Introduction to derivatives

BUSI 588, Final Exam Solutions, Fall 2010

Problem 1 - Oratio Dominica (25 points)

1. The Black-Scholes ABC.
   (a) According to the Black-Scholes model, the price of a put option on an asset with current value $S = 50$, annual volatility $\sigma = 0.2$, one-year maturity, and a strike of $K = 50$ would be \$2.81.
   (b) In order to replicate the put, Black-Scholes would suggest shorting -0.37 units of the underlying asset and lending \$21.08.
   (c) If the underlying asset would go down, Black-Scholes would suggest that we short more of the underlying asset, lending the proceeds.
2. Discuss whether you agree with the following two statements (keep the discussions separate, and use less than 50 words for each).
   (a) Disagree: "A call is equivalent to a put, coupled with a *long* position in the underlying asset and borrowing."
   (b) Agree. "A put is equivalent to shorting the underlying asset and lending, using the money from the cash account to buy more of the underlying asset as the stock price goes up."

Problem 2 - Wyoming (30 points)

1. Using the put-call parity we have that

$$C = P + S - \frac{K}{(1+r_f)^T} = 1.265 + 20 - \frac{20}{1.08} = 2.746$$

   Note that I use the 1-year rate obtained from the 1-year bond (8%, the 6-month bond has a yield of 6%).

2. By inspection, the calls with a strike of \$22.5 are mispriced with respect to the puts with that same strike, so there is a strong violation of the put-call parity. In particular, the puts are relatively expensive. The table below details a trade that generates arbitrage profits.

	Today	In one-year	
		$S^* < 22.5$	$S^* \geq 22.5$
Buy call	-1.65	0	$S^* - 22.5$
Short asset	$+20$	$-S^*$	$-S^*$
Lend PV of 22.5	-20.83	$+22.5$	$+22.5$
Sell put	$+2.55$	$-(22.5 - S^*)$	0
Net	0.07	0	0

3. Forward prices, in the absence of a convenience yield, should be given by

$$F_t = S(1+r_f)^t(1+c)^t$$

where r_f denotes the risk-free rate and c the storage costs. In the case under consideration, the forward prices for potatoes perfectly match the above formula with $c = 0$, whereas those for oranges match it using $c = 20\%$ (note that one needs to use $r_f = 6\%$ for the six-month maturity contracts, and $r_f = 8\%$ for the one-year contracts). Therefore, we learn that it is virtually costless to store potatoes, whereas it costs about 20 cents on the dollar to store oranges.

Problem 3 - Tom Brady (30 points)

1. The risk-neutral probabilities are

$$\hat{p}_u = \frac{1.1-0.8}{1.4-0.8} = 0.5; \qquad \hat{p}_d = 0.5.$$

The value of the GNE security is thus

$$V = \frac{0.5(100)+0.5(0)}{1.1} = 45.45.$$

2. Note that one can manufacture a "synthetic GNE" by trading in the stock and bonds. In particular, let Δ denote the number of units of the stock in our replicating portfolio, and let B be the dollar amount in the riskfree bonds. Then we can try to find Δ and B such that

$$\Delta 140 + B(1.10) = 100;$$

$$\Delta 80 + B(1.10) = 0.$$

My calculations suggest that $\Delta = 5/3$ and $B = -121.21$ do the job.

Since GNE security is the market is underpriced, we buy it and sell the replicating portfolio.

	Today	d state	u state
Buy GNE	−41	0	100
Sell 5/3 units of stock	+166.67	−133.33	−233.33
Lend \$121.21	−121.21	+133.33	+133.33
Net	+4.46	0	0

3. One can find the risk-neutral probabilities in the trinomial world by using the values of the traded securities.

$$\frac{80\hat{p}_d + 105\hat{p}_m + 140\hat{p}_u}{1.10} = 100;$$
$$\frac{0\hat{p}_d + 5\hat{p}_m + 40\hat{p}_u}{1.10} = 16.50;$$

These two equations, together with $\hat{p}_d + \hat{p}_m + \hat{p}_u = 1$, yield $\hat{p}_u = 0.4339$, $\hat{p}_m = 0.1586$, and $\hat{p}_d = 0.4075$. Thus, the value of the GNE security is $V = 0.4339(100)/1.1 = \$39.45$.

Problem 4 - FREESBIE (30 points)

1. Kim Lee's argument ignores the fact that the volatility of the stock makes these options more valuable than their current intrinsic value. Using the Black-Scholes model one can estimate the value of each of the options, with strikes $K = 10$, $K = 15$, and $K = 20$, at 9.63, 7.09, and 6.68 respectively. Summing the option grants, I would estimate the cost to the firm to run around \$3.382m.

2. The Δ of Patricia's portfolio is

$$\Delta = 2000(0.8142) + 4000(0.7263) = 4533,$$

so she could hedge by shorting 4533 units of Eenran's stock.

3. If the traded x units of the stock, and y units of the options with $t = 1/2$ maturity, then the delta and vega of her portfolio would be

$$\Delta = 4533 + x + y(0.4617)$$

$$\text{vega} = 2000(8.58) + 4000(13.77) + y(5.19)$$

Making the above expressions equal to zero, one concludes that she should buy $x = 1886.67$ units of the underlying asset and sell $y = -13906$ call options.

She would need to adjust her position dynamically. If Eenran's stock went down to \$16 over the next few days her hedging portfolio would have to be $x = 717.71$ and $y = -19742.08$, i.e. she would have to sell almost 6000 more calls, and more 1000 units of Eeran's stock.

Problem 5 - Love Wigs Vans and Buttons (35 points)

1. The value of the debt contract is

$$D_S = 600 - C(600, K = 500, \sigma = 0.15, T = 2, r_f = 5\%) = 448.70$$

which carries an implied credit spread of 56 basis points.

2. The value of the debt contract is

$$D_S = 650 - C(650, K = 500, \sigma = 0.175, T = 2, r_f = 5\%) = 449.17$$

so although the debtholders dislike the increase in volality, the increase in asset value compensates and they would not veto the project.

3. The value of the junior debt would be

$$D_J = C(650, K = 500, \sigma = 0.175, T = 2, r_f = 5\%) - C(650, K = 500 + F_J, \sigma = 0.175, T = 2, r_f = 5\%)$$

Equating the above expression to 40 and solving for F_J one finds $F_J = 50.70$. The credit spread for this bond issue would be around 7.6%.

4. The key to this question was to realize that the expected return from an investors perspective would *not* be equal to the yield on the bonds.

   Senior debt can be thought of being equal to $A - C(K = 500)$, so its risk should be

$$\beta_S = \frac{A}{D_S}\beta_A - \frac{C}{D_S}\beta_C = \left(\frac{A}{D_S} - \frac{C}{D_S}3.1\right)\beta_A = 0.08\beta_A$$

   where I use $\beta_C = \Omega_C \beta_A = 3.1\beta_A$.

   For the junior debt, the calculations are similar.

$$\beta_J = \frac{C_1}{D_J}\beta_{C1} - \frac{C_2}{D_J}\beta_{C2} = 0.978\beta_A$$

   where C_1 and C_2 refer to the call options with strikes of $K = 500$ and 550.7 respectively.

   Therefore, the senior debtholders would be receiving an excess expected return that is 0.08 times that of the assets of the firm, i.e. an expected return of 6.24%, whereas the junior debtholders will get a expected return of 19.67% on their investment.

Problem 6 - Messi diamonds (30 points)

1. The profits from producing right away will be given by (in 1000s)

$$\pi_0 = 2500(10) - 12000 - 10(1200) = 1000$$

   where as if you wait

$$\pi_t = \max(2500(10) - 12000 - 10(P_d), 0) = \max(13000 - 10(P_d), 0) = 10\max(1300 - P_d, 0)$$

   i.e. the profits would be those of 10,000 American put options on diamonds with a strike of \$1300. The value of these put options is 3.69m, so you are better off waiting.

2. Profits are now given by

$$\begin{aligned}\pi_t &= \max\left(10\left(1000 + \frac{7}{6}P_d\right) - 12000 - 10P_d, 0\right)\\ &= \max\left(\frac{10}{6}P_d - 2000, 0\right)\\ &= \frac{10}{6}\max(P_d - 1200, 0)\end{aligned}$$

   i.e. the payoffs of the project are those of 10000/6 call options with a strike of \$1200.

   You would clearly wait, since profits would be zero today. The value of the project can be estimated by computing the value of an European call option on diamonds (note for non-dividend paying assets, such as diamonds, it will never be optimal to exercise early). From the put-call parity we have $C = 293.1 - 1200 + 1200/1.05^2 = 404.6$, so the value of the project would be $V = 10000(404.6)/6 = \$674,441$.