Practice Exam 2
Math 31 Section 7

Name: KEY (please print)

Show **all** work!

In problems 1 - 3 below, find $\frac{dy}{dx}$ for the given function y. Please do NOT simplify.

1. $y = \dfrac{\sin(x) + \cos^2(x)}{5x^2}$

$$\boxed{\frac{dy}{dx} = \frac{5x^2\left(\cos x + 2\cos x \cdot \sin x\right) - 10x\left(\sin x + \cos^2 x\right)}{25x^4}}$$

$\frac{dy}{dx}$ ____________________

2. $y = (x^5 + 3x - 1)^{10} \cdot (x^3 - 2x)^8$

$$\boxed{\frac{dy}{dx} = 10(x^5+3x-1)^9(5x^4+3)(x^3-2x)^8 + (x^5+3x-1)^{10} \cdot 8(x^3-2x)^7 \cdot (3x^2-2)}$$

$\frac{dy}{dx}$ ____________________

3. $y = \tan(\sin(2\pi x))$

$$\boxed{\frac{dy}{dx} = \sec^2(\sin(2\pi x)) \cdot \cos(2\pi x) \cdot 2\pi}$$

$\frac{dy}{dx}$ ____________________

4. Using implicit differentiation, determine the equation of the line tangent to the graph of $x^2 - xy + 3 = 3y^3$, at the point $(1, 1)$.

$2x - (x \cdot \frac{dy}{dx} + y) = 9y^2 \cdot \frac{dy}{dx}$

At $(1,1)$, $\frac{dy}{dx} = \frac{2-1}{9-1} = \frac{1}{8}$

$\frac{dy}{dx}(9y^2 - x) = 2x - y$

$\frac{dy}{dx} = \frac{2x-y}{9y^2-x}$.

Equation $\boxed{y - 1 = \frac{1}{8}(x-1)}$

5. Find $\frac{d^4y}{dx^4}$ if $y = \cos(3x)$.

$\frac{dy}{dx} = -\sin(3x) \cdot 3 = -3\sin(3x)$

$\frac{d^2y}{dx^2} = -3 \cdot \cos(3x) \cdot 3 = -9\cos(3x)$

$\frac{d^3y}{dx^3} = 9\sin(3x) \cdot 3 = 27\sin(3x)$

$\frac{d^4y}{dx^4} = 27\cos(3x) \cdot 3 = 81\cos(3x)$

$\frac{d^4y}{dx^4}$ $\boxed{81\cos(3x)}$

6. Find the maximum and minimum values of the function $3x^4 - 4x^2 + 2$ on the interval $(-3, 1]$ if they exist. If one does not exist, say so.

$f(x) = 3x^4 - 4x^2 + 2$.

$f'(x) = 12x^3 - 8x$

$= 4x(3x^2 - 2)$.

$f'(x) = 0$ for $4x = 0$; $x = 0$

or $3x^2 - 2$; $x^2 = \frac{2}{3}$

$x = \sqrt{\frac{2}{3}}$

$f(1) = 3 - 4 + 2 = 1$

$f(\sqrt{2/3}) = 3 \cdot \frac{4}{9} - 4 \cdot \frac{2}{3} + 2 = \frac{2}{3}$

$f(-\sqrt{2}/3)$ = (exactly the same) $= \frac{2}{3}$.

as $x \to -3$, $f(x) \to 3 \cdot (-3)^4 - 4(-3)^2 + 2 = 209$.

Min at $(\sqrt{2}/\sqrt{3}, 2/3)$ & $(-\sqrt{2}/\sqrt{3}, 2/3)$.
No max occurs

7. An object moves according to the equation $s = x^3 - 12x^2 + 6x - 5$. What is its velocity at the moment when it has zero acceleration?

$v(x) = s'(x) = 3x^2 - 24x + 6$.

$a(x) = v'(x) = 6x - 24$.

$a(x) = 0$ for $6x - 24 = 0$

$x = 4$.

$v(4) = 3 \cdot 4^2 - 24 \cdot 4 + 6$

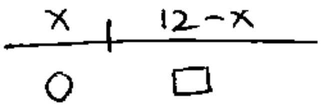
$= 48 - 96 + 6$

$= -42$

Velocity $\boxed{-42}$

8. A piece of wire, 12 inches in length, is to be cut in two (not necessarily halves). One of the resulting pieces will form a circle, the other a square. Where, if at all, should the wire be cut to maximize the sum of the areas inclosed by the figures? (If you determine that the wire should not be cut, what shape should the single piece be formed into?)

Let x be the length of the wire formed into a circle.



$$A = \pi r^2 + s^2$$
$$= \pi\left(\frac{x}{2\pi}\right)^2 + \left(3-\frac{x}{4}\right)^2$$
$$= \frac{x^2}{4\pi} + \left(3-\frac{x}{4}\right)^2.$$

$x = 2\pi r$, $r = \frac{x}{2\pi}$.

$12 - x = 4s$, $s = \frac{12-x}{4} = 3 - \frac{x}{4}$.

Domain: $x \geq 0$, $x \leq 12$.

Maximize $A(x) = \frac{x^2}{4\pi} + \left(3 - \frac{x}{4}\right)^2$ on $[0, 12]$.

$$A'(x) = \frac{x}{2\pi} + 2\left(3-\frac{x}{4}\right)\cdot -\frac{1}{4}$$
$$= \frac{x}{2\pi} - \frac{3}{2} + \frac{x}{8}$$
$$= x\left(\frac{1}{8} + \frac{1}{2\pi}\right) - \frac{3}{2}.$$

$A'(x) = 0$ for $x = \frac{3/2}{(1/8 + 1/2\pi)} \approx 5.2788$

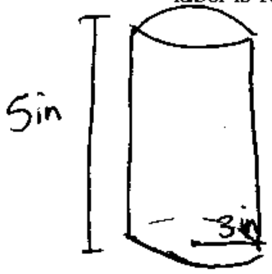
$A(0) = 9$

$A(5.2788) \approx 5.041$

$A(12) = \frac{36}{\pi} \approx 11.459$

The whole wire should be bent into a circle.

9. A 12oz. soup can has, with its label, a radius of 3 inches and a height of 5 inches. The label is .02 inches thick and is glued around the can. Use differentials to approximate the surface area of the can, after the label is removed.



label = .02 in thick.

Let $S(r)$ = surface area of the can of radius r.

We wish to approximate $S(3) - S(3 - .02)$

if $r = 3$, $d = \Delta r = -.02$, this is approximately dS.

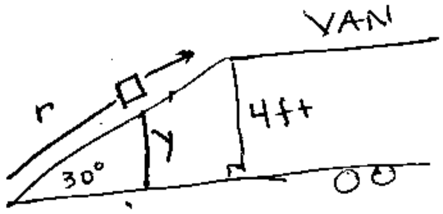
$S = 2\pi r \cdot h$ if radius is r, height is h.

So $h = 5 \Rightarrow S(r) = 10\pi r$.

$$dS = 10\pi \cdot dr$$
$$= 10\pi \cdot .02$$
$$= .2\pi$$

Surface Area $\boxed{.2\pi \text{ in}^2}$

10. A ramp, leading to the bed of a van, makes a 30° angle with the ground. Two movers are pushing a box up the ramp at a rate of 2 $^{in}/_{sec}$. At what rate is the box approaching its final vertical height of 4 feet, at the moment when it is $^3/_4$ of the way up the ramp?



let y = vertical distance of the box
r = distance up the ramp to the box.

$\frac{dr}{dt} = 2$ in/sec.

Find $\frac{dy}{dt}$ when the box is $\frac{3}{4}$ of the way up the ramp.

$\frac{dh}{dt}$ $\boxed{1 \text{ in/sec}}$

$\sin 30° = \frac{y}{r}$

$\frac{1}{2} = \frac{y}{r}$

$y = \frac{r}{2}$.

$\frac{dy}{dt} = \frac{1}{2}\frac{dr}{dt}$.

$\frac{dy}{dt} = \frac{1}{2} \cdot 2$ in/sec $= 1$ in/sec