ECON 310
Assignment #3
Due: Friday, October 26 at 9:00 am
Please write your answers on a separate piece of paper and show all of your work.
Please write legibly!

1. A firm has the production function $\sqrt{KL}$. Using calculus, $MPL = 1/2L^{-1/2}K^{1/2}$, and $MPK = 1/2L^{1/2}K^{-1/2}$. The wage rate is $2 per hour and the rental rate is $8 per hour. The firm decides to set output equal to 8 units. **[8 pts, 2 pts each]**

MPL/MPK = Pl/Pk, so [.5L^(-1/2)K^(1/2)]/ [.5L^(1/2)K^(-1/2)] = 2/8
So, K/L = 2/8, or 8K=2L, so ***L=4K*** *(Equation #1)*
And, we want 8 units so, $\sqrt{KL}$ *=8 (equation #2)*
Substitute equation #1 into #2 to get, $\sqrt{K*(4K)} = \sqrt{4K^2} = 8$*, or 2K=8, so K=4 and L=4*4=16*

i. What is the optimal level of K?__*4*___
ii. What is the optimal level of L?__*16*___
iii. What are the firm's total costs?_*TC= w*L + Pk*K= 16*2 + 4*8 = $64*____
iv. Graph the entire situation. Label both curves, including endpoints, and indicate the optimal bundle.

Isocost curve: 2L + 8K = 64, so Y-intercept (K) = (0,8) and X-intercept (L) = (32,0)
Isoquant curve: Just draw in isoquant that is tangent to isocost at (16,4)

2. A firm discovers that when it uses K units of capital and L units of labor, it is able to produce $\sqrt{KL}$ units of output. Suppose the firm produces 12 units of output using 16 units of capital and 9 units of labor. **[10 pts, 2 pts each]**

i. Compute $MRTS_{LK}$

$$MRTS_{LK} = \Delta K$$
$$\sqrt{16*9} = \sqrt{K*8}$$
$$144 = 8K$$
$$K = 18$$
$$\Delta K = 18 - 16 = 2 \therefore MRTS_{LK} = 2$$

ii. Compute MPL

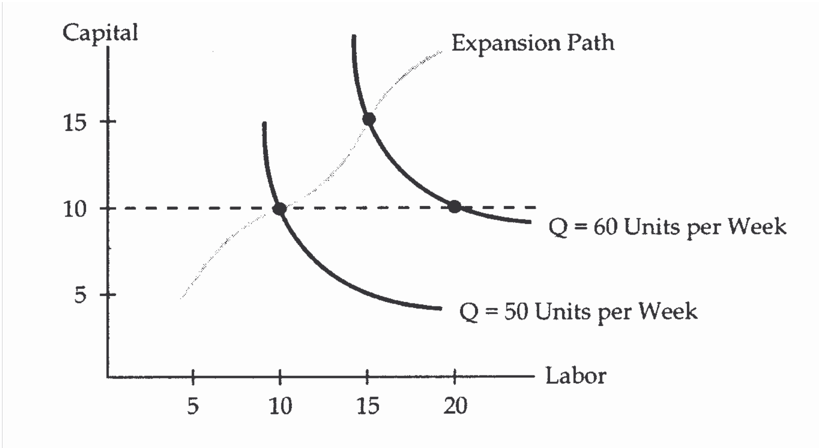
$$\sqrt{16*9} - \sqrt{16*8} = .686$$

iii. Compute MPK

$$\sqrt{16*9} - \sqrt{15*9} = .381$$

iv. Is the equation $MRTS_{LK}$ = MPL/MPK approximately true?
.686/.381 = 1.80, so yes. If units had been smaller, it would have been closer.
v. Suppose that capital costs $3 per machine and labor can each be hired at $1 per unit and that the firm uses 16 units of capital. What is the short run total cost to produce 12 units of output?
*16*3 + 9*1 = $57*

3. Refer to the diagram below. The wage rate (P_L) is assumed to be $30 per hour, the capital rental rate (P_K) is assumed to be $15 per hour, and capital is assumed to be fixed in the short run at 10 units. **[4 pts]**



i. What is the short-run *average* cost of producing 60 units of output per week?

*Pk*K + Pl*L/60 = 15*10 + 30*20 / 60 = $12.50*

ii. What is the long-run *total* cost of producing 60 units of output per week?

*Pk*K + Pl*L=15*15 + 30*15=$675*

iii. By comparing the two points on the expansion path, we can conclude that this technology exhibits

a. decreasing returns to scale.
b. constant returns to scale.
c. increasing returns to scale.
d. zero returns to scale.

iv. In order to produce 50 units of output per week, the firm must spend at least how much?

$450

4. Chapter 7, Numerical Exercises N1 **[4 pts]**, N2 **[4 pts],** & N6 **[8 pts]** *(Note: N6 is problem N7 in version 7 of text)*

N1.

*a. Since firms will produce where P=MC, each firm will produce 5 units at a price of $10. The firms will earn a profit of 5*$10 - $36 = $14. The industry is not in the long-run, because the firms are making a positive economic profit.*

b. In the long-run there will be entry into this industry. The firms will earn zero profit and will produce where P=MC=ATC. This happens when q= 3 and p= 6.

N2.

a. Since the industry is in long-run equilibrium, P=MC=ATC. This happens at P=$15.

b. At P=$15, industry demand is 450 and each firm produces 3 units, so there are 450/3= 150 firms.

*c. At P=$10, each firm produces 2 units. Since there are 150 firms, the quantity 150*2= 300 will correspond to a price of $10.*

N6.

a. In the long-run, P=MC=ATC. Setting the equations for ATC and MC equal, we get Q= 10 and P=$20. The LRS curve will be flat at the break-even price of $20.

b. Putting a price of $20 into the industry demand equation gives a total of 7,000 kites. Since each firm produces 10 kites, there are 700 firms in the industry.

*c. In the SR, it is impossible to produce more kites. Thus, industry Q stays at 7,000 kites. Plugging in 7,000 kites into the demand equation gives a new market price of $40. Each firm will be earning a profit of $40*10 - $20*10 = $200.*

d. In the long-run, the price will be $20 (it must fall so that P=ATC=MC). Now, there will be 8,000 kites in the industry. Each firm produces 10, so there will 800 firms. Thus, 100 firms enter and they will all earn zero profit.

5. In assignment #1, problem 2, the demand curve for a good was given by the equation Q=-12P+2400 (P= -1/12*Q + 200) and the supply curve was given by Q=8P-400 (P=1/8Q + 50). We found the equilibrium price and quantity to be $140 and 720 units. **[12 pts]**

i. Calculate consumer surplus at this equilibrium price and quantity. (1 pt)
*CS= (demand intercept – price)*Q/2*
*CS= (200-140)*720/2 = 21600*

ii. Calculate producer surplus at this equilibrium price and quantity. (1 pt)
*PS= (price-supply intercept)*Q/2*
*PS= (140-50)*720/2 = 32400*

iii. Calculate total social gain at this equilibrium price and quantity. (1pt)
SW= CS+PS = 21600+32400 = 54000

In another part of the problem, we assumed the government imposed an excise tax of $50 per unit. With this tax, we found the new market price to be $160 and the new quantity to be 480 units.

iv. Calculate the post-tax consumer surplus at this new equilibrium price and quantity. (1 pt)
*PTCS= (demand intercept – price demanders actually pay)*new Q/2*
*PTCS= (200-160)*480/2=9600*

v. Calculate the post-tax producer surplus at this new equilibrium price and quantity. (1 pt)
*PTPS= (price suppliers actually receive- supply intercept)*new Q/2 = (110-50)*480/2=14400*

vi. Calculate the tax revenue collected from the government. (1 pt)
*Tax Revenue= amt of tax*new Q = 50*480 = 24000*

vii. Calculate the post-tax total social gain assuming the tax revenue is re-distributed to support public education. (1 pt)
PTSW= PTCS+PTPS+Tax Revenue = 9600+14400+24000=48000

viii. Calculate deadweight loss. (1 pt)
DWL= SW before tax – SW after tax = 54000-48000=6000

ix. Illustrate the effect of this tax graphically. Label the following areas as follows (4 pts):

1. A- consumer surplus
   triangle shape: area below demand curve, above 160 out to Q=480
2. B- producer surplus
   triangle shape: area below p=110 above supply curve out to Q=480
3. C- tax revenue
   square shape: area between p=160 and p=110 out to Q=480
4. D- deadweight loss
   triangle shape: area between supply and demand curves between Q=480 and Q=720