1. Evaluate the following limit: $\lim_{\theta \to 0} \frac{\cot(5\theta)\sin(\pi\theta)}{cos(7\theta)}$

$$\lim_{\theta \to 0} \frac{\cot(5\theta)\cdot\sin(\pi\theta)}{\cos(7\theta)} = \lim_{\theta \to 0} \frac{\pi\cdot\cos(5\theta)}{5\cdot\cos(7\theta)}\cdot\frac{\sin(\pi\theta)}{\pi\theta}\cdot\frac{5\theta}{\sin(5\theta)}$$

$$= \frac{\pi}{5}\cdot\frac{1}{1}\cdot 1\cdot 1 = \boxed{\frac{\pi}{5}}$$

2. Algebraically (not graphically), determine and write equations for all of the asymptotes (both horizontal and vertical) of the graph of $f(x) = \frac{(x-3)(2x^2-1)}{(x^2-9)\sqrt{4x^2+3}}$

VA: First set denominator to 0: $(x^2-9)\sqrt{4x^2+3} = 0$

$(x-3)(x+3)\underbrace{\sqrt{4x^2+3}}_{=0 \text{ has no sol'n.}} = 0$; $x = 3, -3$

$\lim_{x\to 3} f(x) = \lim_{x\to 3} \frac{(x-3)(2x^2-1)}{(x^2-9)\sqrt{4x^2+3}} = \lim_{x\to 3} \frac{2x^2-1}{(x+3)\sqrt{4x^2+3}} = \frac{17}{6\sqrt{39}}$, so there is not a VA at $x=3$

$\lim_{x\to -3^+} f(x) = \lim_{x\to -3^+} \frac{(x-3)(2x^2-1)}{(x-3)(x+3)\sqrt{4x^2+3}} \longrightarrow \frac{-6\cdot 17}{-6\cdot 0^+\cdot\sqrt{39}}$, so

$\lim_{x\to -3} f(x) = \infty$, & there is a $\boxed{\text{VA at } x=-3}$

HA: $\lim_{\substack{x\to\infty \\ \& x\to-\infty}} f(x) = \lim_{\substack{x\to\infty \\ x\to-\infty}} \frac{2x^2-1}{(x+3)\sqrt{4x^2+5}} = \lim_{\substack{x\to\infty \\ x\to-\infty}} \frac{2-\frac{1}{x^2}}{\left(1+\frac{3}{x}\right)\frac{\sqrt{4x^2+5}}{x}}$

as $x\to\infty$, $x>0$ so $\sqrt{x^2} = |x| = x$.

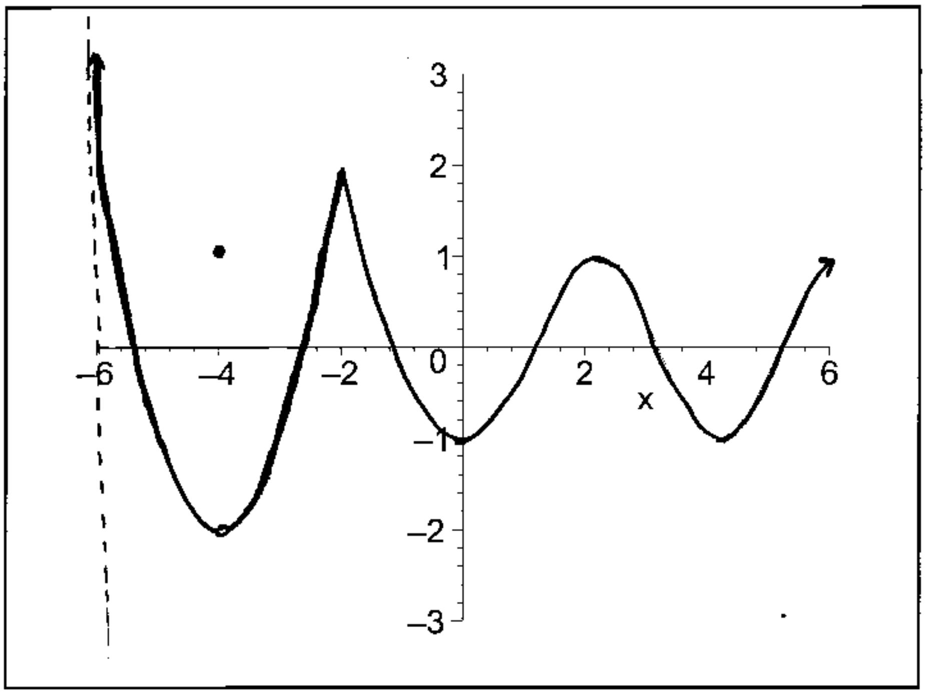
$\therefore \lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{2-1/x^2}{(1+3/x)\sqrt{4+\frac{5}{x^2}}} = \frac{2}{1\cdot\sqrt{4}} = \frac{2}{2} = 1.$

as $x\to-\infty$, $x<0$ so $\sqrt{x^2} = |x| = -x$.

$\therefore \lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{2-1/x^2}{-(1+3/x)\sqrt{4+\frac{5}{x^2}}} = \frac{2}{-1\cdot\sqrt{4}} = \frac{2}{-2} = -1$.

So $\boxed{\text{HA at } y=1, y=-1}$

3. Consider the graph of $f(x)$ given below:




(a) For what values of $x$ is $f(x)$ discontinuous?

$-6, -4.$

(b) For what values of $x$ is $f(x)$ not differentiable?

$-6, -4, -2$

(c) For what values of $x$ is $f'(x) = 0$?

$0, 2, 4.$

4. By using the Intermediate Value Theorem, explain why there is a solution to the equation $4x^4 - 3x^2 + 2x - 5 = 3$ on the interval $[0, 2]$.

Let $f(x) = 4x^4 - 3x^2 + 2x - 5$, continuous on $[0,2]$. Since $f(0) = -5$, $f(2) = 64 - 12 + 4 - 5 = 51$ and $-5 \le 3 \le 51$, there is a value $c$, $0 \le c \le 2$, so that $f(c) = 3$.

5. A ball is rolling down a 2 foot long hill so that its distance from the top of the hill after $t$ seconds is given by $s(t) = \dfrac{4t}{t^2+1}$ feet.

(a) How long does it take for the ball to roll halfway down the hill (s(t)=1)? (Hint: Since $s(1) = 2$, the ball hits the bottom after 1 second, so your answer should be between 0 and 1.)

$$\frac{4t}{t^2+1} = 1$$

$$4t = t^2+1$$

$$t^2-4t+1=0$$

$$t = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \frac{\sqrt{12}}{2} = 2 \pm \sqrt{3}.$$

$2+\sqrt{3} > 1$;

$0 < 2-\sqrt{3} < 1$, so the ball hits the halfway point at $\boxed{2-\sqrt{3} \text{ sec}}$

$\approx .268$ s.

(b) How fast is the ball moving at this time?

want $s'(2-\sqrt{3})$:

$$s'(t) = \frac{(t^2+1)\cdot 4 - 4t(2t)}{(t^2+1)^2} = \frac{-4t^2+4}{(t^2+1)^2}$$

$$s'(2-\sqrt{3}) = \frac{(2-\sqrt{3})^2 \cdot -4 + 4}{((2-\sqrt{3})^2+1)^2}$$

$$= \frac{-4(4-4\sqrt{3}+3)+4}{(4-4\sqrt{3}+3+1)^2}$$

$$= \frac{-28+16\sqrt{3}+4}{64-64\sqrt{3}+48}$$

$$= \frac{16\sqrt{3}-24}{112-64\sqrt{3}} = \boxed{\frac{2\sqrt{3}-3}{14-8\sqrt{3}} \text{ ft/sec}}$$

$\approx 3.23$ ft/sec