

Math 31 - Fall 2003

Practice Problems 3

Name: _____

Instructions:

1. Read carefully and answer **all** questions. Show all work on the exam in the space provided (you may use the backs of pages if necessary).
2. Where appropriate, place your final answer in the boxes provided.
3. Unless otherwise specified, **give exact answers**. For example, write $\sqrt{2}$ not 1.414, write π instead of 3.1415, write $\frac{1}{3}$ instead of 0.3333.
4. Check your answers carefully.
5. Remember to sign the Honor Pledge.

I have neither given nor received any unauthorized help on this exam
and I have conducted myself within the guidelines of the University
Honor Code.

Pledge: _____

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1. Using Newton's Method, estimate the positive real root of $x^2 - 2 = 0$ to 4 decimal places.
Take $x_1 = 1$.

2. Find the (global) max and min of $f(x) = \sin^2 x$ on $(\pi, 2\pi]$.

3. Given $f(x) = 2x^3 - 3x^2$

(a) Find the interval(s) on which $f(x)$ is increasing.

(b) Find the interval(s) on which $f(x)$ is concave down.

(c) The function $g(x)$ has second derivative $g''(x) = \frac{8x^2(3x^2 - 1)}{(x^2 + 1)^3}$. Give the x coordinates of all the inflection points.

4. Find the point on the parabola $y^2 = 2x$ that is closest to the point $(1,4)$. Hint: Consider the square of the distance.

5. (a) Evaluate $\lim_{x \rightarrow \infty} \frac{2x}{9 - x}$

(b) Find the equations of the vertical and horizontal asymptotes of $f(x) = \frac{2x}{9 - x}$. Label them appropriately.

(c) Evaluate $\lim_{x \rightarrow 3^-} \frac{1 - x}{x^2 - 9}$

6. Sketch a graph with all of the following properties:

- (a) $f(0) = 0$, $f(3) = 2$ and $f(4) = 4$
- (b) $f'(x) = 0$ for $x > 3$
- (c) $f'(x)$ does not exist for $x = 3$
- (d) $f'(x) > 0$ for x in the interval $(-\infty, 3)$
- (e) $f''(x) > 0$ for x in the interval $(-\infty, 3)$

7. The current record for weight loss in a human is a drop from 487 pounds to 130 pounds over an 8 month period. Use the Mean Value Theorem to explain how we know that the loss exceeded 44 pounds a month at some time during the 8 month period.

8. Evaluate $\int \frac{x^2 + 1}{\sqrt{x^3 + 3x}} dx$.

9. If $f''(x) = 2\sqrt[3]{x}$, what is $f(x)$? **Be careful with constants!**