Math 30 Sec 4 **Quiz 6** **4 OCTOBER 2002**

Name:____

Show all work on the quiz in the space provided. Correct answers without work will not receive credit. There are to be no calculators used for this quiz.

Potentially useful formulas:

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$

$\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

$\sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$

$\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$

$\tan\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}} = \dfrac{1-\cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1+\cos\theta}$

(5 points) 1. Let α be a third quadrant angle with $\sin\alpha = \frac{-2}{3}$ and let β be an angle in the second quadrant with $\cos\beta = \frac{-12}{13}$. Find the exact value, without using a calculator, of $\sin(\alpha+\beta)$.

Since $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$, we must find the values of $\sin\beta$ and $\cos\alpha$.

α: $x^2 + (-2)^2 = 3^2$ gives $x^2 = 9 - 4 = 5$, so $x = -\sqrt{5}$, negative as α is in the third quadrant.
Then $\cos\alpha = \frac{-\sqrt{5}}{3}$.

β: $(-12)^2 + y^2 = 13^2$ gives $y^2 = 169 - 144 = 25$, so $y = 5$, positive since β is in the second quadrant.
Then $\sin\beta = \frac{5}{13}$.

Now $\sin(\alpha+\beta) = \dfrac{-2}{3}\cdot\dfrac{-12}{13} + \dfrac{-\sqrt{5}}{3}\cdot\dfrac{5}{13} = \dfrac{24}{39} + \dfrac{-5\sqrt{5}}{39} = \dfrac{24-5\sqrt{5}}{39}$.

(5 points) 2. Without using a calculator, find the exact values of

(a) $\sin^{-1}(\cos(\frac{5\pi}{6}))$

(b) $\cot(\sin^{-1}(\frac{2}{7}))$

(a) $\sin^{-1}(\cos(\frac{5\pi}{6})) = \sin^{-1}(\frac{-1}{2}) = \frac{-\pi}{6}$, since $\sin(\frac{-\pi}{6}) = \frac{-1}{2}$ and $\frac{-\pi}{2} \leq \frac{-\pi}{6} \leq \frac{\pi}{2}$.

(b) $\sin^{-1}(\frac{2}{7})$ is the angle θ between $\frac{-\pi}{2}$ and $\frac{\pi}{2}$ with $\sin\theta = \frac{2}{7}$.

So we have a triangle with (relative to θ) opposite side 2, hypotenuse 7, and by the Pythagorean Theorem, opposite side is given by: $x^2 + 2^2 = 7^2$, $x^2 = 49 - 4 = 45$, so $x = \sqrt{45}$, positive since $\sin\theta > 0$ implies θ is in the first quadrant.

Then $\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}} = \frac{\sqrt{45}}{2}$.