Math 31 Sec 7 EXAM II 22 October 2003

Name: KEY

I pledge that I have neither given nor received any unauthorized assistance on this exam.

(signature)

DIRECTIONS

1) Print your name and sign the honor pledge above. If the pledge is not signed, your exam will ***not*** be graded.

2) Check now that your test contains all ***5 pages*** and ***6 problems***.

3) You may use a calculator (except symbolic manipulators such as a TI-89, TI-92, or similar), but your answers must be given in their ***exact*** form. (i.e. $\sqrt{3}$ and not 1.73, π and not 3.14)

4) All work must be shown on this exam. ***No credit will be given for a correct answer without supporting work that leads to the answer***. When it is indicated that calculators are not to be used, clear non-calculator work must be shown.

5) Place a box around ***all*** of your final answers. Include units when necessary.

6) Notation and clarity count. Your job is to communicate mathematically; make what you are thinking clear.

7) Work quickly but thoroughly through the test. If you get stuck on a problem, move on to the next and return to it later after you've completed the problems you know how to do. ***Good Luck.***

(8 each) 1. Consider the equation $x^2 + y^2 = xy + 3$.

(a) Find $\frac{dy}{dx}$.

$$2x + 2y\frac{dy}{dx} = x\cdot\frac{dy}{dx} + 1\cdot y$$

$$2x - y = \frac{dy}{dx}(x - 2y).$$

$$\boxed{\frac{dy}{dx} = \frac{2x-y}{x-2y}}$$

(b) Write the equation of the tangent line to the graph given by this equation at the point $(1, -1)$.

At $(1,1)$, the slope of the tangent line is $\frac{2\cdot 1-(-1)}{1-2(-1)} = \frac{3}{3} = 1$

So the equation of the tangent line is:

$$y-(-1) = 1(x-1) \quad \text{or} \quad y+1 = x-1$$

$$\boxed{y = x-2}$$

(c) Find $\frac{d^2y}{dx^2}$ in terms of x and y. (You should not have any terms containing $\frac{dy}{dx}$!) You do not need to simplify your answer.

$$\frac{dy}{dx} = \frac{2x-y}{x-2y}.$$

$$\frac{d^2y}{dx^2} = \frac{(x-2y)\left(2-\frac{dy}{dx}\right) - (2x-y)\left(1-2\frac{dy}{dx}\right)}{(x-2y)^2}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{(x-2y)\left(2-\frac{2x-y}{x-2y}\right) - (2x-y)\left(1-2\cdot\frac{2x-y}{x-2y}\right)}{(x-2y)^2}}$$

(10) 2. Using differentials, approximate $\sqrt{25.2}$.

Let $y = f(x) = \sqrt{x}$.

$$dy = f'(x)\cdot dx = \frac{1}{2\sqrt{x}}\cdot dx$$

For $x = 25$, $dx = \Delta x = .2$, $dy = \frac{1}{2\sqrt{25}}\cdot .2 = \frac{1}{10}\cdot\frac{2}{10} = .02$.

$$\Delta y = f(25+.2) - f(25) \approx dy = .02$$

so $\sqrt{25.2} \approx \sqrt{25} + .02 = \boxed{5.02}$

(5 each) 3. A projectile is fired directly upward from the ground with initial velocity v_0 feet per second. Its height after t seconds is given by $s(t) = -16t^2 + v_0 t$.

(a) What is the velocity function for the projectile?

$v(t) = s'(t) = -32t + v_0$

$\boxed{v(t) = -32t + v_0}$

(b) When does the projectile reach its maximum height?

When $v(t) = 0$:

$-32t + v_0 = 0$

$v_0 = 32t$

$t = \frac{v_0}{32}$

$\boxed{\text{After } \frac{v_0}{32} \text{ seconds, the projectile reaches its maximum height.}}$

(c) What is the maximum height that it reaches?

$s\left(\frac{v_0}{32}\right) = -16 \cdot \frac{v_0^2}{32^2} + v_0 \cdot \frac{v_0}{32}$

$= -\frac{v_0^2}{64} + \frac{v_0^2}{32}$

$= \frac{v_0^2}{64}$

$\boxed{\text{The maximum height reached is } \frac{v_0^2}{64} \text{ feet.}}$

(d) What must its initial velocity, v_0, be if the projectile reaches a maximum height of 1 foot?

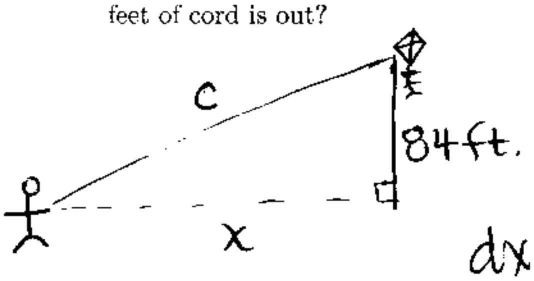
$\frac{v_0^2}{64} = 1$

$v_0^2 = 64$

$v_0 = 8$

$\boxed{\text{Its initial velocity must be 8 ft/sec.}}$

(15) 4. A child is flying a kite. If the kite is 84 feet above the child's hand level and the wind is blowing the kite horizontally away from the child at 5 feet per second, how fast is the child letting out cord when 91 feet of cord is out?



let c be the amount of cord out
let x be the horizontal distance from the child to the kite.

$\frac{dx}{dt} = 5$ ft/sec. Find $\frac{dc}{dt}$ when $c = 91$ ft.

$x^2 + 84^2 = c^2$.

$2x \frac{dx}{dt} = 2c \frac{dc}{dt}$

$\frac{dc}{dt} = \frac{x}{c} \cdot \frac{dx}{dt}$.

When $c = 91$ft, $x^2 + 84^2 = 91^2$

$x^2 = 91^2 - 84^2 = 1225$

$x = 35$ ft.

At this moment, $\frac{dc}{dt} = \frac{35 \text{ ft}}{91 \text{ ft}} \cdot 5 \frac{\text{ft}}{\text{sec}}$

The cord is being let out at a rate of $\frac{175}{91}$ ft/sec.

(8 each) 5. Let $f(x) = cos^2(3x^2 + 1)$.

(a) Find $f'(x)$.

$f'(x) = 2\cos(3x^2+1)(-\sin(3x^2+1))(6x)$

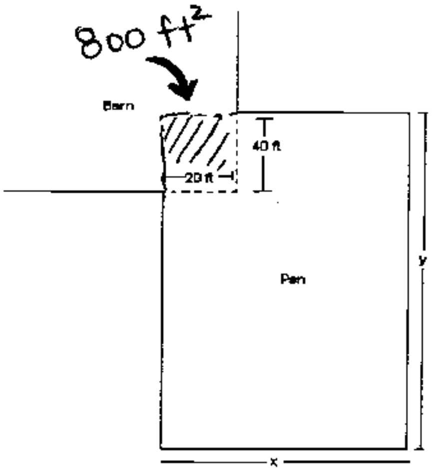
$\boxed{f'(x) = -12x\cos(3x^2+1) \cdot \sin(3x^2+1)}$

(b) Find $f''(x)$.

$f''(x) = -12\cos(3x^2+1)\sin(3x^2+1) - 12x(-\sin(3x^2+1))(6x)\sin(3x^2+1)$
$\quad - 12x\cos(3x^2+1) \cdot \cos(3x^2+1)(6x)$

$\boxed{f''(x) = -12\cos(3x^2+1)\sin(3x^2+1) + 72x^2\sin^2(3x^2+1) - 72x^2\cos^2(3x^2+1)}$

(15) 6. A farmer has 300 feet of fencing he wishes to use to enclose a rectangular pen. He wants the pen to fit a 20 x 40 square foot corner of his barn. (The corner must be used and does not need fencing.) What are the dimensions which give the pen a maximum area?



As in the diagram, let the dimensions of the pen be $x \times y$.

Area of the pen is $xy - (20)(40)$ square feet.

$x + y + (x-20) + (y-40) = 300$

$2x + 2y - 60 = 300$

$2x + 2y = 360$

$x + y = 180$; $y = 180 - x$

Area is $x(180-x) - 800$.

want to maximize $A(x) = 180x - x^2 - 800$ on $[20, 140]$, since $x \geq 20$, $y = 180 - x \geq 40$

$x \leq 180 - 40 = 140$.

Stationary points: $A'(x) = 0$.

$A'(x) = 180 - 2x$

$A'(x) = 0$ for $180 - 2x = 0$; $2x = 180$; $x = 90$.

$A(20) = 20 \cdot 160 - 800 = 3200 - 800 = 2400$

$A(90) = 90 \cdot 90 - 800 = 8100 - 800 = 7300$ * Max

$A(140) = 140 \cdot 40 - 800 = 5600 - 800 = 4800$

The dimensions which give the largest pen area are 90 ft x 90 ft.