

Name: _____

Show all work on the quiz in the space provided. Correct answers without work will not receive credit. There are to be no calculators used for this quiz.

Potentially useful formulas:

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 1 - 2 \sin^2 \theta = 2 \cos^2 \theta - 1$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\tan \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \frac{1 - \cos \theta}{\sin \theta} = \frac{\sin \theta}{1 + \cos \theta}$$

(2 points) 1. Find the exact value, without using a calculator, of $\cos(\frac{3\pi}{8})$.

Since $\frac{3\pi}{8} = \frac{3\pi}{4} / 2$, we use the half-angle formula for cosine:

$$\cos \frac{3\pi}{8} = \sqrt{\frac{1 + \cos(\frac{3\pi}{4})}{2}} = \sqrt{\frac{1 + (-\frac{\sqrt{2}}{2})}{2}} = \sqrt{\frac{2 - \sqrt{2}}{4}} = \frac{\sqrt{2 - \sqrt{2}}}{2}$$

where the positive square root is used since $\frac{3\pi}{8}$ is in the first quadrant.

(5 points) 2. Let θ be an angle in the second quadrant with $\sin \theta = \frac{6}{7}$. Without using a calculator, find the exact values of

a) $\sin(2\theta)$

b) $\cos(2\theta)$

c) $\tan(2\theta)$.

First we must find the values of $\cos \theta$ and $\tan \theta$. Pythagorean theorem gives: $6^2 + x^2 = 7^2$ so $x^2 = 49 - 36 = 13$ and $x = -\sqrt{13}$, negative as θ is in the second quadrant. Then:

$$\sin 2\theta = 2 \sin \theta \cos \theta = 2 \frac{6}{7} \frac{-\sqrt{13}}{7} = \frac{-12\sqrt{13}}{49}$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = \left(\frac{-\sqrt{13}}{7}\right)^2 - \left(\frac{6}{7}\right)^2 = \frac{13}{49} - \frac{36}{49} = \frac{-23}{49}$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2 \frac{-6}{\sqrt{13}}}{1 - \left(\frac{6}{\sqrt{13}}\right)^2} = \frac{\frac{-12\sqrt{13}}{13}}{1 - \frac{36}{13}} = \frac{\frac{-12\sqrt{13}}{13}}{\frac{-23}{13}} = \frac{12\sqrt{13}}{23}$$