:

$$U\left(c,t\right) = -exp\left[-\rho t - \gamma c\right]$$

where γ is the coefficient of absolute risk aversion and ρ the time preference parameter.

Under the incomplete information set, $\mathcal{F}_t$, cash-flows can be written as $dD_t = \alpha_{Dt}dt + \Phi d\mathbf{v}_t$, where $\alpha_{Dt} = [\alpha_{1D,t}, ..., \alpha_{nD,t}]'$ and $\alpha_{iD,t} \equiv \theta_{iG}\pi_t + \theta_{iB}(1-\pi_t)$. The investor's optimization problem is solved by expressing dD_t in terms of the Brownian motion $d\mathbf{v}_t$ and including π_t as a state variable. Pricing formulas are obtained by imposing a market clearing condition on the available shares of the risky assets.

2.2 Asset Prices and Returns

The following proposition shows that asset prices that solve the investor problem and clear the market are non-linear functions of the investor conditional belief and dividends.

Proposition 3. *[Ribeiro and Veronesi (2002)] The following asset prices solve the investor problem and clear the market:*

$$P_{it} = p_{0i} + \frac{D_{it}}{r} + p_{\pi i}\pi_t + p_{1i} + S_i\left(\pi_t\right) \tag{3}$$

where S_i is *the solution to a differential equation given in the Appendix and*

$$\begin{aligned} p_{0i} &= \frac{\theta_{iB}}{r^2} + \frac{\left(\theta_{iG} - \theta_{iB}\right)}{r^2\left(r+\lambda+\mu\right)}\mu \\ p_{\pi i} &= \frac{\left(\theta_{iG} - \theta_{iB}\right)}{r\left(r+\lambda+\mu\right)} \\ p_{1i} &= -\frac{\gamma\sigma_{i,m}}{r^2} \end{aligned}$$

[1] In what follows, we refer to news as shocks to dv_t times the signal to noise ratio $\Delta\theta'\Phi'^{-1}$. This normalization will help us compare news across assets and simplify our exposition. For instance, a shock to cash-flows from an asset with a very volatile process is not as informative as a shock of the same magnitude to an asset with more stable cash-flows. Also, a positive shock to an anti-cyclical asset is actually bad news about the state of the economy. Thus, by considering news as $\Delta\theta'\Phi'^{-1}dv_t$, we do not need to be more specific about the cash-flow structure of the assets.

[2] In fact, NBER cycles imply an unconditional mean of around $\pi_s = 0.80$

for $i = 1, ..., n$. The market portfolio is the aggregate portfolio $P_{mt} = \sum_{i=1}^{n} \omega_i P_{it}$.

Proof. See Appendix. $\square$

The S_i function in equation (3) discounts cyclical assets and inflates anti-cyclical assets, generating a premium for holding risky assets. This discount (inflation) reaches a minimum (maximum) in the interior of $\pi_t \in (0, 1)$ if the asset is cyclical (anti-cyclical).

From asset prices, excess returns, variances and covariances can be obtained by direct application of Ito's lemma, as the following proposition shows.

Proposition 4. *Define excess return as $R_{it}^e \equiv \frac{dP_{it}}{P_{it}} + \frac{D_{it}}{P_{it}}dt - rdt$. Then the following continuous process describes excess returns in terms of the model's parameters:*

$$R_{it}^e = \alpha_{iR,t}dt + \sigma_{iR,t}dv_t \tag{4}$$

$$\begin{aligned}
\alpha_{iR} &= \frac{1}{P_{it}}\left[\frac{\gamma}{r}e_i'\Sigma\omega - rS_i(\pi_t) + S_i'(\pi_t)\alpha_\pi + \frac{1}{2}S_i''(\pi_t)\sigma_\pi^2\right] \\
\sigma_{iR} &= \frac{1}{P_{it}}\left[\frac{e_i'\Phi}{r} + \left[S_i'(\pi_t) + p_{\pi i}\right]\pi_t(1-\pi_t)\Delta\theta'\Phi'^{-1}\right]
\end{aligned}$$

for $i = 1, ..., n$ assets, where e_i is a $(n \times 1)$ vector of zeros and one at the ith *row. For the market portfolio, set $i = m$ and ω. Expected excess returns are then given by $E_t[R_{it}^e] = \alpha_{iR}dt$ and excess return covariance between assets i and j, where $i, j = 1, ..., n, m$, $E_t\left[R_{it}^e R_{jt}^e\right]$, by:*

$$\sigma_{ij,R} = \frac{1}{P_{it}P_{jt}}\left[(A_{ij} + M_{ij}(\pi_t))\pi_t^2(1-\pi_t)^2 + (B_{ij} + N_{ij}(\pi_t))\pi_t(1-\pi_t) + C_{ij}\right]dt$$

where

$$\begin{aligned}
A_{ij} &= \frac{\Delta\theta_i\Delta\theta_j}{r^2(r+\lambda+\mu)^2}\Delta\theta'\Sigma^{-1}\Delta\theta \\
B_{ij} &= 2\frac{\Delta\theta_i\Delta\theta_j}{r^2(r+\lambda+\mu)} \\
C_{ij} &= \frac{1}{r^2}cov_t(dD_{it}, dD_{jt}) \\
M_{ij}(\pi_t) &= \Delta\theta'\Sigma^{-1}\Delta\theta\left[S_i'(\pi_t)S_j'(\pi_t) + \frac{S_i'(\pi_t)\Delta\theta_j + S_j'(\pi_t)\Delta\theta_i}{r(\lambda+\mu+r)}\right] \\
N_{ij}(\pi_t) &= \frac{\left[S_i'(\pi_t)\Delta\theta_j + S_j'(\pi_t)\Delta\theta_j\right]}{r}
\end{aligned}$$

The excess return variance of asset i follows by setting both subscripts above to i.

Proof. It follows by applying Ito's lemma to the definition of excess returns. $\square$

If the investor is risk-neutral, the discounting function S is zero and expected returns are proportional to the cash-flow covariance of the asset with the market portfolio, normalized by prices. If we instead assume the investor is risk averse, expected returns will also depend on the conditional probability π_t through the S function. Increases in the discounting of prices, $-rS_i(\pi_t)$, and in their sensitivity to π_t, $S_i'(\pi_t)\,\alpha_\pi$, imply higher expected returns. Also, higher uncertainty will command higher expected returns through the term $\frac{1}{2}S_i''(\pi_t)\,\sigma_\pi^2$. In addition to time-varying expected returns, the model also implies that return covariance and volatility are stochastic.

Expected returns can also be expressed in terms of the exposure of the asset to the common sources of risk, or risk factors. In this representation, the risk premium of an asset should equal its quantity of risk, the conditional beta, times the price of such risk. This decomposition is convenient as it splits the difficult task of estimating returns into two separate ones, the estimation of conditional betas and the price of risk. The price of risk is the same for all assets; conditional betas are functions of second moments which, in theory, should be easier to estimate (Merton, 1980).

Proposition 5. *Expected returns have the following factor representation:*

$$E_t\left[R_{it}^e\right] = \lambda_{mt}\beta_{im,t} + \lambda_{\pi t}\beta_{i\pi,t} \tag{5}$$

where the prices of risk are given by:

$$\begin{aligned}\lambda_{mt} &= r\gamma P_{mt}\sigma_{mR,t}^2\\ \lambda_{\pi t} &= \left[f'(\pi_t) - r\gamma S_m'(\pi_t)\right]\sigma_{\pi t}^2\end{aligned}$$

and conditional betas, defined as $\beta_{im,t} \equiv \frac{\sigma_{im,R}}{\sigma_{m,R}^2}$ *and* $\beta_{i\pi,t} \equiv \frac{\sigma_{i\pi,R}}{\sigma_{\pi,R}^2}$, *have the following expressions:*

$$\beta_{im,t} = \frac{P_{mt}}{P_{it}} \times \frac{\left(A_{im} + M_{im}(\pi_t)\right) h(\pi_t)^2 + \left(B_{im} + N_{im}(\pi_t)\right) h(\pi_t) + C_{im}}{\left(A_{mm} + M_{mm}(\pi_t)\right) h(\pi_t)^2 + \left(B_{mm} + N_{mm}(\pi_t)\right) h(\pi_t) + C_{mm}} \tag{6}$$

$$\beta_{i\pi,t} = \frac{1}{P_{it}}\left[p_{i\pi} + S_i'(\pi_t) + \frac{\Delta\theta_i}{rh(\pi_t)\,H}\right] \tag{7}$$

where A, B, C, M and N are given in proposition (4). The functions f and S are solutions to differential equations given in the Appendix.

Proof. The expression for expected returns (5) follows by rewriting the optimal demand for shares, equation (14) in Appendix, in terms of expected returns and substituting for the market clearing condition, $X_t^* = \omega$. Then, scaling the terms by the variances $\sigma_{m,R}^2$ and $\sigma_{\pi,R}^2$, we obtain betas and prices of risk. The expressions of betas in terms of the parameters of the model follow after substituting for the covariances and variances given in (4). □

The first component of expression (5) is the usual conditional CAPM term, with variable beta and price of risk. The conditional market beta is defined as the ratio of the conditional covariance of asset and market excess returns normalized by the conditional variance of the market excess returns, $\beta_{im,t} = \sigma_{im,R}/\sigma_{m,R}^2$. This measure of risk captures the responsiveness of asset returns to changes in market returns. An asset with a high market beta will be riskier as it amplifies the volatility, or risk, of the investor's portfolio. Indeed, the price of market risk is positive as all elements in λ_{mt} are greater than zero, and assets with high betas reward the investor with higher returns.

Since we found expressions for returns (see proposition (4)), we can substitute the formulas and link conditional market betas to the parameters of the model and its underlying state variables. As equation (6) shows, the market beta is a non-linear function of π_t and depends upon the discounting function S that can only be obtained numerically. In Section 3 we investigate betas by solving the model for calibrated parameters and computing the S function numerically. However, we can obtain some results by considering the case of a risk-neutral investor, which obviates the S function. This case is developed below.

The second term in expression (5) results from the time-varying nature of the investment opportunity set (Merton, 1973). Note that the drift and diffusion terms of stock returns in equation (4) are functions of the random variable π_t and are thus stochastic. Assets that can help the investor hedge against future changes in profitability should be more expensive, i.e. have lower expected returns. The exposure of an asset to this source of risk is measured by its factor loading, defined as $\beta_{i\pi,t} \equiv \sigma_{i\pi,R}/\sigma_\pi^2$, which can be rewritten as equation (7). We observe that assets that are very sensitive to changes in π_t, and have a large state shift risk, i.e. a large $\Delta\theta_i$, also have large betas. Additionally, as uncertainty is reduced, i.e. $h(\pi_t)$ gets closer to zero, betas of cyclical assets increase and those of anti-cyclical assets decrease. In the limit, when $h(\pi_t)$ is zero, the betas diverge to infinity[3].

The price of a unit of such risk is given by $\lambda_{\pi t}$ and it can be positive or negative, depending on the function f and the market discount S_m function. For the parameters selected in the next section, the price of risk is positive at lower values of π_t and negative for higher values.

2.3 The Risk Neutral Case

In this section we further investigate how conditional market betas differ for assets with distinct cash-flow structures. In order to do so, we will assume the investor is risk neutral and thus avoid the calculation of S. Note that the risk neutral betas are also time-varying and nonlinear in π_t, as the risk neutral investor still has to predict the true drift.

Consider first the dynamics of the risk neutral return volatility. Setting S equal to zero, the

[3] As we will see in the next section, however, the price of this risk is also zero when $h(\pi_t)$ is zero.

risk neutral variance of asset i follows directly from proposition (4):

$$\sigma_{iRN}^2 = \left(\frac{1}{P_{it}^{RN}}\right)^2 \left[A_{ii}h\left(\pi_t\right)^2 + B_{ii}h\left(\pi_t\right) + C_{ii}\right]$$

where $P_{it}^{RN} = p_{0i} + D_{it}/r + p_{\pi i}\pi_t$ denotes risk neutral prices and the constants A_{ii}, B_{ii} and C_{ii} are the same ones defined in proposition (4). Note that these constants are positive for both cyclical and anti-cyclical assets and so variance is increasing in uncertainty. Furthermore, return volatility of assets with a higher state shift risk is more responsive to changes in uncertainty.

From corollary (2) we know the following: i) good markets are more frequent than bad markets and ii) in good markets, negative news is followed by an increase in uncertainty whereas positive news implies a decrease in uncertainty. If we combine this observation with the above result that volatility increases with uncertainty we have that conditional return volatility of all assets should respond asymmetrically to news. That is, on average, bad news will be followed by a larger increase in volatility than good news. This predicted volatility asymmetry has long been observed in stock returns and was originally attributed to a leverage effect[4]: negative stock returns reduce the equity value of the firm and increase the debt-to-equity ratio and the riskiness of the firm, which ultimately increase variance. Here, the mechanism behind is closer to the volatility feedback hypothesis of Campbell and Hentschel (1992): negative shocks increase the required risk premium which further depreciate price to compensate the increase in the expected return.

Consider now the risk neutral covariance of an asset i with the market:

$$\sigma_{im,RN} = \frac{1}{P_{it}^{RN}P_{mt}^{RN}} \left[A_{im}h\left(\pi_t\right)^2 + B_{im}h\left(\pi_t\right) + C_{im}\right]$$

For cyclical assets, the constants A_{im} and B_{im} are positive, since $\Delta\theta_i > 0$ and $\Delta\theta_m > 0$, as the market is by definition cyclical. On the other hand, these constants are negative for anti-cyclical assets. For simplicity, assume that C_{im}, the covariance of the asset and market cash-flows, is positive. Then, the covariance of asset and market returns increase with uncertainty if the asset is cyclical but decrease if the asset is anti-cyclical. Since the economy is in the good state most of the time, covariances will also respond asymmetrically to shocks: negative news has a stronger positive effect upon covariances of cyclical assets than positive news. The opposite is true for anti-cyclical assets.

Finally, we consider how risk neutral conditional betas respond to news. With S equal to

[4] Black (1976)

zero, the market betas from equation (6) are simplified to:

$$\beta_{im,t}^{RN} = \frac{P_{m,t}^{RN}}{P_{it}^{RN}} \times \frac{A_{im}h(\pi_t)^2 + B_{im}h(\pi_t) + C_{im}}{A_{mm}h(\pi_t)^2 + B_{mm}h(\pi_t) + C_{mm}} \tag{8}$$

As discussed above, for cyclical assets and assuming C_{im}, both numerator and denominator are increasing functions of uncertainty. Depending upon which term responds more strongly to uncertainty, the asset's beta will be either increasing or decreasing with uncertainty. An inspection of the constants A_{im}, A_{mm}, B_{im} and B_{mm} indicates that assets with smaller $\Delta\theta_i$ than that of the market's, $\Delta\theta_m$, have a decreasing beta in uncertainty and vice-versa. This pattern is maintained after scaling equation (8) by the ratio of prices. For the anti-cyclical asset, the market covariance is declining in uncertainty and, as a result, conditional betas decline as uncertainty increases. As with the other moments, conditional betas are also expected to respond asymmetrically to news. For assets with large state shift risk, $\Delta\theta_i > \Delta\theta_m$, conditional betas should on average increase more after negative news than after positive news. However, for anti-cyclical assets, $\Delta\theta_i < 0$, or assets with low state shift risk, $0 < \Delta\theta_i < \Delta\theta_m$, conditional betas increase on average more after positive news than after negative news.

We summarize the findings about risk neutral moments as follows: first, conditional variance increases with uncertainty, irrespective of the asset's cash flow structure. Second, the conditional covariance of asset returns and market returns increases with uncertainty if the asset is cyclical and decreases if it is anti-cyclical. Finally, conditional betas of assets with a large state shift risk than the average will increase with uncertainty and decrease otherwise. Since these moments are monotonic functions of uncertainty and uncertainty responds asymmetrically to news, risk neutral conditional variance, covariance and betas also respond asymmetrically to news.

In this section, we showed that the above results holds on the assumption that the investor is risk-neutral. However, as we will see in the next section, similar patterns are observed when the investor is risk averse. Namely, that second moments of returns are also monotonic functions of a "risk adjusted" uncertainty, that attains a maximum point slightly to the right of $\pi_t = 0.5$.

3 Simulation

In this section, we investigate the asset pricing equations by calibrating the model with the parameter values implied by the U.S. data. This will allows us to numerically compute the S and f functions and to consider the more general case of a risk averse investor. More importantly, we will be able to assess whether the model can replicate some of the stylized facts of U.S. stock returns for a reasonable selection of parameters.

The cash-flow parameters are based on the log-dividend growth of the five book-to-market sorted portfolios. For each portfolio, log-dividend growth series are computed from the difference of monthly returns with and without dividend payouts as in Bansal, Dittmar, and Lundblad (2005). In order to avoid seasonal variations in dividend payouts, growth is on a year over year basis. The sample starts in January 1956 and ends in December 2010 and was obtained from the website of Kenneth French[5].

The particular choice of the book-to-market portfolios is motivated by two observations. First, portfolios sorted on the book-to-market characteristic serve as a benchmark to the performance of asset pricing models. Book-to-market portfolios provided one of the first pieces of irrefutable evidence of the failure of the unconditional CAPM (Basu, 1977). Largely based on the poor performance of the unconditional CAPM on these portfolios, conditional (Hansen and Richard, 1987) and multifactor models were suggested (Fama and French, 1993). Second, these portfolios have very distinct cash-flow structures. The portfolio with the lowest book-to-market ratio, the so-called growth portfolio, is usually associated with firms in new industries and with future profitability prospects. On the other hand, the portfolio with the highest book-to-market ratio, referred to as the value portfolio, is associated with firms whose shares are at a discount relative to their current profitability level. These distinct cash-flow structures will result in different pricing equations.

Also needed for the calibration are the parameters contained in the Markov switching transition matrix. Since the model requires the profitability of all assets to jointly switch from one state to the other, it is natural to relate these parameters to the business cycles fluctuations. The NBER cycles data[6] during January 1956 and December 2010 indicate that i) 83.3% of the months were expansionary and ii) the average duration of a recession was 11.22 months against 62.22 months for expansions. These numbers imply the following two-state Markov transition parameters[7]: $\lambda = 0.0165$, the probability of going from the good state to the bad state, and $\mu = 0.0843$, the probability of switching to the good state from the bad state.

In Panel A of Table II we compute averages of the log-dividend growth series for the NBER recession and expansion sub-samples. The sample moments do not vary much across sub-samples and portfolios. This could be a result that dividend payouts are relatively persistent and its dynamics may not necessarily coincide with the business cycles. In Panel B of Table II, we compute sample means of the assets' dividend growth for the 16.7% months with the worst market dividend growth as well as for the 83.3% months with the best growth. We observe that the value portfolio dividend growth varies the most across the two sub-samples, it shifts from −14.2% to 9.7%, and also is the most volatile, 16% in the sub-sample with the longest duration.

[5]http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

[6]http://www.nber.org/cycles.html

[7]The duration of an expansion is $1/\lambda$ and of a recession is $1/\mu$. Additionally, the proportion of expansionary months is given by $\mu/(\mu+\lambda)$. Matching the sample values to these measure result in a system of equation that uniquely define $\lambda$and μ.

On the other hand, the growth portfolio varies the least, from 2.3% to 5.2%, but is also more volatile than the other intermediate portfolios, at 12.4%. In addition to the increase in the size of $\Delta\theta$ as we move from growth to value portfolios, the unconditional mean also increases from growth to value portfolios, 4.7% and 5.9% respectively.

The observed increase in $\Delta\theta$ across portfolios points to this being an economically meaningful phenomenon. It has long been recognized that value firms are particularly susceptible to economic downturns (Fama and French, 1993). Growth firms, on the other hand, are less sensitive to current economic condition. In our framework, this translates into a lower state-shift risk for growth portfolios, a small $\Delta\theta$, and a high state shift risk for value portfolios, a larger $\Delta\theta$ with a negative θ_B. Furthermore, the cash-flow of growth firms should be more difficult to forecast than those of firms in already established industries. This would be captured in our model by a higher volatility of cash-flows, irrespective of the state of the economy. Note that these portfolios are rebalanced every year and so they track the same firm characteristic, not the same firms over the years.

So far we have based our analysis of the cash-flow structure on the imposition that expansions should account for 83.3% of the months. We can avoid this by estimating a Markov switching model. This approach lets the econometric model to find the parameters in the transition matrix, μ and λ, jointly with the state dependent drifts. While most papers have focused on Markov switching model of stock returns (see Perez-Quiros and Timmermann (2000) and Guidolin and Timmermann (2008)), fewer have investigated the switching properties of the cash-flow distribution (Timmermann, 2001). In order to find the good and bad state drifts and the transition parameters for two recurrent states we estimate a joint Markov switching model of the five log-dividend growth series. Denote the two possible states by $S_t = [1, 2]$ and the log-dividend growth time series by $\Delta d_t = [\Delta d_{1t}, ..., \Delta d_{nt}]'$. In our first Markov switching specification (Model 1) we allow only the constant to shift across states, fixing the variances:

$$\begin{aligned} \Delta d_t &= \alpha_i + \epsilon_t, \epsilon_t \sim N\left(0, diag\left(\sigma\right)\right) \\ Prob\left(s_t = i | s_{t-1} = j\right) &= p_i, \quad i = 1, 2 \end{aligned} \tag{9}$$

Following Perez-Quiros and Timmermann (2000), we also estimate a Markov switching model that allows the transition matrix to be time-varying. The time variation is obtained by projecting the transition coefficients on lags of the Composite Leading Indicator[8] (CLI) variable. Even though our theoretical model uses a constant transition matrix, here the introduction of the CLI variable will be helpful to identify whether the good and bad states implied by the

[8]The OECD composite leading indicator (CLI) is designed to provide early signals of turning points (peaks and troughs) between expansions and slowdowns of economic activity. Source: www.oecd.org/std/cli.

data are related to business cycles fluctuations. More specifically, we fit the following model:

$$\begin{aligned} \Delta d_t &= \alpha_i + \epsilon_t, \epsilon_t \sim N(0, \Sigma_i) \\ Prob\left(s_t = i | s_{t-1} = j\right) &= F\left(a_i + b_i \Delta CLI_{t-1}\right), \quad i = 1, 2 \end{aligned} \tag{10}$$

where F is the normal cdf, which ensures the estimated parameter is a proper probability. We estimate 3 versions of this model. In the first version, both parameters a_i and b_i are included (Model 2). In a second, we set b_i to zero (Model 3) and finally our last version a_i equals to zero (Model 4).

The estimation results for the Markov switching models in Table III show a similar pattern to that observed in the non-parametric estimations presented in Table II. As before, the value portfolio cash-flow is more susceptible to changes in the state of the economy than the growth portfolio cash-flow. In fact, all the Markov switching models indicate that the growth portfolio has actually a larger constant in state 2 than in state 1. Note that the states are related to opposite ends of the business cycle, as the sign on the CLI variable is positive for state 1 and negative for the state 2. Additionally, the implied parameters for the transition matrix are also similar to the ones indicated by the NBER cycles. The λ implied by Model 2 is 0.026, which is slightly higher than the λ implied by the NBER data, 0.016. The probability of switching to an expansionary state from a recessionary one, μ, is slightly lower, 0.064 versus 0.084.

Based on the above discussion, we calibrate the parameters of our model in the following way. For the transition matrix of the Markov switching process, we set the constants λ and μ equal to the values implied by the NBER cycles, 0.0165 and 0.0843, respectively. For the cash-flow parameters of our five assets, which we will denote by A1, A2, A3, A4 and A5, we follow the general pattern followed by the mean and standard deviation of the dividend growth rates of the five book-to-market portfolios. The parameter values do not coincide because we selected the parametrization that matches as close as possible the real data on excess returns[9]. As can be seen in Table IV, the first asset, A1, resembles the growth portfolio. Its cash-flow has the lowest state shift risk among the assets, it is slightly anti-cyclical, $\Delta\theta_1 = -0.01$, it has the lowest unconditional mean, 4.2%, and the largest volatility $\sigma_1 = 0.16$. At the other extreme is portfolio A5 that is calibrated after the value portfolio. This asset's cash-flow has the highest state shift risk, with $\Delta\theta_5 = 0.23$, but also the highest unconditional mean at 6.2% and the lowest volatility at $\sigma_5 = 0.09$. Note that this diffusion term is smaller than the one implied by the data. This was needed to match sample and theoretical returns[10]. The correlation

[9]It should be noted that the model is sensitive to different choices of parameters. One of the difficulties is that, due to the assumption that cash-flows follow an arithmetic Brownian motion, the resulting asset prices can have negative signs.

[10]If we imposed a higher variance for A5 cash-flow and kept the state-risk spread, this would have resulted in a very risky asset with an incompatible high expected returns. We preferred to keep the state-risk spread but reduce the diffusion risk.

parameters, ρ_{ij}, were set equal to 0.25, 0.15, 0.10 and 0.05 for $|i-j|$ equal to 1, 2, 3 and 4, respectively.

The remaining parameters were set as follows: the risk free rate is $r = 0.045$, a relatively high value but still lower than the average one month treasury bill rates over that period (4.9%). For the risk aversion parameter γ, we follow Veronesi (2004) who sets it at $\gamma = 1$ for a similar simulation exercise, marginally increasing it by 0.1 so that the calibration numbers get closer to the sample counterparts.

3.1 Ex Ante Moments

Table IV shows the moments implied by the model at the unconditional mean $\pi_t = \pi_s$ and for fixed cash-flow levels set at 1[11]. The numbers in Panels B and C show that, for reasonable parameters, the model's implied asset returns and standard deviation are very close to their sample counterparts. The dividend yields preserve the ranking order, lower for A1 and higher for A5, but not the spread observed in real data. A higher dispersion of dividend yields could be obtained at the expense of a higher spread in the expected returns.

In Figure 1 we show the theoretical formulas for all values of π_t. In the first plot, we have the prices of all assets. We observe that asset A1 has the highest price on almost all the domain of π_t. This asset is the least profitable, but because it is also less susceptible to changes in the economic conditions, it is also less risky. On the other hand, A5 is the most profitable one, but because it has a higher state shift risk, it is more heavily discounted. Note also that the price of this asset is very responsive to changes in economic conditions.

The plot with conditional expected returns indicates that assets' risk premia vary substantially with π_t. In particular, we observe that for asset A5, it can go from 3% to 14% as we go from low to high uncertainty. For all the other cyclical assets, A2, A3 and A4, expected returns are also increasing with uncertainty. However, for the anti-cyclical asset A1, the expected return is relatively flatter and declining with uncertainty.

As we saw in proposition (5), an asset's expected return can also be expressed in terms of the exposure of the asset to the common sources of risk. Figure 1 shows that the price of state uncertainty, $\lambda_{\pi t}$, can take on both signs. When the economy is close to the bad state, the price is positive but also close to zero. On most of its domain, however, the price of this risk is negative and reaches a minimum at around $\pi_t = 0.70$. Furthermore, at points of very low uncertainty, the price is zero. To assess if this factor is a relevant component of expected returns, we also have to look at the conditional betas, $\beta_{i\pi,t}$. As can be seen in Figure 1, these conditional betas are very low and close to zero for most values of π_t. When the uncertainty is very low, the betas increase substantially in absolute value but at the same time the price of

[11]When the levels of cash-flow are at one, the drift parameters better approximate the log-dividend changes that were used to calibrate them.

such risk converges to zero.

In order to have a better idea of the relative importance of the risk factors, consider the asset with the highest exposure to the common sources of risk, A5. The state uncertainty premium for this asset reaches a maximum (in absolute terms) at $\pi_t = 0.75$ when $\lambda_{\pi t}\beta_{5\pi,t} = -1\%$. At the same point, the risk premium for its exposure to market risk is much larger, around $\lambda_{mt}\beta_{5m,t} = 15\%$. This has also been observed by Merton (1980), who pointed out that the market portfolio was likely to be the most important factor in determining expected returns.

Now consider the market risk factor. The conditional market betas have a similar shape to that observed when the investor is risk neutral, as determined analytically in the previous section. Note first that the conditional variance of all assets increases as π_t moves away from 0 and 1 and uncertainty increases, as was shown in the risk neutral case. Similarly, the covariance of asset and market returns increases with uncertainty for cyclical assets (A2, A3, A4 and A5) but decreases for anti-cyclical assets (A1). Finally, as we concluded in the previous section, the conditional market betas increase with uncertainty for large state shift assets (A4 and A5) but decrease for lower state shift assets (A1, A2 and A3).

The difference with respect to the risk-neutral case arises at the precise point where these functions attain a maximum or a minimum. In the risk-neutral case, the maximum and minimum values coincided with the point of maximum uncertainty, at $\pi_t = 0.5$. However, when the investor is risk-averse, the functions reach the extreme value to the right of that point, around $\pi_t = 0.60$ in our calibration. This rightward shift is also observed in the single asset case considered in Veronesi (1999) and increases with the level of risk aversion. Thus, the second moments of returns should be regarded as monotonic functions (either increasing or decreasing) in a "risk-adjusted" uncertainty, that attains a maximum to the right of the "risk-neutral" uncertainty, $\pi_t = 0.5$. Because of this rightward shift is relatively small and for ease of exposition, in what follows we will consider the risk-averse functions of second moments of returns as monotonically increasing or decreasing in uncertainty.

Finally, we observe that the price of market risk is positive and increasing with uncertainty. It reaches a maximum at $\pi_t = 0.60$ of about 8% and a minimum of 3% at $\pi_t = 1$. For the unconditional mean of the state probability, $\pi_s = 0.83$, the price of market risk is close to 6.5% and very close to its historical sample mean[12].

We now turn to the question of which of these two terms, λ_{mt} or $\beta_{im,t}$, is the most relevant one to the dynamics of expected returns. An inspection of Figure 1 shows that for the assets at the two extremes of the spectrum, A1 and A5, both the conditional betas and prices of risk are equally important. Consider for instance a shift of the economy from $\pi_1 = 0.90$ to $\pi_2 = 0.50$. The market price of risk, λ_{mt}, increases from 4.93% to 7.81%, a change of 158%. At the same

[12]The price of market risk is often estimated by the sample mean of the market portfolio excess return over a long period.

time, the A5 conditional beta also change significantly, from 1.327 to 1.868, an increase of 141%. The change in the A1 market beta is also important but in the opposite direction, it decreases from 0.72 to 0.42. On the other hand, most of the dynamics of A2, A3 and A4 are dictated by the price of market risk, as the variation of their market betas is relatively smaller than the variation in the price of risk. This illustrates the importance of a correct specification of market betas. For some assets, it is indispensable to allow market betas to vary. For other assets, this may not be the case and the incorrect specification of a time varying beta could even do more harm than good, as was pointed out by Ghysels (1998).

So far we have considered the theoretical implications of the model at all possible values of π_t. However, the investor's belief about the state of the economy is a random variable, and does not take all values with the same probability. In the next section, we will address this issue by generating random draws of this variable and computing ex-post sample moments.

3.2 Ex Post Moments

In the previous section we analyzed the ex-ante formulas of returns and betas for all possible values of the state variable, π_t, and for fixed cash-flows, D_t. In this section, we generate time series of cash-flows for all assets and compute the resulting optimal investor belief, π_t. The simulated data will allow us to assess which are the relevant values of the ex-ante formulas and to compute ex-post moments.

For the simulation exercise, we generate a sample of six years of daily data. We avoid selecting a longer time period because of our assumption that cash-flows follow an arithmetic Brownian motion. As the cash-flow level moves away from its starting value, one, the drift of the stochastic process becomes a worse approximation of percentage changes, the scale used for the calibration. On the other hand, we cannot select a smaller sample because of the duration of recessions and expansions implied by the transition matrix parameters. The six year time frame permits an expansionary period of five years and a recession of one year that coincide with the average durations and proportions of good and bad months, 83.3% and 16.7% respectively.

We will consider three cases with different economic conditions. In case A we have a bull market, the economy is in the good state the whole period. In case B we have normal economic conditions, there is one expansionary period that lasts five years and one recession that lasts one year. Finally, in case C we have a bear market, two out of the six years are recessionary. For each case, 500 histories are generated.

In the first column of Table V we show the averages and standard deviations across histories of the ex-ante and the ex-post excess returns. As expected, cases A and C result in biases in excess returns. In case A, the sample is favorable to the value portfolio. The annual average realized return of asset A5 is around 9% and the average annual expected returns is 4%. On the other hand, in case C, the value portfolio underperforms, with an annual average realized return of 4.8% against an average expected return of 7.6%. The opposite holds for the growth portfolio. It underperforms in case A but overperforms in case C. When the economic conditions are the ones implied by the transition parameters, as in case B, expected returns generally match realized returns. The values do not coincide, however, because of the approximation imposed by the assumption that cash-flows follow arithmetic Brownian motions, a point also observed byVeronesi (2004).

Table V shows that these pricing biases are also captured by the unconditional CAPM pricing error, the alphas. We observe that in bull markets, as a result of this overperformance in terms of excess returns, the value portfolio has a substantial positive alpha across histories. Additionally, growth portfolios display negative alphas, a result of the lower profitability across histories. The reverse result holds for the bear market. For case B, the alphas are close to zero. As we observed, a value premium result in the unconditional CAPM when an unfavorable sample is selected. Ang and Chen (2007) have also pointed to this potential problem in empirical applications for U.S. stock data. The authors show that the alphas in the unconditional CAPM model are insignificant if the sample period considered is long enough. They observe that most studies on the value premium consider only the post 1963 sample and this omission turns out to be crucial.

Since our goal is not to address the value premium puzzle but to derive theoretical implications about conditional betas, the next section will only consider the unbiased histories generated under case B.

3.3 Conditional Moments

We have seen that conditional market betas can vary substantially with the economic conditions, particularly the betas of assets A1 and A5. In order to see to which extent this time variation is relevant, we need to understand how news are incorporated into uncertainty and betas. In this section, we verify this by means of an econometric model fitted to the simulated data. The framework of choice for this task is the family of GARCH models. Multivariate GARCH models capture time variation in covariances by a deterministic auto-regressive function of past return shocks.

We start our analysis of conditional moments with the conditional variance. We fit an univariate GARCH model to the simulated sample excess returns. In order to allow for asymmetric responses we introduce an extra term in the conditional variances. Let $u_{it} = R^e_{it} - E\left[R^e_{it}\right]$ denote the mean-adjusted excess returns. Then, the conditional volatility is specified as

$$\sigma_{it+1} \quad = \quad \kappa + \delta\sigma_{it} + \gamma u_{it} + \gamma_{-} 1_{[u_{it}<0]} u_{it}$$

where $1_{[u_{it}<0]}$ is an indicator. In Table VI we show the averages, standard deviations and quantiles of the estimated parameters across the 500 histories for each asset. The following remarks are in order: first, we observe that conditional volatilities are very persistent, the δ's are high and close to one, a well known stylized fact about stock returns.

Second, negative shocks to returns are more important to future volatility than positive shocks. The coefficients on the negative shocks term, γ_{-}, are much higher than the coefficients on the symmetric shock term, γ. Note that for all assets, including the anti-cyclical one, A1, it is the case that negative shocks are followed by higher volatility than positive shocks. This is another well known stylized fact and has been referred to simply as volatility asymmetry.

Finally, assets with cash-flows that are more exposed to shifts have stronger asymmetries. The average leverage coefficient across the 500 histories for asset $A5$ is 0.089 while for asset $A1$ it is only 0.016. This was expected, as the A5 pricing equation is the most responsive to changes in uncertainty. Furthermore, shocks to A5 are also the most informative ones, as it has the highest signal to noise ratio among the assets. These two characteristics of A5 magnify the asymmetric response of uncertainty to news, which drives the results.

We now turn our attention to the covariance dynamics of asset and market returns. The theoretical expressions show that the covariance of cyclical assets increases with uncertainty while it decreases with uncertainty for anti-cyclical assets. Furthermore, because uncertainty responds asymmetrically to news, we also expect positive and negative shocks to returns to have distinct impacts on the conditional covariance.

In order to assess whether such dynamics are present in the simulated data, we estimate conditional covariances using an asymmetric multivariate GARCH model. More precisely, we follow the BEKK specification of Engle and Kroner (1995) and introduce asymmetric terms as in Hafner and Herwartz (1998). For computational convenience, we will focus on bivariate models of asset excess returns and market excess returns.

Denote demeaned excess returns by $u_{it} = R^{e}_{it} - E\left[R^{e}_{it}\right]$ for $i = 1, ..., n,m$, and $u_t = [u_{it}, u_{mt}]'$. Let $y_t = \sigma\left(u_\tau, \tau < t\right)$ be the conditional information set generated by past observations. The conditional joint distribution is assumed to follow $u_t|y_t \sim \left(0, \Sigma_{t|t-1}\right)$ with conditional covariance given by

$$\begin{aligned} \Sigma_{t|t-1} &= C'C + A'\Sigma_{t-1|t-2}A + B'u_{t-1}u'_{t-1}B \\ &\quad +1_{[u_{it-1}<0]}D'_1 u_{t-1}u'_{t-1}D_1 + 1_{[u_{mt-1}<0]}D'_2 u_{t-1}u'_{t-1}D_2 \end{aligned} \tag{11}$$

where A, B, D_1 and D_2 are 2×2 matrices and C an upper triangular matrix. Matrices D_1 and D_2 are new to the BEKK formulation and add the necessary flexibility to capture asymmetric responses of the covariance matrix to shocks to the portfolio and the market. Assuming that the joint distribution is normal, parameters are estimated by maximizing the log-likelihood function. The asymptotic distribution of the estimates is generally unknown and the results can only provide a description of the data set (Herwartz and Lutkepohl, 2000).

The estimated parameters of equation (11) for a simulated history are shown in Table VII. We note that the introduction of asymmetries affects all assets. The likelihood ratio test indicates that the difference in the likelihoods of the symmetric BEKK and the asymmetric BEKK with matrices D_1 and D_2, is statistically significant after controlling for the inclusion of the extra parameters. Except for asset A5, asymmetry at the asset level captured by the matrix D_1 is less important than market asymmetries. As shown above, this asset is very informative about the state of the nature and so shocks to this asset's past returns are more likely to be related to changes in profitability level.

We can also see that asymmetries are relevant by noting that the parameters on matrices D_1 and D_2 are significant and relatively large. In order to have a more precise notion about the dynamics implied by these parameters, we compute the impulse response function (IRF) for covariances. To do that, we rewrite BEKK parameters in vector form:

$$vec\left(\Sigma_t\right) = \overline{C} + \overline{A}vec\left(\Sigma_{t-1}\right) + \overline{B}vec\left(u_t u_t'\right) + \overline{D_1}1_{[u_{it-1}<0]}vec\left(u_t u_t'\right) + \overline{D_2}1_{[u_{mt-1}<0]}vec\left(u_t u_t'\right)$$

where $\overline{C} = (C \otimes C)' vec\left(I_2\right)$, $\overline{A} = (A \otimes A)'$,$\overline{B} = (B \otimes B)'$, $\overline{D_1} = (D_1 \otimes D_1)'$ and $\overline{D_2} = (D_2 \otimes D_2)'$. vec is an operator that stacks the columns of a matrix and I_2 is a (2×2) identity matrix. In our case, $vec\left(\Sigma_t\right)$ will then be a (4×1) vector, with the first element being the asset return conditional variance, the second the conditional covariance of the asset return with the market return and the last term the market return conditional variance. Hafner and Herwartz (1998) define the impulse response function as $V_t\left(\xi_0\right) = E\left[vec\left(\Sigma_t\right)|\xi_0, \Sigma_0\right]$, which can be computed by starting the above auto-regression at the long run value of the covariance matrix, Σ, and perturbing it with standardized shocks, ξ_0. At $t = 1$ we have

$$V_1\left(\xi_0\right) = \overline{C} + \left(\overline{B} + 1_{[\xi_{0,i}<0]}\overline{D_1} + 1_{[\xi_{0,m}<0]}\overline{D_2}\right) vec\left(\Sigma^{1/2}\xi_0\xi_0'\Sigma^{1/2}\right) + \overline{A}vec\left(\Sigma\right)$$

and for $t \geq 2$

$$V_t\left(\xi_0\right) = \overline{C} + \left(\overline{A} + \overline{B} + \frac{\overline{D_1}}{2} + \frac{\overline{D_2}}{2}\right) V_{t-1}\left(\xi_0\right)$$

Figure 2 shows the impulse response functions of the covariance of assets A1 and A5 returns with the market return. In the first panel we show the responses of the covariance to shocks on the asset returns for a period of 22 days. We note that negative shocks to the anti-cyclical asset A1 do not change its covariance by much. Large positive news, however, tends to decrease the covariance but only for a short period of time. On the other hand, negative news tends to increase substantially the covariance of A5 with the market and for a long period of time. Note that positive news does not change the covariance of A5 and market returns by much.

Consider now the second panel of Figure 2. There we see how shocks at the market level change conditional covariances. First, we observe for A5 a similar pattern to that which arises when the shocks are originated by the asset itself. The IRF for A1 is different from the previous plot. Indeed, shocks to the market are more likely to be originated from other, more informative assets, which are also cyclical assets. As a result, negative shocks to these assets indicate, on average, that uncertainty has increase and, as we have seen from our theoretical equations, the covariance of anti-cyclical assets declines with uncertainty.

Finally, we investigate the dynamics of the conditional market betas. The betas can be computed from the parameters of the bivariate asymmetric BEKK models fitted above. In order to illustrate how news are incorporated into the betas, we compute a beta IRF. Following Hafner and Herwartz (1998), we define the beta IRF as the ratio of the covariance IRF and market variance IRF:

$$\beta_{it}(\xi_0) = \frac{V_{im,t}(\xi_0)}{V_{m,t}(\xi_0)}$$

where $V_{im,t}(\xi_0)$ and $V_{m,t}(\xi_0)$ are the second and fourth elements of the vector $V_t(\xi_0)$. In the first panel, we induce shocks from -2 to 2 standard deviations to asset returns and track the response of market betas for 22 days. We observe that negative shocks result in opposite responses of the betas. The conditional beta of the anti-cyclical asset A1 declines with negative shocks. Note that the decrease in the market beta of A1 was a result of the increase in the market variance, given that, as shown above, the covariance of the asset with market returns did not change with negative news. On the other hand, negative shocks to asset A5 increase its conditional betas. Both changes are persistent and last well over the 22 days, particularly after shocks above one standard deviation. Finally, positive shocks to returns reduce both conditional betas and again illustrate the asymmetric response of betas to news. For A1, positive and negative news reduces betas, but to a lesser degree if shocks are positive. For our cyclical asset, A5, negative news increases betas whereas positive news decreases it. Also, the change induced by positive news is short lived if compared to the persistence of negative shocks.

In the third panel of Figure 3 we show the betas IRF for a combination of market and asset return shocks after 22 days. First we observe that the most important changes to conditional betas occur when both shocks to the market and to the asset are negative. On this quadrant, $(-,-)$, A1 conditional beta decline and A5 betas rise after 22 days. In the opposite quadrant, $(+,+)$, the resulting changes are also the opposite. The market betas seem to respond the least when both asset and market shocks are positive. For asset A5, negative shocks at the asset level are more relevant to the increase in beta. On the other hand, for asset A1, negative shocks at the market level are more important to the decline in betas.

4 Conclusion

In this paper we showed how different cash-flow structures combined with uncertainty about the state of the economy can generate enough cross-sectional variation in asset returns to match the sample moments of the book-to-market portfolios. The key elements behind the cross-sectional variation are the opposite features of the value and growth cash-flow structures. In particular, the value portfolio cash-flows are very susceptible to changes in the overall economic conditions while the growth portfolio cash-flows are the least responsive to fluctuation from business cycles.

The model allows for expected returns to be decomposed into asset exposure to two common sources of risk: the market portfolio and the conditional probability of the good state, π_t. The calibration showed that the market portfolio is by far the most important risk factor of the two. Thus, a well specified conditional CAPM can provide an accurate description of expected returns.

The factor decomposition of expected returns uncovers rich time-series dynamics of conditional betas and prices of risk. First, we observed that time variation in both market betas and the price of risk are important in determining the value and growth portfolios expected returns. For the other intermediate portfolios, most of the time variation in expected returns originated in the price of risk. This indicates that the correct specification of conditional market betas may be important not only to potentially address the value premium puzzle, by permitting time-varying betas to capture substantial changes in risk, but also to avoid overfitting betas when time-variation is less important.

Another feature of the conditional market betas implied by the model is their asymmetric response to positive and negative news. This asymmetric response to news is a consequence of two results. First, uncertainty about the state of the economy increases with negative news and decreases with positive news when markets are good (that is, when the economy is more likely to be in the good state, $\pi_t > 0.5$). However, if markets are bad, i.e. $\pi_t < 0.5$, the opposite is true. Since the economy is more frequently in the good state, from the very nature of business cycles, negative news is more frequently followed by an increase in uncertainty than positive news. Second, conditional betas are monotonic functions of uncertainty.

Combining the two results we have that conditional market betas are more responsive to negative news than to positive news. More specifically, if the asset's profitability is very susceptible to changes in the economy, i.e. a larger $\Delta\theta_i$ than the market average, its conditional beta will increase in response to negative news. On the other hand, if the asset's profitability is less susceptible than the average portfolio, then its conditional beta will decrease in response to negative news.

Similar asymmetric responses are also observed in the variances and covariances of returns. Conditional variances will increase after negative news irrespective of the asset cash-flow structure. This is a well known stylized fact about stock returns and has been attributed to a leverage effect (Black (1976)) and to a volatility feedback effect (Campbell and Hentschel (1992)). Conditional covariances will increase after negative news if the asset is cyclical but decrease if the asset is anti-cyclical. This excess comovement of stock returns after downturns has also been observed in real data, for instance on international stock markets (Ribeiro and Veronesi (2002)).

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5 Appendix

In this appendix we solve the investor problem and derive the asset pricing equations. This model was also derived by Ribeiro and Veronesi (2002). The problem of the representative investor has two parts. In the first part, the investor optimally infer the conditional means of the cash-flow processes. In the second part, the investor maximize the utility function subject to the intertemporal budget constraint, with choice variables consumption, $\{c_t\}$, and demand for assets, $\{X_t\}$, $X_t = [x_{1t} \ldots x_{nt}]'$. The maximization is solved using the Bellman-Hamilton-Jacobi equation with two state variables, wealth, W_t, and the belief π_t.

Recall the assumptions about the available assets in this economy. There are n risky assets in this economy that pay a continuous stream of cash-flows: $dD_t = \theta_t dt + \Phi d\xi_t$. The random vector θ_t, is not observed by the investor, who only knows the values it can take, $[\theta_G, \theta_B]$, and that it follows a 2 state Markov process with the following infinitesimal transition matrix:

$$M = \begin{bmatrix} -\lambda & \lambda \\ \mu & -\mu \end{bmatrix}$$

with $\lambda = Prob\,(\theta_{t+dt} = \theta_B | \theta_t = \theta_G)$ and $\mu = Prob\,(\theta_{t+dt} = \theta_G | \theta_t = \theta_B)$. The lemma (1) shows that the investor's optimal beliefs about the state of the economy conditional on $\mathcal{F}_t = \sigma\,(D_\tau, \tau < t)$ can be represented by the following stochastic differential equation:

$$d\pi_t = (\lambda + \mu)(\pi_s - \pi_t)\,dt + \pi_t\,(1 - \pi_t)\,\Delta\theta'\Phi'^{-1}dv_t$$

Under this incomplete information set, $\mathcal{F}_t$, cash-flows can be written as $dD_t = \alpha_{Dt}dt + \Phi d\mathbf{v}_t$, where $\alpha_{Dt} = [\alpha_{1D,t}, \ldots, \alpha_{nD,t}]'$ and $\alpha_{iD,t} \equiv \theta_{iG}\pi_t + \theta_{iB}(1 - \pi_t)$.

With the optimal beliefs already defined, we now turn to the utility maximization problem. First, since the risk free is inelastically supplied, the budget constraint is given by:

$$\begin{aligned} dW_t &= X_t'\,(dP_t + D_t dt) + (W_t - X_t'P_t)\,rdt - c_t dt \\ &= X_t'\,(dP_t + D_t dt - rP_t dt) + (W_t r - c_t)\,dt \end{aligned}$$

where $X_t = [x_{1t} \ldots x_{nt}]'$ are the demand for asset shares and $P_t = [P_{1t} \ldots P_{nt}]$ the asset prices.

As in Veronesi (1999), first conjecture a functional form for prices and then find are parameters that solve the problem. The conjectured form is linear in D_t but possibly non-linear in π_t, through the function S_i:

$$P_{it} = p_{i0} + p_{i\pi}\pi_t + p_{iD}D_{it} + p_{i1} + S_i(\pi_t)$$

and by Ito's lemma we obtain:

$$dP_{it} = \alpha_{ip}dt + \sigma_{ip}dv_t \quad i = 1, ..., n$$

$$\begin{aligned}
\alpha_{ip} &= \left(p_{i\pi} + S_i'(\pi_t)\right)\alpha_\pi + p_{iD}m_{it} + \frac{1}{2}S_i''(\pi_t)\,h(\pi_t)^2 H \\
\sigma_{ip} &= h(\pi_t)\left(p_{i\pi} + S_i'(\pi_t)\right)\Delta\theta'\Phi'^{-1} + p_{iD}\sigma_i
\end{aligned}$$

with the simplifying notation $\alpha_\pi \equiv (\lambda + \mu)(\pi_s - \pi_t)$, $h(\pi_t) \equiv \pi_t(1 - \pi_t)$ and $H \equiv \Delta\theta'\Sigma^{-1}\Delta\theta$. Furthermore, denote the vector of price changes by: $dP_t = \alpha_p dt + \Phi_p dv_t$, where $\alpha_p = [\alpha_{1p}, ..., \alpha_{np}]'$ and Φ_p is a $n \times n$ matrix that stacks the row vectors σ_{ip}, and $\Sigma_p = \Phi_p\Phi_p'$. Substitute the conjecture prices into the budget constraint to obtain:

$$dW_t = \left[X_t'(\alpha_p + D_t - rP_t) + W_t r - c_t\right]dt + X_t'\Phi_p dv_t$$

Risk Neutral Prices

The parameters $p_0 = [p_{10}, ..., p_{n0}]'$, $p_\pi = [p_{1\pi}, ..., p_{n\pi}]'$ and $p_D = [p_{1D}, ..., p_{nD}]'$ are found by solving for risk neutral prices, $P_{i,t}^{RN}$:

$$P_{i,t}^{RN} \equiv E_t\left[\int_0^\infty e^{-rs}D_{i,t+s}ds\right] = \int_0^\infty e^{-rs}E_t\left[D_{i,t+s}\right]ds$$

where the equality follows from Foubini's theorem. Since, $D_{i,t+s} = D_{it} + \int_0^s \alpha_{iD,t+\tau}d\tau + \sigma_i(v_{t+s} - v_t)$, the only conditional expectation that matters is $\int_0^s E_t\left[\alpha_{iD,t+\tau}\right]d\tau$. For this, we need the eigendecomposition of the infinitesimal transition matrix M to compute the transition matrix over τ periods. The eigenvalues of M are 0 and $-(\lambda + \mu)$ with corresponding eigenvectors $[1\,1]'$ and $\left[-1\,\frac{\mu}{\lambda}\right]'$. The transition matrix over τ is:

$$\begin{aligned} P(\tau) &= \begin{bmatrix} 1 & -1 \\ 1 & \frac{\mu}{\lambda} \end{bmatrix} \begin{bmatrix} e^{0\tau} & 0 \\ 0 & e^{-(\lambda+\mu)\tau} \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & \frac{\mu}{\lambda} \end{bmatrix}^{-1} \\ &= \frac{1}{(\lambda+\mu)} \begin{bmatrix} \mu+\lambda e^{-(\lambda+\mu)\tau} & \lambda-\lambda e^{-(\lambda+\mu)\tau} \\ \mu-\mu e^{-(\lambda+\mu)\tau} & \lambda+\mu e^{-(\lambda+\mu)\tau} \end{bmatrix} \end{aligned}$$

and so $E_t\left[\alpha_{iD,t+\tau}\right] = \left[\pi_t\, 1-\pi_t\right] P(\tau) \left[\theta_{iG}\,\theta_{iB}\right]' = \theta_{is} + \Delta\theta_i \left(\pi_t - \pi_s\right) e^{-(\lambda+\mu)\tau}$, where $\pi_s = \mu/\left(\mu+\lambda\right)$ and $\theta_{is} = \theta_{iG}\pi_s + \theta_{iB}\left(1-\pi_s\right)$. Now, the conditional expectation of cash-flows are:

$$\begin{aligned} E_t\left[D_{i,t+u}\right] &= D_{it} + \int_0^s \left[\theta_{is} + \Delta\theta_i \left(\pi_t - \pi_s\right) e^{-(\lambda+\mu)\tau}\right] d\tau \\ &= D_{it} + \theta_{is} s + \frac{\Delta\theta_i \left(\pi_s - \pi_t\right)}{\lambda+\mu} \left[e^{-(\lambda+\mu)s} - 1\right] \end{aligned}$$

and finally, risk neutral prices are found by continuously discounting expected dividends at the risk free rate:

$$\begin{aligned} P_{i,t}^{RN} &= \int_0^\infty e^{-rs} \left[D_{it} + \theta_{is} s + \frac{\Delta\theta_i \left(\pi_s - \pi_t\right)}{\lambda+\mu} \left[e^{-(\lambda+\mu)s} - 1\right]\right] ds \\ &= \frac{D_{it}}{r} + \frac{\theta_{is}}{r^2} - \frac{\Delta\theta_i \left(\pi_s - \pi_t\right)}{\lambda+\mu} \left[\frac{1}{r} - \frac{1}{(\lambda+\mu+r)}\right] \\ &= p_{i0} + p_{i\pi}\pi_t + p_{iD} D_{it} \end{aligned}$$

where

$$\begin{aligned} p_{i0} &= \frac{\theta_{iB}}{r^2} + \frac{\Delta\theta_i \mu}{r^2\left(\lambda+\mu+r\right)} \\ p_{i\pi} &= \frac{\Delta\theta_i}{r\left(\lambda+\mu+r\right)} \\ p_{iD} &= \frac{1}{r} \end{aligned}$$

Risk Averse Prices

To solve for the risk aversion case, we need to solve the investor problem:

$$\begin{aligned} J(W_t, \pi_t, t) &= \max_{\{c_t, X_t\}} E\left[\int_0^\infty U(c_s, s)\, ds\right] \\ \text{s.t. } dW_t &= \left[X_t'(\alpha_p + D_t - rP_t) + W_t r - c_t\right] dt + X_t'\Phi_p dv_t \text{ (Budget Constraint)} \\ X_t &= [\omega_1 \ldots \omega_n]' \equiv \omega \qquad \text{(Market Clearing)} \end{aligned}$$

This problem is solved using the Hamilton-Bellman-Jacobi equation:

$$\begin{aligned} 0 &= \max_{c_t, X_t}\left[U(c_t, t) + J_t + J_W \frac{E_t[dW_t]}{dt} + J_\pi \frac{E_t[d\pi_t]}{dt} + \frac{1}{2} J_{WW} \frac{E_t[dW_t^2]}{dt}\right. \\ &\quad \left. + \frac{1}{2} J_{\pi\pi} \frac{E_t[d\pi_t^2]}{dt} + J_{W\pi} \frac{E_t[dW_t d\pi_t]}{dt}\right] \end{aligned}$$

where we have that:

$$\begin{aligned} E_t[dW_t] &= \left[X_t'(\alpha_p + D_t - rP_t) + W_t r - c_t\right] dt \\ E_t\left[dW_t^2\right] &= X_t'\Sigma_p X_t dt \\ E_t[d\pi_t] &= \alpha_\pi dt \\ E_t\left[d\pi_t^2\right] &= h(\pi_t)^2 H dt \\ E_t[dW_t d\pi_t] &= X'\Phi_p \sigma_\pi' dt \end{aligned}$$

A solution to problem, c_t^* and X_t^*, satisfy the first order conditions:

$$\begin{aligned} 0 &= U_c(c_t^*, t) - J_W \\ 0 &= J_W(\alpha_p + D_t - rP_t) + J_{WW}\Sigma_p X_t + J_{W\pi}\Phi_p \sigma_\pi' \end{aligned} \tag{12}$$

In order to advance, we have to conjecture a functional form for the value function. Following the univariate model of Veronesi (1999), we set $J(W_t, \pi_t, t) = -exp(-\rho t - r\gamma W_t - g(\pi_t) - \beta)$ where $g(\pi_t)$ is a function to be determined and β a constant to be defined. Substituting the partial derivatives of the conjecture value function and of the utility function, $U(c_t, t) = -exp(-\rho t - \gamma c_t)$, on the first order conditions we obtain:

$$c_t^* = \frac{1}{\gamma}(r\gamma W_t + g(\pi_t) + \beta - ln(r)) \tag{13}$$

$$X_t^* = \frac{1}{r\gamma}\Sigma_p^{-1}(\alpha_p + D_t - rP_t) - \frac{g'(\pi_t)}{r\gamma}\Sigma_p^{-1}\Phi_p\sigma_\pi' \tag{14}$$

We have an extra equation that will help to identify the problem. Evaluate the HJB equation at the maximum and set it equal to zero:

$$\begin{aligned} 0 &= -exp\left(-\rho t-\gamma c_t^*\right)-\rho J-r\gamma J\left[X_t^{*\prime}\left(\alpha_p+D_t-rP_t\right)+W_tr-c_t\right]-g'\left(\pi_t\right)J\alpha_\pi+ \\ &\quad \frac{1}{2}\left(r\gamma\right)^2 JX_t^{*\prime}\Sigma_p X_t^*+\frac{1}{2}\left(-g''\left(\pi_t\right)+g\left(\pi_t\right)^2\right)Jh\left(\pi_t\right)^2 H+r\gamma g'\left(\pi_t\right)JX^{*\prime}\Phi_p\sigma_\pi' \end{aligned} \tag{15}$$

Before we proceed, we can simplify the expression for $\alpha_p+D_t-rP_t$ by substituting the parameters that were obtained for the risk neutral price, p_{i0}, $p_{i\pi}$ and p_{iD}.

$$\begin{aligned} \alpha_{ip}+D_{it}-rP_{it} &= \left(p_{i\pi}+S_i'\left(\pi_t\right)\right)\alpha_\pi+p_{iD}D_{it}+\frac{1}{2}S_i''\left(\pi_t\right)h\left(\pi_t\right)^2 H+D_{it} \\ &\quad -r\left(p_{i0}+p_{i\pi}\pi_t+p_{iD}D_{it}+p_{i1}+S_i\left(\pi_t\right)\right) \\ &= -rp_{i1}-rS_i\left(\pi_t\right)+S_i'\left(\pi_t\right)\alpha_\pi+\frac{1}{2}S_i''\left(\pi_t\right)Hh\left(\pi_t\right)^2 \end{aligned}$$

Take the above simplification, the expression for c_t^* from the first order condition (13) and the market clearing $X_t^*=\omega$ and substitute them in the equality (15) to get:

$$\begin{aligned} 0 &= r-\rho-r\gamma\left[\omega'\left(-rp_1-rS\left(\pi_t\right)+S'\left(\pi_t\right)\alpha_\pi+\frac{1}{2}S''\left(\pi_t\right)Hh\left(\pi_t\right)^2\right)-\frac{g\left(\pi_t\right)}{\gamma}-\frac{\beta}{\gamma}+\frac{ln\left(r\right)}{\gamma}\right]+ \\ &\quad r\gamma g'\left(\pi_t\right)\left[\omega'\left(h\left(\pi_t\right)\left(p_\pi+S'\left(\pi_t\right)\right)\Delta\theta'\Phi'^{-1}+p_D\Phi\right)\left(\Phi^{-1}\Delta\theta h\left(\pi_t\right)\right)\right]+ \\ &\quad \frac{1}{2}\left(r\gamma\right)^2\left[\omega'\left(h\left(\pi_t\right)\left(p_\pi+S'\left(\pi_t\right)\right)\Delta\theta'\Phi'^{-1}+p_D\Phi\right)\left(h\left(\pi_t\right)\left(p_\pi+S'\left(\pi_t\right)\right)\Delta\theta'\Phi'^{-1}+p_D\Phi\right)'\omega\right]+ \\ &\quad \frac{1}{2}\left(-g''\left(\pi_t\right)+g\left(\pi_t\right)^2\right)h\left(\pi_t\right)^2 H-g'\left(\pi_t\right)\alpha_\pi \end{aligned}$$

where we have used the notation $p_{m1}\equiv\omega'p_1$, $p_{m\pi}\equiv\omega'p_\pi$, $\Delta\theta_m\equiv\omega'\Delta\theta$. Also, let $\sigma_\omega^2\equiv\omega'\Sigma\omega$ and $\sigma_{i\omega}\equiv e_i'\Sigma\omega$ denote the variance of the market portfolio cash-flow and covariance of the market and asset i cash-flows, where e_i is a vector with zeros and one the ith position. Note that in the above equation the $S=\left[S_1,...,S_n\right]'$ vector of functions is multiplied by the market clearing vector ω and so the equality only depends on $S_m\equiv\omega'S$. After some simplifications and substituting $f\left(\pi_t\right)=g\left(\pi_t\right)+r\gamma S_m\left(\pi_t\right)$ we get the following nonlinear differential equation for $f\left(\pi_t\right)$:

$$0 = -f''\left(\pi_t\right)Q_3\left(\pi_t\right)+\left(f'\left(\pi_t\right)\right)^2 Q_3\left(\pi_t\right)+f'\left(\pi_t\right)Q_2\left(\pi_t\right)+f'\left(\pi_t\right)r+Q_0\left(\pi_t\right)$$

where

$$\begin{aligned}
Q_3(\pi_t) &= \frac{1}{2}h^2(\pi_t)H \\
Q_2(\pi_t) &= \gamma h(\pi_t)\Delta\theta_m + r\gamma h(\pi_t)^2 H\frac{\Delta\theta_m}{r(r+\mu+\lambda)} - \alpha_\pi \\
Q_0(\pi_t) &= \frac{1}{2}H\left(\frac{r\gamma h(\pi_t)\Delta\theta_m}{r(r+\mu+\lambda)}\right)^2 + r\gamma^2 h(\pi_t)\frac{\Delta\theta_m^2}{r(r+\mu+\lambda)}
\end{aligned}$$

where some extra terms in $Q_0(\pi_t)$ were eliminated after choosing appropriately the parameters β and p_1:

$$\begin{aligned}
\beta &= \frac{\rho}{r} + ln(r) + \frac{\gamma^2}{2r}\sigma_\omega^2 - 1 \\
p_{i1} &= -\frac{\gamma}{r^2}e_i'\Sigma\omega
\end{aligned}$$

which in vector notation is $p_1 = -\frac{\gamma}{r^2}\Sigma\omega$. This non-linear differential equation f is the same one in Veronesi (1999) and it was shown there it has a unique solution on the relevant domain, $\pi_t \in (0,1)$.

Next, we have to find the individual discounting functions, S_i. In order to do so, we use the first order conditions (14) for asset demands, X_t^*, and the market clearing condition $X_t^* = \omega$ to get the equalities:

$$\begin{aligned}
r\gamma\Sigma_p X_t^* &= (\alpha_p + D_t - rP_t) - g'(\pi_t)\Phi_p\sigma_\pi' \\
r\gamma\Sigma_p\omega &= \left(-r - \frac{\gamma}{r^2}\Sigma\omega - rS(\pi_t) + S'(\pi_t)\alpha_\pi + \frac{1}{2}S''(\pi_t)Hh(\pi_t)^2\right) \\
&\quad - (f'(\pi_t) - r\gamma S_m'(\pi_t))\Phi_p\sigma_\pi'
\end{aligned}$$

If we left multiply both sides of the above expression by e_i, $i = 1,...,n$, we get individual expression for S_i:

$$\begin{aligned}
r\gamma\sigma_{im,p} &= \left(-r - \frac{\gamma}{r^2}\sigma_{im} - rS_i(\pi_t) + S_i'(\pi_t)\alpha_\pi + \frac{1}{2}S_i''(\pi_t)Hh(\pi_t)^2\right) \\
&\quad - (f'(\pi_t) - r\gamma S_m'(\pi_t))\sigma_{ip}\sigma_\pi'
\end{aligned}$$

that if we substitute for $\sigma_{im,p}$, σ_{ip}, σ_{im} and σ_π and rearrange the terms, we observe that the market discount function $S_m'(\pi_t)$ cancels out and a differential equations for each asset $i = 1,...n$ is obtained:

$$0 = S_i''(\pi_t)P_3(\pi_t) + S_i'(\pi_t)P_2(\pi_t) + S_i'(\pi_t)r + P_{i0}(\pi_t)$$

where

$$
\begin{aligned}
P_3(\pi_t) &= -\frac{1}{2}h^2(\pi_t)\,H \\
P_2(\pi_t) &= \gamma h(\pi_t)\,\Delta\theta_m + r\gamma h(\pi_t)^2\,H\frac{\Delta\theta_m}{r(r+\mu+\lambda)} + h(\pi_t)^2\,Hf'(\pi_t) - \alpha_\pi \\
P_{i0}(\pi_t) &= \gamma h(\pi_t)\frac{\Delta\theta_i\Delta\theta_m}{r(r+\mu+\lambda)}\left(2+\frac{h(\pi_t)\,H}{(r+\mu+\lambda)}\right) + f'(\pi_t)\,\Delta\theta_i h(\pi_t)\left(\frac{h(\pi_t)\,H}{r(r+\mu+\lambda)}+\frac{1}{r}\right)
\end{aligned}
$$

This differential equation is essentially the same one in Veronesi (1999). We refer the reader to that paper for a proof that a solution exists on relevant domain, $\pi_t \in (0,1)$. Note that only the last term, $P_{i0}(\pi_t)$, varies across assets. Furthermore, we observe that if two assets have the same $\Delta\theta_i$ they will share the same discounting function.

Graphs and Tables

TABLE I

Descriptive Statistics: 5 Book to Market Sorted Portfolio (1950 - 2010)

Descriptive statistics computed over a monthly sample of annualized log-excess returns, log-dividend growth and dividend yield. Risk free rate is the return on the 1-month t-bill. Market is the value weighted return on the CRSP market portfolio.

		Mean	Stdev	Skew	Kurt	Min	10th	50th	90th	Max
Log Returns	**Low 20**	0.084	0.180	-0.651	3.510	-0.635	-0.174	0.106	0.294	0.567
	Qnt 2	0.097	0.163	-0.615	3.695	-0.517	-0.116	0.115	0.294	0.488
	Qnt 3	0.108	0.157	-0.991	5.453	-0.666	-0.094	0.128	0.286	0.509
	Qnt 4	0.114	0.167	-1.122	5.958	-0.747	-0.090	0.141	0.298	0.483
	High 20	0.129	0.182	-0.730	3.996	-0.623	-0.117	0.160	0.321	0.594
Log Excess Returns	**Low 20**	0.035	0.181	-0.692	3.487	-0.713	-0.226	0.055	0.244	0.485
	Qnt 2	0.048	0.164	-0.624	3.548	-0.595	-0.175	0.068	0.244	0.406
	Qnt 3	0.059	0.159	-0.909	4.841	-0.679	-0.154	0.084	0.234	0.469
	Qnt 4	0.065	0.166	-0.986	5.359	-0.760	-0.138	0.092	0.247	0.482
	High 20	0.080	0.182	-0.564	3.786	-0.636	-0.164	0.110	0.265	0.593
Log Dividend Growth	**Low 20**	0.047	0.121	0.006	6.382	-0.402	-0.083	0.045	0.172	0.535
	Qnt 2	0.046	0.115	-0.502	5.697	-0.467	-0.087	0.057	0.158	0.451
	Qnt 3	0.055	0.107	-0.303	5.119	-0.425	-0.056	0.052	0.188	0.375
	Qnt 4	0.048	0.130	-2.387	14.972	-0.843	-0.083	0.069	0.167	0.327
	High 20	0.058	0.227	-3.018	17.559	-1.496	-0.151	0.100	0.263	0.407
Dividend Yield	**Low 20**	0.020	0.008	0.383	2.396	0.006	0.011	0.019	0.032	0.040
	Qnt 2	0.030	0.011	0.448	2.520	0.012	0.017	0.031	0.047	0.066
	Qnt 3	0.037	0.014	0.443	2.712	0.013	0.018	0.037	0.057	0.079
	Qnt 4	0.041	0.016	0.147	2.192	0.013	0.019	0.042	0.065	0.083
	High 20	0.040	0.016	0.402	2.840	0.007	0.019	0.038	0.062	0.094
Risk Free		0.049	0.027	0.794	3.836	0.000	0.018	0.048	0.085	0.142
Market	**Excess Ret**	0.044	0.167	-0.838	4.020	-0.612	-0.181	0.079	0.229	0.458
	Div Growth	0.050	0.064	-0.167	5.478	-0.206	-0.021	0.048	0.127	0.263

TABLE II

High and Low Dividend Growth (Sample Moments)

Panel A describes the NBER recession and expansion cycles. Panel B, moments of the log-dividend growth series were computed for the NBER recessionary and expansionary subsamples. In Panel B, we also compute sample averages of the assets' dividend growth for the 16.7 % months with the worst market dividend growth as well as for the 83.3 % months with the best growth.

Panel A : NBER Cycles (1956-2010)

	Sample (months)			Implied Transition Matrix	
	Rec.	Exp.		Exp.	Rec.
Avg. Duration	11.22	62.22	Exp.	0.984	0.016
Proportion	0.17	0.83	Rec.	0.080	0.920

Panel B: Log-Dividend Growth Sample Moments

NBER Series Threshold

		Mean			Std. dev	
	Full	**Rec.**	**Exp.**	**Full**	**Rec.**	**Exp.**
Low 20	0.047	0.027	0.051	0.121	0.074	0.128
Qnt 2	0.046	0.007	0.053	0.111	0.121	0.107
Qnt 3	0.057	0.049	0.058	0.103	0.119	0.100
Qnt 4	0.051	0.056	0.050	0.127	0.064	0.135
High 20	0.059	0.097	0.052	0.223	0.110	0.239
Mkt	0.051	0.037	0.054	0.066	0.052	0.068

Market Threshold

		Mean			Std. dev	
	Full	**Highest**	**Lowest**	**Full**	**Highest**	**Lowest**
Low 20	0.047	0.023	0.052	0.121	0.100	0.124
Qnt 2	0.046	0.004	0.054	0.111	0.144	0.101
Qnt 3	0.057	-0.002	0.068	0.103	0.137	0.092
Qnt 4	0.051	-0.031	0.067	0.127	0.214	0.094
High 20	0.059	-0.142	0.097	0.223	0.344	0.167
Mkt	0.051	-0.045	0.070	0.066	0.047	0.052

TABLE III

High and Low Dividend Growth (Markov Switching)

A multivariate Markov switching model is fitted to the 5 book-to-market portfolios log-dividend growth series on the monthly sample from 1956 to 2010. Model 1 has fixed covariance matrix and a constant transition matrix, all other models allow covariance matrix to be state dependent. Models 2 through 4 allow the transition matrix parameters depend on a constant and a leading indicator variable (CLI). The transition matrix parameters shown in the table are time averages of the estimated values.

		Model 1		Model 2		Model 3		Model 4	
		State 1	State2	State 1	State2	State 1	State2	State 1	State2
Constant	Low 20	0.038	0.059*	0.025*	0.094*	0.032*	0.075*	0.027*	0.072*
	Qnt 2	0.082*	-0.010*	0.040*	0.058*	0.062*	0.013	0.042*	0.050*
	Qnt 3	0.096*	-0.007*	0.055*	0.054*	0.069*	0.028*	0.065*	0.041*
	Qnt 4	0.101*	-0.032*	0.060*	0.022	0.068*	0.010	0.056*	0.038*
	High 20	0.139*	-0.064*	0.108*	-0.054*	0.140*	-0.102*	0.120*	-0.022
Variance	Low 20	0.121*		0.093*	0.156*	0.137*	0.072*	0.087*	0.149*
	Qnt 2	0.105*		0.117*	0.109*	0.063*	0.171*	0.124*	0.101*
	Qnt 3	0.094*		0.079*	0.151*	0.080*	0.141*	0.079*	0.133*
	Qnt 4	0.112*		0.080*	0.198*	0.096*	0.172*	0.079*	0.174*
	High 20	0.206*		0.096*	0.357*	0.101*	0.305*	0.092*	0.309*
Transition Matrix	Const.			-1.99*	-1.56*	-1.99*	-1.65*		
	CLI (t-1)			12.75	-12.55			77.25*	-19.92*
	State 1	0.975*	0.025	0.974	0.026	0.977	0.023	0.498	0.502
	State 2	0.034	0.966*	0.064	0.936	0.050	0.950	0.494	0.506
LogLikelihood		2198.6		2607.8		2646.1		2389.0	
AIB		-4359.2		-5127.5		-5208.2		-4694.0	
BIC		-4243.3		-4859.1		-4952.0		-4437.7	

Significant at 1% (*) and at 5% (**).

TABLE IV

Calibration

Panel A shows the parameters that were used to calibrate the model. Mu and lambda are the parameters of the transition matrix, gamma the risk aversion parameter and r the risk free rate. In Panel B, shows the theoretical values implied by the model for a fixed cash-flow of 1 and at the unconditional mean of beliefs. Panel C presents the empirical sample moments implied by the 5 book-to-market porftolios monthly returns from 1956 to 2010.

Panel A: Calibrated Parameters

Cash-Flows	A1	A2	A3	A4	A5	Market
Good State Drift	0.040	0.059	0.066	0.074	0.100	0.068
Bad State Drift	0.050	0.000	-0.020	-0.040	-0.130	-0.035
Std.	0.160	0.135	0.130	0.130	0.090	
Economy	mu	0.08	risk free	0.045		
	lambda	0.02	gamma	1.10		

Panel B: Theoretical Moments

	A1	A2	A3	A4	A5	Market
Excess Returns	3.49%	4.85%	5.51%	5.95%	8.22%	5.57%
Std Deviation	0.151	0.135	0.141	0.152	0.213	0.104
Dividend Growth	0.042	0.049	0.052	0.055	0.062	0.051
Dividend Yield	0.0415	0.0418	0.0424	0.0421	0.0443	0.0424
Market Beta	0.55	0.84	0.98	1.08	1.59	0.062
Hedging Beta	-0.01	0.07	0.10	0.13	0.29	-0.027
Risk Neutral Beta	0.89	0.97	1.01	1.02	1.09	

Panel C: Empirical Sample Moments

	A1	A2	A3	A4	A5	Market
Excess Returns	3.47%	4.77%	5.86%	6.47%	7.97%	4.42%
Std Deviation	0.181	0.164	0.159	0.166	0.182	0.167
Dividend Growth	0.047	0.046	0.055	0.048	0.058	0.050
Dividend Yield	0.020	0.030	0.037	0.041	0.040	0.030
Market Beta	1.031	0.930	0.873	0.866	0.926	

TABLE V
Time Series Simulations

We simulate 6 years of daily data for 500 histories of the economy in three cases. In Case A there is no recession. In case B, there is one year of recession. In case C two years of recessions. The Table shows the averages and standard deviations across these histories of the following expected or ex-ante variables: excess returns, market betas, hedging betas and the prices of risk. The Table also shows the averages and standard deviations of realized or ex-post excess returns and the alphas and betas of the unconditional CAPM.

	CASE A		CASE B		CASE C		CASE A		CASE B		CASE C	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
	Expected Excess Returns						**Realized Excess Returns**					
A1	0.035	*0.002*	0.034	*0.001*	0.034	*0.002*	0.035	*0.052*	0.038	*0.052*	0.040	*0.054*
A2	0.037	*0.002*	0.041	*0.003*	0.044	*0.003*	0.050	*0.038*	0.045	*0.042*	0.040	*0.045*
A3	0.039	*0.003*	0.045	*0.004*	0.049	*0.004*	0.056	*0.038*	0.048	*0.046*	0.038	*0.045*
A4	0.039	*0.004*	0.047	*0.005*	0.053	*0.006*	0.062	*0.038*	0.053	*0.041*	0.040	*0.042*
A5	0.040	*0.007*	0.062	*0.013*	0.076	*0.013*	0.091	*0.024*	0.075	*0.025*	0.048	*0.030*
Market	0.033	*0.003*	0.054	*0.005*	0.046	*0.005*	0.056	*0.021*	0.048	*0.023*	0.036	*0.025*
	Expected Market Beta						**Expected Hedging Beta**					
A1	0.972	*0.293*	0.826	*0.298*	0.734	*0.285*	-0.031	*0.023*	-0.026	*0.022*	-0.021	*0.020*
A2	1.005	*0.002*	0.958	*0.003*	0.917	*0.003*	0.168	*0.123*	0.148	*0.121*	0.122	*0.107*
A3	1.052	*0.087*	1.039	*0.105*	1.027	*0.100*	0.247	*0.181*	0.221	*0.179*	0.185	*0.162*
A4	1.042	*0.086*	1.068	*0.099*	1.084	*0.105*	0.315	*0.230*	0.285	*0.227*	0.241	*0.206*
A5	1.070	*0.341*	1.331	*0.462*	1.505	*0.475*	0.618	*0.435*	0.616	*0.463*	0.542	*0.431*
Price of Risk	0.039	*0.012*	0.047	*0.017*	0.051	*0.017*	-0.011	*0.011*	-0.012	*0.013*	-0.015	*0.014*
	Unconditional CAPM alphas						**Unconditional CAPM Betas**					
A1	-0.012	*0.050*	0.006	*0.049*	0.018	*0.054*	0.867	*0.210*	0.689	*0.166*	0.611	*0.179*
A2	-0.005	*0.034*	0.001	*0.037*	0.007	*0.037*	0.997	*0.126*	0.929	*0.126*	0.906	*0.133*
A3	-0.002	*0.033*	0.000	*0.041*	0.002	*0.038*	1.038	*0.118*	1.017	*0.154*	1.010	*0.143*
A4	0.002	*0.035*	0.001	*0.035*	0.000	*0.037*	1.067	*0.135*	1.097	*0.147*	1.112	*0.157*
A5	0.024	*0.033*	0.002	*0.037*	-0.012	*0.042*	1.162	*0.206*	1.509	*0.221*	1.649	*0.263*

TABLE VI

Asymmetric Volatility

We estimate univariate GJR-GARCH models of all 5 assets and the market portfolio. The Table shows the averages, standard deviations, minimum, maximum and quantiles of all the estimated parameters across the 500 histories.

		Mean	Stdev	Min	0.05	0.25	0.5	0.75	0.95	Max
Persistence	A1	0.939	0.163	0.000	0.608	0.979	0.985	0.988	0.990	0.993
	A2	0.945	0.162	0.000	0.678	0.979	0.984	0.987	0.990	0.993
	A3	0.958	0.126	0.000	0.953	0.976	0.982	0.985	0.989	0.991
	A4	0.979	0.009	0.824	0.967	0.976	0.980	0.984	0.987	0.991
	A5	0.952	0.008	0.921	0.937	0.947	0.953	0.958	0.964	0.973
	M	0.966	0.007	0.922	0.953	0.962	0.967	0.971	0.977	0.983
News	A1	0.006	0.012	0.000	0.000	0.000	0.001	0.007	0.026	0.083
	A2	0.005	0.009	0.000	0.000	0.000	0.001	0.007	0.019	0.063
	A3	0.005	0.007	0.000	0.000	0.000	0.002	0.008	0.018	0.057
	A4	0.004	0.006	0.000	0.000	0.000	0.001	0.007	0.016	0.030
	A5	0.002	0.005	0.000	0.000	0.000	0.000	0.001	0.012	0.038
	M	0.008	0.008	0.000	0.000	0.000	0.007	0.013	0.022	0.034
Leverage	A1	0.016	0.017	-0.078	-0.006	0.012	0.017	0.022	0.035	0.102
	A2	0.017	0.014	-0.063	0.002	0.012	0.018	0.022	0.031	0.131
	A3	0.022	0.011	-0.043	0.006	0.016	0.021	0.026	0.039	0.102
	A4	0.026	0.009	-0.013	0.011	0.020	0.026	0.031	0.039	0.082
	A5	0.089	0.017	0.036	0.060	0.077	0.088	0.100	0.117	0.135
	M	0.036	0.012	-0.011	0.017	0.029	0.036	0.045	0.055	0.071

TABLE VII

Asymmetric Covariance

A bivariate asymmetric BEKK model is estimated for each asset and the market portfolio excess returns. The returns were obtained from a simulated history, with 6 years of daily data (1584 observations). Standard deviation are in parenthesis. The maximum log-likelihood of two other specifications is also shown. The log-likelihood ratio (LR) test is performed with respect to the model with both asymmetries D1 and D2. The constant estimated were multiplied by 100. Standard deviations are in brackets.

	Constant		A (Auto)		B (News)		D1 (A<0)		D2 (M<0)		Max LogLik		LR test	
Asset 1 and Market	0.234	0.014	0.930	0.011	0.006	-0.045	-0.040	0.023	-0.232	0.005	**Both**	11539.2	**Chi2**	**P-value**
	[0.086]	[0.029]	[0.042]	[0.006]	[0.023]	[0.020]	[0.075]	[0.028]	[0.071]	[0.008]	**Only M<0**	11538.2	2.0	0.73
		0.000	0.023	0.975	-0.196	0.071	-0.137	0.036	0.015	-0.238	**Full BEKK**	11514.3	49.8	0.00
		[0.004]	[0.012]	[0.006]	[0.071]	[0.042]	[0.070]	[0.069]	[0.021]	[0.033]				
Asset 2 and Market	0.126	0.040	0.991	0.003	-0.013	-0.003	-0.129	-0.005	-0.052	-0.016	**Both**	11610.3	**Chi2**	**P-value**
	[0.146]	[0.059]	[0.031]	[0.014]	[0.009]	[0.023]	[0.114]	[0.014]	[0.098]	[0.030]	**Only M<0**	11608.2	4.1	0.39
		-0.036	-0.021	0.977	-0.005	0.024	0.078	0.019	0.309	0.246	**Full BEKK**	11587.1	46.3	0.00
		[0.016]	[0.016]	[0.010]	[0.015]	[0.012]	[0.165]	[0.017]	[0.150]	[0.053]				
Asset 3 and Market	0.046	0.048	0.995	0.000	-0.009	0.000	-0.029	0.000	-0.008	0.000	**Both**	11977.5	**Chi2**	**P-value**
	[0.029]	[0.022]	[0.012]	[0.008]	[0.016]	[0.006]	[0.041]	[0.014]	[0.024]	[0.009]	**Only M<0**	11975.6	3.9	0.42
		0.026	-0.009	0.981	0.072	0.011	-0.141	-0.077	0.161	0.219	**Full BEKK**	11952.7	49.6	0.00
		[0.022]	[0.022]	[0.011]	[0.139]	[0.059]	[0.069]	[0.071]	[0.052]	[0.035]				
Asset 4 and Market	0.070	0.035	0.974	-0.001	0.125	0.053	0.010	0.088	0.150	0.017	**Both**	12092.8	**Chi2**	**P-value**
	[0.028]	[0.036]	[0.022]	[0.009]	[0.068]	[0.082]	[0.026]	[0.055]	[0.123]	[0.080]	**Only M<0**	12091.2	3.2	0.52
		0.044	0.005	0.979	-0.012	-0.062	0.222	0.044	-0.025	0.166	**Full BEKK**	12063.4	58.8	0.00
		[0.024]	[0.037]	[0.014]	[0.013]	[0.094]	[0.106]	[0.137]	[0.073]	[0.071]				
Asset 5 and Market	0.017	-0.085	0.958	0.003	-0.114	0.035	-0.282	-0.087	0.022	0.056	**Both**	11500.9	**Chi2**	**P-value**
	[0.106]	[0.056]	[0.037]	[0.027]	[0.182]	[0.072]	[0.150]	[0.068]	[0.034]	[0.058]	**Only M<0**	11492.2	17.4	0.00
		0.000	0.033	0.960	0.149	-0.076	0.118	0.123	0.431	0.154	**Full BEKK**	11458.5	84.8	0.00
		[0.115]	[0.112]	[0.058]	[0.153]	[0.094]	[0.311]	[0.133]	[0.095]	[0.104]				

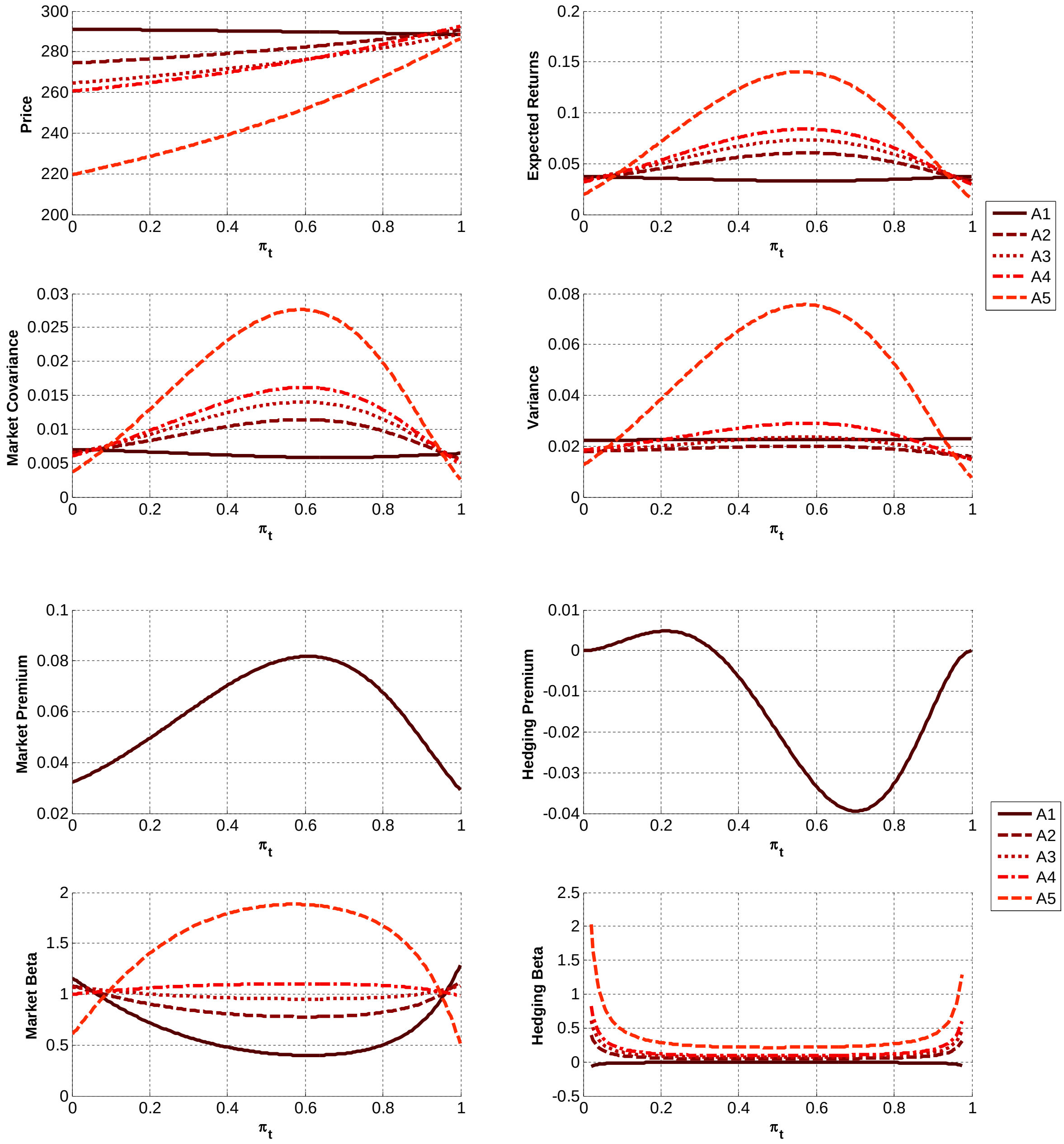

Figure 1: Theoretical variables implied by the model

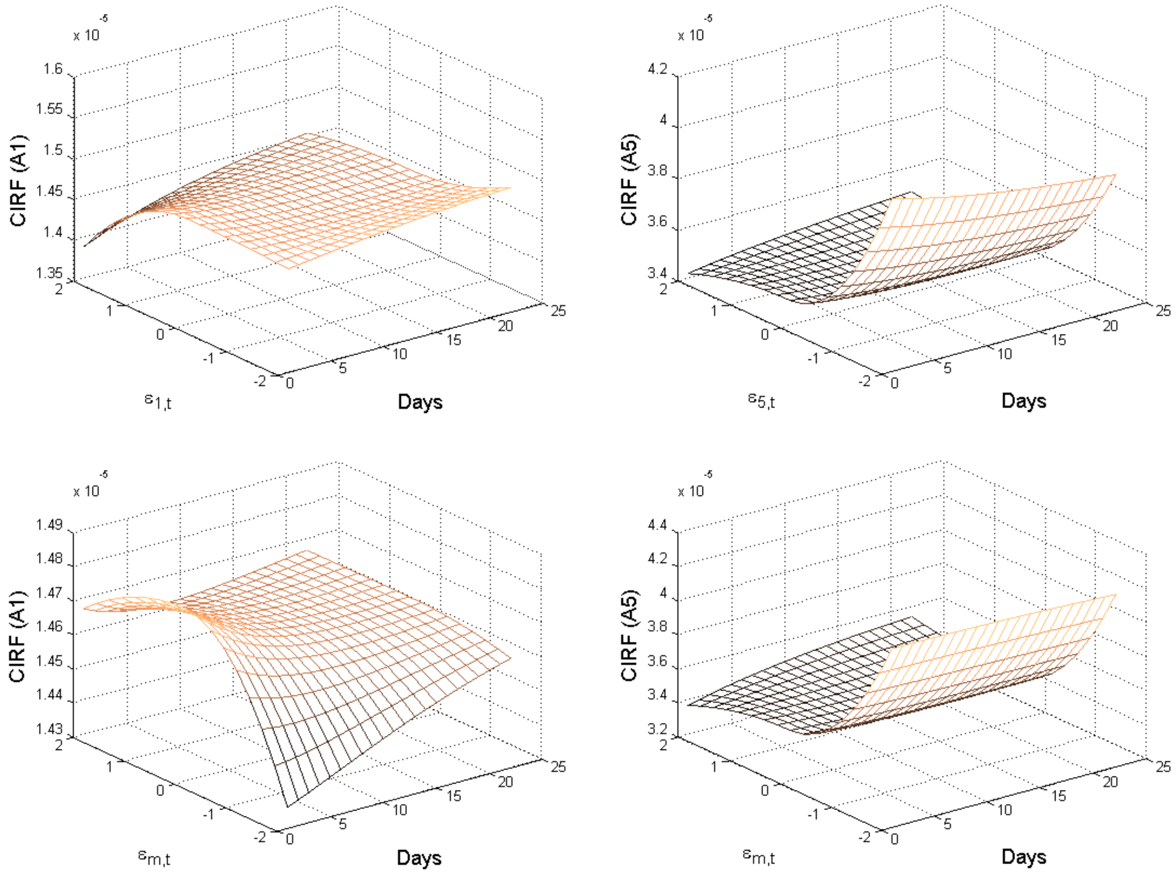

Figure 2: Covariance impulse response functions

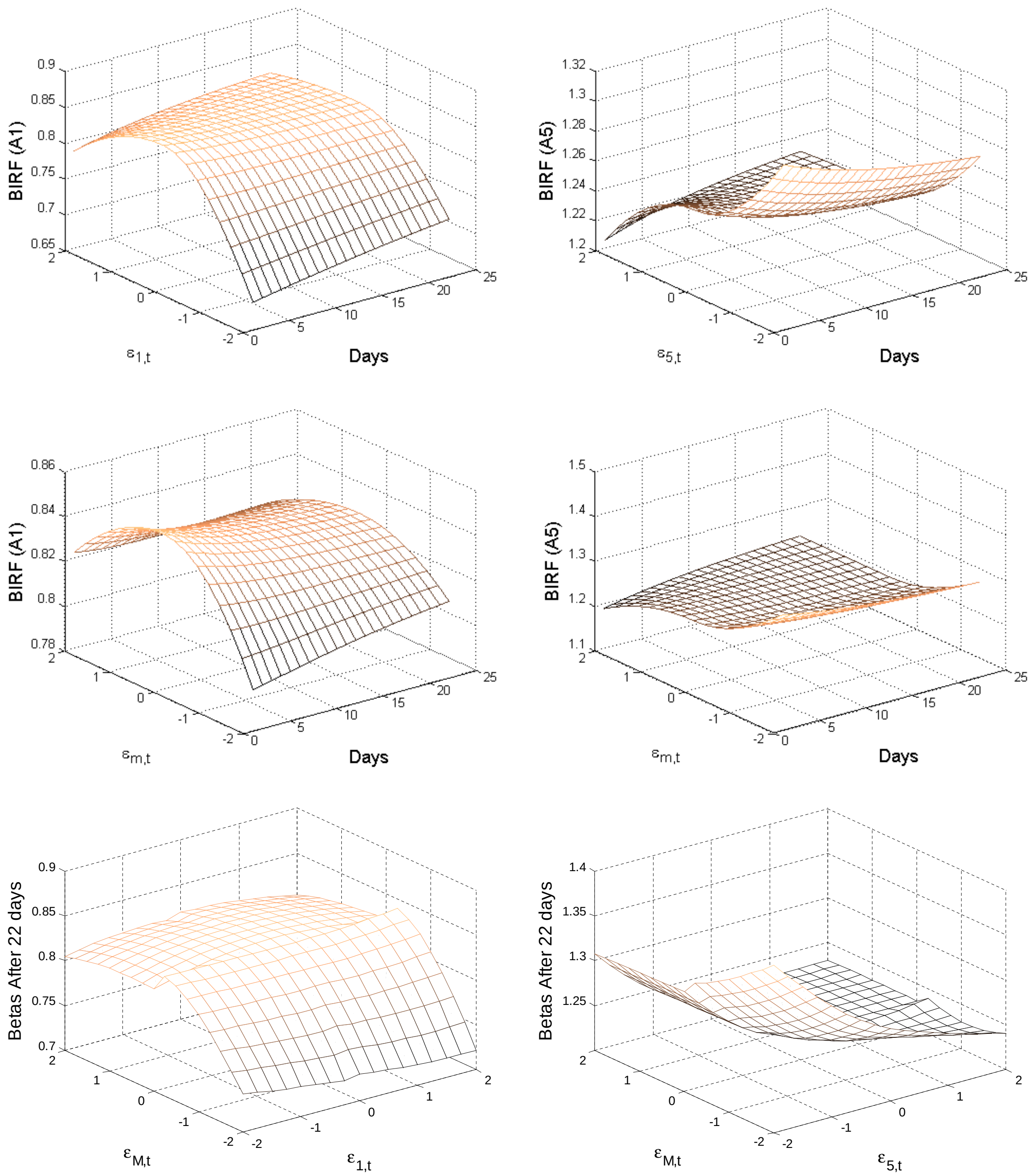

Figure 3: Betas impulse response functions.