Introduction to derivatives

BUSI 588, Final Exam, Fall 2011

Name: _____ **ID#:** _____ **Section#:** _____

- Please include the front page of the exam stapled with the rest of your exam answers. Turn in page 2 separately. Also, please return separately pages 3–7 of the exam.
- This exam contains a total of five questions of which you are to answer *all five*.
- You have 2.5 hours to answer the questions, plus an additional 15 minutes to format and/or print your exam (i.e. make sure things print out nicely, your solutions are in order, ...).
- The exam is closed book. You may not consult your notes, readings, or any material distributed in class. No collaboration is allowed. You may not use any pre-programmed macros or spreadsheets, other than a plain vanilla Black-Scholes calculator.
- You are free to submit your answers in any "hard-copy" combination of the following three forms: (1) handwritten sheets, (2) a single word-file, and (3) a single excel file (a workbook with possibly several spreadsheets). If I were to do this exam, I would format files in such a way that I can organize them neatly in a short amount of time. Perhaps handwritten plus spreadsheets, perhaps just spreadsheets.
- If you are using a computer, please please please do save your files frequently.
- Turn in your answers in paper sized 216mm by 279mm (if you print your answers they will be in 216 by 279mm paper). Do not use the exam itself to answer the questions.
- The honor code applies.
- Try to have some fun.
- Please sign below to confirm you have read and understood the above instructions.

_____

Question #	**Maximum Score**	**Actual Score**
1. Oratio Dominica	35	
2. Fifty-cents	25	
3. Trader Smurf	30	
4. Goldensacks Bank	30	
5. Johannes Brahms	30	
Total score	**150**	

Team evaluation form.

Name: _____

List the other members in your study group. Evaluate their contribution to the group assignments. Assign each one a score based on the following scale. (Responses will be mean-adjusted so it does not benefit anyone to inflate scores.)

3 = group member did significantly more work than the average group member
2 = group member did roughly the same amount of work as the average group member
1 = group member did significantly less work than the average group member
0 = group member did not participate in group work

Group member	Grade (0-3)
1. Me	
2.	
3.	
4.	

If you assigned a group member a score of zero, briefly explain.

Problem 1 - Oratio Dominica (35 points)

1. The Black-Scholes ABC.

   (a) According to the Black-Scholes model, how would you replicate a put option on an asset with current value $S = 50$, annual volatility $\sigma = 0.2$, one-year maturity, and a strike of $K = 50$. Assume the annual risk-free rate is 5%?

   (b) How would the replicating portfolio change if the underlying asset went down?

2. Assume throughout this problem that the term structure is flat at 5%.

   (a) The Bogotá Stock Exchange (BSE) index is currently trading at 1,500. You find out that forward contracts on the BSE with one year maturity are trading at a forward price of 1529.13. Can you estimate the the dividend yield of the BSE index?

   (b) You actually find out more about derivative markets in Colombia. Aureliano Buendía, running a financial firm going for the name of Macondo Inc, quotes the following prices on options on aluminum with one year maturity.

	Calls		Puts	
Strike price	Bid	Ask	Bid	Ask
$K = 45$	8.25	8.35	1.15	1.20
$K = 50$	5.15	5.25	2.80	2.90
$K = 55$	2.95	3.05	5.05	5.15

   Aluminum is currently trading at \$50. Is there an arbitrage opportunity? How would you take advantage of it?

Problem 2 - Fifty-cents (25 points)

Fifty–cents, a violist–turned–rapper, is desperately trying to purchase some put options on Eminem stock. He would like to essentially eliminate the downside risk of his portfolio, which at the moment is long Eminem. Unfortunately, options are not traded on Eminem, so it is not that simple to achieve the downside protection Fifty–cents is yearning for.

You have been hired to offer Fifty–cents some advice as to what investment strategy to follow in order to get him some downside protection. Fifty–cents argues that he does not mind spending some money in order to guarantee himself that his stock holdings do not drop in value below \$40, i.e. he would like to be able to sell his shares for at least \$40 in three months time.

The current price of Eminem is \$50, and you know that the market expects this stock to up by 40% or down by 40% over each of the following months (with equal probabilities). The evolution of Eminem's stock price over the next three months is thereby given by the following binomial tree:

50	70	98	137.2
	30	42	58.8
		18	25.2
			10.8

There is a riskless asset that pays a riskless return of 1% each month, and it is expected to stay at 1% over the next three months.

1. Assess what you would consider to be a fair price for a three–month put option on Eminem with a strike of \$40.

2. How can Fifty–cents achieve the downside protection he is looking for by trading dynamically on the Eminem stock and cash? Be explicit about the amount of shares Fifty–cents needs to hold at each point in time, as a function on the evolution of Eminem's stock

   Note: I would like you to tell Fifty–cents how much stock and bonds to hold at $t = 1$ in two different states, and at $t = 2$ in three different states.

3. Under the given assumptions, what is the expected return on Enimem's stock? Under the assumptions of the CAPM model, what would you assess the systematic risk of Eninem stock to be? What would you assess the systematic risk of the put on Eninem to be?

   Hint: This is kind of a tongue–twister (worth less points than much of the exam coming up). Qualitative answers suffice. You may want to assume that the market portfolio has a higher expected return than the risk–free rate (i.e., the market risk–premium is positive).

Problem 3 - Trader Smurf (30 points)

Trader Smurf has just finished a very busy day. After much trading in the CBOE, he has closed the day with the following inventory of options on mushrooms: a long position in 100,000 calls with a strike of $K = 100$ and 6-month maturity, a short position in 80,000 puts with a strike price of $K = 90$ and 3-month maturity, and a long position in 50,000 puts with a strike of $K = 95$ and 6-month maturity. Trader Smurf also has about $500,000 in cash after all this hard work, earning the risk-free rate of interest (5%).

Mushrooms, a very liquid commodity in the world of Smurfs, are currently trading at $96. The volatility of monthly returns from investing in mushrooms over the recent past has been 8.66%.

1. Estimate the value of Trader Smurf's portfolio using Black-Scholes.
2. How can Trader Smurf hedge his position by trading on the underlying asset (mushrooms)?
3. If Trader Smurf could trade both on the underlying asset (mushrooms) and the at-the-money options that he owns (those with $K = 95$), what trading strategy would you recommend in order to be hedged against movements on the underlying asset?
4. If Trader Smurf could trade both on the underlying asset (mushrooms) and the at-the-money options that he owns (those with $K = 95$), what trading strategy would you recommend in order to be hedged against movements on the underlying asset and movements in volatility of mushroom returns?

Problem 4 - Goldensacks Bank (30 points)

Wolfgang, senior loan offier at Goldensacks Bank, was considering the decision of whether to give a large loan to Tar Heels Chess Corp, an up and coming firm specialized in board games. After some analysis, Wolfgang had concluded that the value of the firm's current assets (namely the present value of all future cash flows, with the exception of the project discussed below) is around \$50m. The value of the firm will fluctuate due to the nature of the business and general market conditions, and Wolfgang knows that 50% is a good estimate for the volatility of the returns in the firm.

The firm has outstanding debt with a face value of $F = 30$ and two year maturity. This debt includes a covenant specifying that all future debt issues will be junior to theirs, i.e. they will have to get fully paid before any other debt gets paid. This senior debt is held by a hedge fund going by the name of Save-the-whales. Senior debtholders also have the right to veto any new investment in Tar Heels Chess Corp.

The project that the firm is considering to fund requires a \$8m investment, which will be financed by Goldensacks Bank with junior debt with a two year maturity. The project is a profitable investment opportunity, and after going through the usual due-diligence, all parties (Wolfgang, the firm and the principals from Save-the-whales) agree that its NPV is about \$4m, i.e. the firm will increase its value from \$50 to \$62m after the investment.

Assume that the risk-free rate is 10%, and ignore taxes and/or other frictions.

1. What is the value of the senior debt prior to the firm making the investment decision? How about its credit spread?[1]

2. What is the value of the senior debt if Wolfgang decides to finance the investment? Will the current senior bondholders, Save-the-whales, want to veto Wolfgang's investment?

3. What is the lowest rate r_d (or equivalently face value F) that Wolfgang would consider setting on the junior bond? Note that it is a risky bond, so r_d should be higher than the risk-free rate.

4. If you were the CFO of the firm, would you see any value in eliminating the covenant that forces Wolfgang's debt to be junior? Why? How much would you be willing to pay to the Save-the-whales fund in order to have the covenant removed? Be specific about the different seniority rules you consider plausible (i.e. the debt from Save-the-whales being junior to the Goldensacks debt, both debt issues having equal seniority with some sharing rule in the event of default, ...).

[1] The credit spread is defined as the difference between the yield to maturity on the bond and the risk-free rate. The yield-to-maturity r_d is defined by $D = F/(1 + r_d)^T$, where D denotes the market value of the bond, F its face value, and T its time to maturity.

Problem 5 - Johannes Brahms (30 points)

Johannes Brahms is considering to sell bronze bracelets with the sweet face of Clara Schumann. His compositions have faired well - but it is time to make some real money. Franz Schubert, Johannes' only friend with some marketing kills, has informed him that he can sell these for $400 each to about 100,000 people. Each bracelet contains 20 grams of bronze.

Currently, the price of bronze is $8.50 per gram. The cost of the bronze is not the only variable cost, Johannes estimates he will need to pay the bracelet engravers about $150 per chain. The fixed costs of starting off the project are $5m. Assume that if Brahms takes the project at date t the cash flows (both costs and revenues) will occur virtually instantaneously, so no discounting of the revenues is needed once you decide to take on the project (at date t).

Johannes has to make a decision on whether to release this product to the market right now, or wait. You think that the most you can wait is five years, since after that the glamour of the Schumman's will have disappeared.

Assume that the riskless interest rate is 10%, and that there are no taxes. The expected return on bronze is 25%, and its volatility is 30% (both in annual terms).

1. How much is the project worth if you invest immediately?
2. How much is the project worth if you decide to invest in 5-years time?
3. How much is the project worth if you decide to invest at any point in the following 5-years? How does the decision of whether to exercise early at date t depend on the price of bronze at date t?

   Hint: use the binomial model. Formula refresher: the standard calibration of the binomial model uses $u = e^{\sigma\sqrt{T/n}}$ and $d = 1/u$ as the up and down moves, where σ denotes the volatility of the stock, T the maturity date for the option, and n the number of steps in the tree. Given time constraints a 5-step tree would suffice for the purposes of this fun exam.