Math 31 - Fall 2003
Practice Problems 3

Name: KEY

Instructions:

1. Read carefully and answer **all** questions. Show all work on the exam in the space provided (you may use the backs of pages if necessary).
2. Where appropriate, place your final answer in the boxes provided.
3. Unless otherwise specified, **give exact answers.** For example, write $\sqrt{2}$ not 1.414, write π instead of 3.1415, write $\frac{1}{3}$ instead of 0.3333.
4. Check your answers carefully.
5. Remember to sign the Honor Pledge.

I have neither given nor received any unauthorized help on this exam and I have conducted myself within the guidelines of the University Honor Code.

Pledge: ______________________

1. Using Newton's Method, estimate the positive real root of $x^2-2=0$ to 4 decimal places. Take $x_1=1$.

$f(x) = x^2-2$.

$f'(x) = 2x$

Iteration scheme: $x_{n+1} = x_n - \frac{x_n^2-2}{2x_n}$

$\boxed{1.4142}$

$x_1 = 1$

$x_2 = 1 - \frac{1-2}{2\cdot 1} = 1 - \frac{-1}{2} = 1.5$

$x_3 = 1.5 - \frac{2.25-2}{3} = 1.5 - \frac{.25}{3} \approx 1.41666666666$

$x_4 \approx 1.41421568627$

$x_5 \approx 1.41421356237$

$\therefore$ the positive root is approximately 1.4142

2. Find the (global) max and min of $f(x) = sin^2 x$ on $(\pi, 2\pi]$.

$f'(x) = 2\sin x \cos x$

$f'(x) = 0$ for $\sin x = 0$ or $\cos x = 0$

$x = 2\pi$ or $x = \frac{3\pi}{2}$

$\boxed{\text{max value is } 1 \quad \text{min value is } 0}$

$f'(x)$ exists on $(\pi, 2\pi]$, so the only critical points are $x = 3\pi/2$, $x = 2\pi$.

$f(3\pi/2) = (-1)^2 = 1$

$f(2\pi) = 0^2 = 0$

$\therefore$ Max. value of f is 1, min. value of f is 0.

3. Given $f(x) = 2x^3 - 3x^2$

(a) Find the interval(s) on which $f(x)$ is increasing.

$f'(x) = 6x^2 - 6x = 6x(x-1)$

$\boxed{(-\infty, 0] \cup [1, \infty)}$

$f'(x) = 0$ for $x = 0, 1$.

$f'(x)$: + 0 − 1 +

INCR 0 DEC 1 INCR

f is increasing on $(-\infty, 0] \cup [1, \infty)$

(b) Find the interval(s) on which $f(x)$ is concave down.

$f''(x) = 12x - 6 = 6(2x-1)$

$f''(x) = 0$ for $x = \frac{1}{2}$

$\boxed{(-\infty, \frac{-1}{2})}$

$f''(x)$: $-$ | $+$

DOWN $\frac{1}{2}$ UP

$\therefore$ f is concave down on $(-\infty, \frac{-1}{2})$

(c) The function $g(x)$ has second derivative $g''(x) = \dfrac{8x^2(3x^2-1)}{(x^2+1)^3}$. Give the x coordinates of all the inflection points.

$g''(x) = 0$ for $8x^2(3x^2-1) = 0$

$x = 0$ or $x^2 = \frac{1}{3}$

$x = \pm \frac{1}{\sqrt{3}}$

$\boxed{x = \frac{1}{\sqrt{3}},\ x = \frac{-1}{\sqrt{3}}}$

$+$ | $-$ | $-$ | $+$

$-\frac{1}{\sqrt{3}}$ 0 $\frac{1}{\sqrt{3}}$

Inflection points occur at $x = \pm \frac{1}{\sqrt{3}}$

4. Find the point on the parabola $y^2 = 2x$ that is closest to the point (1,4). Hint: Consider the square of the distance.

$x = \frac{y^2}{2}$.

Consider $D = (x-1)^2 + (y-4)^2$

$= (\frac{y^2}{2} - 1)^2 + (y-4)^2$.

$\boxed{(2,2)}$

$\frac{dD}{dy} = 2(\frac{y^2}{2} - 1) \cdot y + 2(y-4)$

$= (y^2-2)y + 2y - 8$

$= y^3 - 2y + 2y - 8$

$= y^3 - 8$.

At $y = 2$, $4 = 2x$

$x = 2$.

$\frac{dD}{dy} = 0$ for $y = 2$.

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$\frac{dD}{dy}$: $-$ | $+$

2

so $y = 2$ is a min.

5. (a) Evaluate $\lim_{x\to\infty} \frac{2x}{9-x}$

$\lim_{x\to\infty} \frac{2x}{9-x} = \lim_{x\to\infty} \frac{2}{\frac{9}{x}-1} = \frac{2}{-1} = -2$

$\boxed{-2}$

(b) Find the equations of the vertical and horizontal asymptotes of $f(x) = \frac{2x}{9-x}$. Label them appropriately.

As in (a), $\lim_{x\to\infty} f(x) = -2$, there is a horizontal asymptote at $y=-2$.

$9-x=0$ for $x=9$: check $\lim_{x\to 9^+} f(x) = \lim_{x\to 9^+} \frac{2x \to 18}{9-x \to 0^-} = -\infty$

so there is a vertical asymptote at $x=9$

$\boxed{\text{VA: } x=9 \quad \text{HA: } y=-2}$

(c) Evaluate $\lim_{x\to 3^-} \frac{1-x}{x^2-9}$

$\lim_{x\to 3^-} \frac{1-x}{x^2-9} = \lim_{x\to 3^-} \frac{1-x \nearrow -2}{\underset{\downarrow\, 6}{(x+3)}\underset{\downarrow\, 0^-}{(x-3)}} = +\infty$

$\boxed{+\infty}$

6. Sketch a graph with all of the following properties:

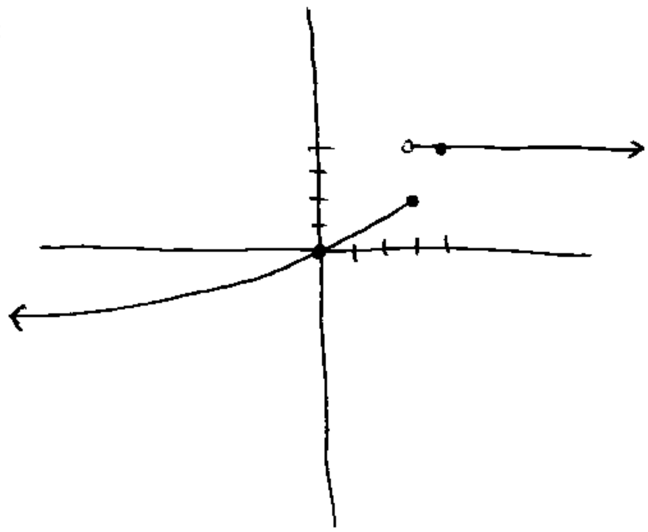
(a) $f(0) = 0$, $f(3) = 2$ and $f(4) = 4$

(b) $f'(x) = 0$ for $x > 3$

(c) $f'(x)$ does not exist for $x = 3$

(d) $f'(x) > 0$ for x in the interval $(-\infty, 3)$

(e) $f''(x) > 0$ for x in the interval $(-\infty, 3)$



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7. The current record for weight loss in a human is a drop from 487 pounds to 130 pounds over an 8 month period. Use the Mean Value Theorem to explain how we know that the loss exceeded 44 pounds a month at some time during the 8 month period.

let $w(t)$ = weight in pounds of this person at t months.
w is continuous on $[0,8]$, differentiable on $(0,8)$
[$w'(t)$ represents the change in weight over time]
By the Mean Value Theorem, there is at least one c in $(0,8)$ so that $w'(c) = \frac{w(8)-w(0)}{8-0} = \frac{130-487}{8}\ \frac{\text{lbs}}{\text{mo}}$
$= \frac{-357}{8}$ lb/mo. $= -44.625$ lbs/mo ; loss of 44.625 in one month
Thus the loss must have exceeded 44 pounds a month at some time during that period.

8. Evaluate $\int \frac{x^2+1}{\sqrt{x^3+3x}}dx$.

Let $u = x^3+3x$. $du = (3x^2+3)\,dx$

so $\int \frac{x^2+1}{\sqrt{x^3+3x}}dx = \int \frac{1/3}{\sqrt{u}}du = \frac{1}{3}\int u^{-1/2}du = \frac{1}{3}\cdot\frac{u^{1/2}}{1/2} + C$

$= \boxed{\frac{2}{3}\sqrt{x^3+3x} + C}$

9. If $f''(x) = 2\sqrt[3]{x}$, what is $f(x)$? **Be careful with constants!**

$f'(x) = \int 2\sqrt[3]{x}\,dx = 2\cdot\frac{x^{4/3}}{4/3} + C_1 = \frac{3}{2}x^{4/3} + C_1$

$f(x) = \int\left(\frac{3}{2}x^{4/3} + C_1\right)dx = \frac{3}{2}\cdot\frac{x^{7/3}}{7/3} + C_1x + C_2$

$\boxed{f(x) = \frac{9}{14}x^{7/3} + C_1x + C_2}$