Optimal contracts with privately informed agents and active principals*

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Abstract

This paper considers an optimal contracting problem between an informed risk-averse agent and a principal, when the agent needs to perform multiple tasks, and the principal is active, namely she can influence some aspect of the agency relationship on top of the contract itself (i.e. capital budgets, task assignments). The paper shows asymmetric information makes incentives and investment decisions substitutes for the principal under rather general conditions. This result yields novel implications for the capital budgeting literature that studies informational problems. In particular the nature of the private information, e.g. whether it affects the marginal value of investment, the value of the agent's effort choice, or potential interactions between effort and investment, plays a crucial role in the optimal contracts. The paper also shows that asymmetric information considerations yield a positive relationship between the noise of the performance measure and the action scope of managers.

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1 Introduction

Managerial compensation and the allocation of productive tasks to agents in an organization are among the most important control variables investors have at their disposal to induce effort and optimal capital investments. The large principal-agent literature has successfully dealt with many aspects of this problem: from perk consumption to moral hazard and adverse selection, economists have developed many insights into the properties of optimal contractual arrangements in agency settings. The novel aspect of this paper is to analyze a more general version of the standard principal agent framework which allows the principal to take an active role in the agency relationship, in addition to writing contracts. Examples of the types of actions the principal may take include: a capital allocation rule (Bernardo, Cai, and Luo, 2001), transfer-pricing decisions among divisions (Besanko and Sibley, 1991), control of the set of projects the agent has access to (Hirshleifer and Suh, 1992; Sung, 1995), task assignment decisions (Holmstrom and Milgrom, 1991), setting part of the firm's strategy (Dow and Raposo, 2002), design of the compensation measure (Feltham and Xie, 1994), auditing and capital allocation (Harris and Raviv, 1996), and/or a productive effort choice that complements the actions of the agent.

The paper considers a standard agency problem where, on top of moral hazard considerations, the agent possesses some private information about his ability or about the profitability of the division he controls. The principal can influence the agent's decisions by the compensation package she offers to the agent, and by other actions she may take that indirectly impact the agency relationship. A crucial aspect of the model is the active role played by the principal besides setting wages. In particular, I allow the principal to offer the agents a menu of wage-action combinations from which they can choose, where the action can be any of the aspects of the agency relationship mentioned above. The set of implementable mechanisms is enlarged significantly once the principal can screen agents through both the contracts offered and the additional actions that impact the agency relationship, compared to models where principals are inactive (i.e. they simply set wages). For example, standard monotonicity properties of the contracts can be violated without violating the usual incentive compatibility contraints. More importantly, the paper shows that, under fairly general conditions, asymmetric information makes wages and other actions substitutes for the principal. This substitutability between incentives and other complementary actions is driven by the efforts of the principal to reduce the informational rents earned by agents under the equilibrium contracts.

These results are then applied to two different models in sections 4 and 5: one in which both the agent and the principal take a one-dimensional action, and one in which the agent's effort choice is multi-dimensional. The first model, presented in section 4, is interpreted as an agency relationship in which the agent exerts effort to increase expected output, whereas the principal offers each of the agents a menu of capital budgets and salaries. The point of departure from the literature is to consider a more general agency setting, since the models studied in the literature are quite specialized. The main result is to show that the nature of the private

information, e.g. whether it affects the marginal value of investment, the value of the agent's effort choice, or potential interactions between effort and investment, may change standard features of the optimal contracts. The substitutability created by private information can, for example, make incentives and capital bugets be non-monotonically related. The paper solves analytically for a set of special cases, and it gives conditions under which different comparative statics will prevail, thereby generating a rich set of empirical implications.

The multi-task feature of most real-world incentive problems has sprung substantial recent interest (Holmstrom and Milgrom, 1991). The second model, discussed in section 5, adds to this literature by analyzing the effects of private information in the task assignment decision. Action restrictions are shown to be optimal in a third-best world in a (weak) generic sense, even for cases in which the second-best solution does not involve any restrictions on the agent's action set. Therefore private information considerations bring new trade-offs to the optimality of such action restrictions, thereby complementing their generic optimality shown in Holmstrom and Milgrom (1991). Moreover, the distortion in the optimal contingent compensation from the second-best to the third-best solution[1] is not necessarily downwards, as in the case where the principal is inactive: the optimal contracts can actually be more aggresive in the presence of private information than in the case of symmetrically informed parties. The paper also develops a new set of empirical predictions with respect to action scope (number of tasks assigned to managers) and the underlying contracting variables. When the noise in measurable output is large, the principal will optimally allow the agents to tackle as many tasks as possible; whereas when the noise is small, action restrictions will generally be optimal.

There is a large literature in finance, started with the seminal Leland and Pyle (1977) paper, which analyzes the effects of asymmetric information in financial contracting settings. The analysis in this paper follows the tradition in the capital budgeting literature started with Harris, Kriebel, and Raviv (1982) and Antle and Eppen (1985).[2] The model in section 4 is formally very close to that in Bernardo, Cai, and Luo (2001), who also jointly model optimal capital budgeting and compensation under private information and moral hazard, but allowing for more general relationships between information, investment, effort and output.[3] The paper also contributes to the moral hazard literature where there is an investment decision on top of an effort elicitation problem,[4] by studying the effects of asymmetric information on the optimal contracts and capital budgeting decisions. Also closely related is the literature that discusses

[1] As usual in the literature, I will refer to the case where the action taken by the agent is unobservable but there is no private information as the second-best world, and the case where on top of the unobservability of the action there is adverse selection as the third-best world.

[2] For some recent work in this area see Milbourn and Thakor (1996), Bernardo, Cai, and Luo (2001), Stoughton and Zechner (2003), Bernardo, Cai, and Luo (2004) and Sung (2005). The literature on capital budgeting with other types of frictions is also large - see Harris and Raviv (1996), Harris and Raviv (1998), Core and Qian (2002), Goldman (2002) and the references therein.

[3] See DeMarzo and Duffie (1997),Chemmanur and Fulghieri (1997), DeMarzo (2005), He (2005) for some recent work in the signalling literature in which the informed party can signal her quality through multiple actions, very close in spirit to the main contribution of this paper. Daniel and Titman (1995) surveys prior literature related to this class of models in finance, including an analysis with multiple actions.

[4] See Hirshleifer and Suh (1992), Hermalin (1993), Sung (1995).

deviations from the standard NPV rule due to contracting imperfections (e.g. Berkovitch and Israel, 2004). The optimal capital budgeting schemes in the paper depart from naive NPV rules due to the distortions that arise both due to asymmetric information and moral hazard considerations. The analysis of the task allocation decisions in section 5 is closely related to that of Holmstrom and Milgrom (1991), who ignored in their analysis private information considerations. Other recent papers that study the potential optimality of action restrictions are Szalay (2002) and Carrillo (2002). Szalay (2002) looks at the optimality of (ex-post) task allocation decisions from the point of view of motivating effort decisions ex-ante. Carrillo (2002) studies a model driven by career concerns of the managers, and the task assignment decisions play the role of screening devices. In contrast, this paper's driving forces are the trade-off between production inefficiencies and informational rents considerations.

The rest of the paper is organized as follows. Section 2 presents the model and solves for the optimal contracts and actions in the case with symmetric information. Section 3 characterizes the set of implementable mechanisms and reduces the principal's problem to a simple pointwise maximization. Further results are contained in sections 4 and 5, where the previous results are specialized to the two settings described above. Section 6 considers other extensions of the basic model. All proofs are relegated to the appendix.

2 The model and benchmark solutions

In this section I describe the model and relate it to those in the literature. The solutions when there is not adverse selection nor moral hazard (first-best world) and when there is not adverse selection (second-best world) are then discussed.

2.1 Elements of the model

Output is driven by both the agent's and principal's actions, and is also dependent on the private information of the agent. I assume that the principal is risk-neutral. She accrues benefits $B(e, a, \theta)$ from the interaction with the agent and their productive actions, where $e \in E \subset \mathbb{R}^n$ is an action taken by the agent, and $a \in A \subset \mathbb{R}^m$ is an action conducted by the principal. The action a has a personal cost for the principal of $c_P(a)$. The agent has CARA preferences with risk-aversion parameter R, and incurs an additively separable cost given by $c(e, \theta)$. Define $C(e, a, \theta) \equiv c(e, \theta) + c_P(a)$.

The variable θ indexes the agent's type, privately known by the agent. For simplicity in the exposition I assume that there is a continuum of types with total mass normalized to one, namely I assume that $\theta \in [\underline{\theta}, \bar{\theta}] \equiv \Theta$, and that the principal knows that in the population θ is distributed according to the distribution function $F(\theta)$ with associated density $f(\theta)$. The inverse of the hazard rate $H(\theta) \equiv (1 - F(\theta))/f(\theta)$ is assumed to be bounded and monotonic.

There is a contractible signal given by

$$Y = \mu(e, a, \theta) + \sigma\epsilon; \tag{1}$$

with $\epsilon \sim N(0, 1)$. The principal and the agent can only use linear functions of Y as contracts, which I will denote by $w \equiv \alpha + \beta Y$.[5] The agent's certainty equivalent utility can be expressed as

$$U_A(e, a, \alpha, \beta, \theta) = \alpha + \beta\mu(e, a, \theta) - \frac{R}{2}\beta^2\sigma^2 - c(e, \theta). \tag{2}$$

The principal's utility can analogously be written as

$$U_P(e, a, \alpha, \beta, \theta) = B(e, a, \theta) - \alpha - \beta\mu(e, a, \theta) - c_P(a). \tag{3}$$

The following assumptions[6] will be referred to as "standard assumptions:"

- The variable θ orders the types unambiguously by their efficiency. In particular I assume $C_\theta < 0$ and $C_{\theta e} \leq 0$,[7] i.e. higher values of θ are associated with lower cost and lower marginal cost for the agent's effort choice.
- The function $\mu(\cdot)$ is assumed to satisfy $\mu_\theta \geq 0$, i.e. higher values of θ are associated with higher values of the signal. Moreover, $\mu_{\theta e} \geq 0$, i.e. agents with higher values of θ have a higher marginal value of effort.
- The marginal value of the principal's action is increasing in θ, i.e. $\mu_{\theta a} \geq 0$.

These assumptions are neither more restrictive nor much more general than those made in the literature: they are the standard Spence-Mirrlees single-crossing property when the signal and cost functions are modeled (separately) to be dependent on θ. Section 6 discusses how generalizations of these assumptions could be addressed. For now it is just worth noticing that since the main results of the paper are statements regarding weak generic properties of the model,[8] these "standard assumptions" are not necessary for the results.

By the revelation principle (e.g. Myerson, 1979) the principal can restrict attention to a direct mechanism of the form $\{\alpha(\theta), \beta(\theta), a(\theta)\}$, where α is the fixed wage offered, β the bonus

[5] If the agent were risk-neutral the restriction to linear contracts would be without loss of generality by standard arguments (see Laffont and Tirole, 1986). This restriction to linear contracts has been shown to be without loss of generality in a dynamic version of this model in Holmstrom and Milgrom (1987) in the pure moral hazard case. Sung (2005) shows that introducing private information does not destroy the optimality of linear contracts in this type of continuous-time setting. The only result that depends on risk-aversion is Corollary 3 in section 5.

[6] I use the notation $f_x(\cdot)$ to denote the derivative of the function f with respect to x. For functions of one single real variable I will also use the notation $f'(x)$. For notational simplicity I omit the arguments of the functions when there is no room for ambiguity.

[7] Note that $C_{e\theta}$ is a vector, so $C_{e\theta} < 0$ means that $\frac{\partial^2 C}{\partial\theta\partial e_i} < 0$ for $i = 1, \ldots, n$.

[8] A property is said to hold in a weakly generic sense if there is an open set of the model's parameters such that this property holds.

coefficient, and a the action that the principal controls. The optimal menu of contracts solves

$$\max_{\{\alpha,\beta,a\}} \quad \int_{\underline{\theta}}^{\bar{\theta}} \left(-\alpha(x) - \beta(x)\mu(e(x), a(x), x) + B(e(x), a(x), x) - c(a(x))\right) f(x)dx \tag{4}$$

such that

$$U_A(e(\theta), a(\theta), \alpha(\theta), \beta(\theta), \theta) \geq \bar{u}, \qquad \forall \theta \in \Theta; \tag{5}$$

$$U_A(e(\theta, \theta), a(\theta), \alpha(\theta), \beta(\theta), \theta) \geq U_A(e(\theta, \hat{\theta}), a(\hat{\theta}), \alpha(\hat{\theta}), \beta(\hat{\theta}), \theta), \qquad \forall \theta, \hat{\theta} \in \Theta; \tag{6}$$

$$e(\theta, \hat{\theta}) \in \arg\max_{e} U_A(e, a(\hat{\theta}), \alpha(\hat{\theta}), \beta(\hat{\theta}), \theta), \qquad \forall \theta, \hat{\theta} \in \Theta. \tag{7}$$

The constraint (5) is the standard individual rationality constraint. It is assumed that the reservation utility of the agent is $\bar{u}$, independent of the agent's type. Constraint (6) is the incentive-compatibility constraint, which states that the agent is better off announcing his true signal and receiving the allocation $\{\alpha(\theta), \beta(\theta), a(\theta)\}$ than announcing a different type $\hat{\theta}$ when his true type is θ. Lastly, equation (7) requires the effort choice for the agent to maximize his expected utility.

The Holmstrom and Milgrom (1987) setting is obtained when θ has a degenerate distribution (i.e. there is no private information), $n = 1$ and $A = \emptyset$. For n arbitrary and $A = \{I_i\}_{i=1}^n$, with I_i denoting the indicator variable taking on the value 1 if task i is assigned to the agent, a special case of the model discussed in Holmstrom and Milgrom (1991) is recovered.[9] Special cases of $n = m = 1$ are discussed in Besanko and Sibley (1991), and Bernardo, Cai, and Luo (2001), who consider settings where on top of the standard contractual agreement the principal is *active*, i.e. takes an action that affects some of the parameters in the agency model (transfer pricing policy in Besanko and Sibley (1991) and capital allocation in Bernardo, Cai, and Luo (2001)). The two novelties with respect to this literature are the simultaneous consideration of multi-task agency problems with private information when the principal can also influence the agency relationship in a fairly general agency setting.

The model just described is quite general. Nevertheless, some simplifying assumptions have been made, mostly for clarity in the exposition: σ^2 is assumed to be independent of e, a and θ; $c(\cdot)$ and $c_P(\cdot)$ only depend on the respective effort levels of the agent and the principal, and the private information of the agent affects only the mean value of output and the cost function of the agent. Section 6 discusses the effects of these assumptions on the paper's results.

2.2 First-best and second-best benchmarks

In a first best world, where both e and θ were observable, the principal's problem is reduced to the maximization of (4) such that (5) is satisfied. The contract would stipulate paying the agent a fixed wage, since imposing risk on the agent is suboptimal. The optimal actions would

[9] Holmstrom and Milgrom (1991) deal with the more general case where the principal has k contractible signals, whereas I limit attention to contracting environments with one signal.

be given by

$$\max_{e,a} \quad \Phi^{\text{FB}}(e,a,\theta) \equiv B(e,a,\theta) - C(e,a,\theta).$$

When effort e is unobservable, but θ is common knowledge, the principal maximizes (4) such that (5) is satisfied, and the agent's action solves (7). This reduces to the model studied by Holmstrom and Milgrom (1991), who showed the first-order approach is valid in this context. Using (7) the optimal action for the agent is given by

$$\beta\mu_e(e,a,\theta) = c_e(e,\theta). \tag{8}$$

This first-order condition defines the function $e(\beta,a,\theta)$, which maps an arbitrary menu/signal pair to the effort chosen by the agent. I will also use the notation $e(\theta,\hat{\theta})$ introduced in (7) as referring to $e(\beta(\hat{\theta}),a(\hat{\theta}),\theta)$. Using (8) one can eliminate the effort choice from the principal's problem and maximize over β the second-best surplus

$$\max_{\beta,a} \quad \Phi^{\text{SB}}(\beta,a,\theta) \equiv B(e(\beta,a,\theta),a,\theta) - C(e(\beta,a,\theta),a,\theta) - \frac{R}{2}\sigma^2\beta^2, \tag{9}$$

to find the optimal contract. The above equation captures the standard features of the classic moral hazard problem: now the principal discounts the value of the actions e and a by the risk they impose on the risk-averse agent.

The following example presents a simple setting that will illustrate the main trade-offs with respect to action restrictions in a second-best world.

Example 1. Consider the linear-quadratic case given by $B(\cdot) = e^\top b$, $\mu(e) = e^\top\mu$ and $c(e) = e^\top Ce$, where $b,\mu \in \mathbb{R}^n$ are some fixed vectors, $e \in \mathbb{R}^n$ is the effort choice of the agent, and $C \in \mathbb{R}^{n\times n}$ is a symmetric matrix, as in Holmstrom and Milgrom (1991). Let I be the diagonal matrix with a typical element the indicator variable I_i as defined before. It is straightforward to show that the optimal effort is given by $e = \beta C_I^{-1}\mu$, where C_I^{-1} denotes the inverse of the relevant cost matrix with the task assignment I.[10] Using this expression to eliminate e from the principal's objective function and maximizing over β we get that the optimal contract is given by

$$\beta = \frac{\mu^\top C_I^{-1} b}{R\sigma^2 + \mu^\top C_I^{-1}\mu}. \tag{10}$$

The optimal task allocation solves

$$\max_{I_i} \frac{\left(\mu^\top C_I^{-1} b\right)^2}{R\sigma^2 + \mu C_I^{-1}\mu}. \tag{11}$$

From the above equation it is easy to give conditions under which it will be optimal to

[10] Let $\hat{C}_I$ be the submatrix of C composed of the rows and columns of C such that $I_i = 1$. Then C_I^{-1} has 0's for all elements j,k such that $I_j = 0$ or $I_k = 0$, and the submatrix for which $I_j = I_k = 1$ is given by $\hat{C}_I^{-1}$.

restrict a set of actions. For example, if $n = 2$ and C is the identity matrix, then $I_1 = 0$ is optimal as long as

$$(\mu_2 b_2 \mu_1)^2 \geq (R\sigma^2 + \mu_2^2)((\mu_1 b_1)^2 + 2\mu_1\mu_2 b_1 b_2).$$

Note that for $R = 0$ the second-best solution coincides with the first-best if $n = 1$, but this is not the case in general.[11] To see this just note that as $b_2 \uparrow \infty$ the above condition is automatically satisfied, i.e. it is optimal to restrict access to action 1. The intuition is that as the value of action 2 grows, it becomes optimal to avoid having the agent divert attention from action 2 to action 1.[12] Finally note that if $b_i = \mu_i$, i.e. when the signal is just a noisy version of output, it will never be optimal to restrict the agent from any task.[13] □

3 Implementability and optimal contracts

In this section the main results needed to characterize the optimal mechanism are discussed. The method of solution to the problem is standard: I use the first-order approach[14] to express the optimal effort level as a function of the contract, thereby reducing the problem to one in which there is no agency relationship and the principal has to design a mechanism to induce a particular allocation rule. It turns out that under the assumptions of CARA preferences, linear contracts and Gaussian returns the setting reduces to one with quasi-linear preferences. The model is thus reduced to a multi-dimensional allocation rules mechanism design problem with a one-dimensional information variable.[15] This section extends the results in the literature to the agency model introduced above.

The existence of private information conditions the set of allocations, combinations of contracts and actions, that the principal can achieve. A mechanism $\{\alpha(\theta), \beta(\theta), a(\theta)\}$ is said to be implementable if it satisfies (6). The next proposition gives necessary and sufficient conditions for implementability in this model.[16]

[11] This seems to be a novel observation: typically moral hazard models exhibit the property that efficiency is not affected if there are no risk-sharing considerations. For $\mu = b$ the optimal contract has $\beta = 1$, i.e. the agent becomes the residual claimant. If $\mu \neq b$ the second-best will differ from the first-best outcome even under risk-neutrality in a generic sense.

[12] Note that $e_2 = (b_1\mu_1 + b_2\mu_2)\mu_2/(\mu_1^2 + \mu_2^2)$ in a second-best world with both tasks assigned and $e_2 = b_2$ when one task 2 is allowed.

[13] This particular result does depend on the assumption that C is equal to the identity matrix. As shown in Holmstrom and Milgrom (1991) it is possible to have optimal action restrictions in this setting if the cross-partials in the cost function are non-zero.

[14] It is worth noticing that under the restriction to linear contracts the first-order approach to moral-hazard problems is always valid, i.e. the second-order conditions are immediately satisfied.

[15] For a survey of the growing multi-dimensional mechanism design literature see Rochet and Stole (2000). García (2005) gives the set of necessary and sufficient conditions for implementability in the type of model considered here in the case without an agency relationship.

[16] The above characterization of the set of feasible mechanisms will be sufficient for the capital budgeting application of section 4, but the restriction to differentiable mechanisms is with loss of generality. In the general case, which is necessary to consider in the analysis of section 5, the above conditions should be substituted for statements holding for almost all $\theta \in \Theta$, i.e. at all those points where the mechanism is differentiable. The reader interested in the technical details is referred to Rochet (1985) and Stole (1997). The precise conditions boil

Proposition 1. *The following two conditions are sufficient for a differentiable mechanism* $\{\alpha(\theta), \beta(\theta), a(\theta)\}$ *to be implementable:*

$$\left(\mu - \beta R\sigma^2\right)\frac{d\beta}{d\theta} + \beta\mu_a\frac{da}{d\theta} + \frac{d\alpha}{d\theta} = 0, \qquad \textit{for all } \theta \in \Theta; \tag{12}$$

$$\left[\mu_\theta + (\beta\mu_{\theta e} - c_{\theta e})\frac{de}{d\beta}\right]\frac{d\beta}{d\theta} + \left[\beta\mu_{\theta a} + (\beta\mu_{\theta e} - c_{\theta e})\frac{de}{da}\right]\frac{da}{d\theta} \geq 0, \qquad \textit{for all } \theta, \hat{\theta} \in \Theta. \tag{13}$$

where $e(\beta, a, \theta)$ *is defined by (8). Equation (12) and (13) holding for* $\hat{\theta} = \theta$ *constitute necessary conditions for implementability.* □

The proof of the previous proposition starts by solving for the effort choice as a function of the mechanism, and thereby eliminate the agency relationship from the problem. By inspection it becomes clear that u is quasi-linear, and therefore we can apply standard techniques to characterize the implementability of the mechanisms (Fudenberg and Tirole, 1991). Equation (12) is the first-order condition to the agent's information revelation problem. Condition (13) is equivalent to the second-order condition to this problem, which explains why we need the condition holding for all $\theta, \hat{\theta} \in \Theta$.

Proposition 1 gives a precise statement about the set of mechanisms that the principal can achieve. Condition (12) will be used next to calculate the informational rents the agent gets under any feasible mechanism. Condition (13) puts a further constraint on the mechanisms that the principal can use: this is what the literature usually refers to as the "monotonicity constraint." In standard models this constraint yields the property that β and e must be increasing in the agent's announced type. As it will become clear in the next examples, equation (13) imposes a restriction in the sign of the derivative of the elements of the allocation rule (β and a), but this constraint is not as strict as in standard models. Once the principal's action has a non-trivial effect on output the set of mechanisms that are implementable grows substantially: for example note that it could even be feasible to have a decreasing $\beta(\theta)$ function, since (13) does not rule it out.

The following examples consider some special cases to gain some further intuition into the restrictions that private information imposes on the set of contract/action pairs the principal can implement.

Example 2. Consider the case where μ is independent of θ, i.e. when private information only affects the costly effort function of the agent (his efficiency or ability). If $e \in \mathbb{R}$, then (13) becomes

$$-C_{\theta e}\frac{de}{d\hat{\theta}} \geq 0;$$

where $\frac{de}{d\hat{\theta}} = \frac{de}{d\beta}\frac{d\beta}{d\theta} + \frac{de}{da}\frac{da}{d\theta}$.[17] Since $C_{\theta e} \leq 0$ the above equation immediately implies that $\frac{de}{d\hat{\theta}} \geq 0$,

down to requiring that the indirect utility of the agent, $U(\theta) \equiv U_A(e(\theta), a(\theta), \alpha(\theta), \beta(\theta), \theta)$ is non-decreasing and convex in θ.

[17] Using the notation introduced before this is equivalent to $de/d\hat{\theta} \equiv \frac{de(\theta,\hat{\theta})}{d\hat{\theta}}|_{\hat{\theta}=\theta}$.

i.e. implementability requires that the effort function must be monotonically increasing in the agent's announced type. Note that this does not imply that β is increasing in the announced type: if $de/da > 0$ and $da/d\theta$ are sufficiently large the implementability condition is satisfied for *any* function β. On the other hand, when $e(\beta, a, \theta)$ is independent of a, implementability does imply that β must be a monotone function of θ.

It is interesting to note that in the general case where $e \in \mathbb{R}^n$ for $n \geq 2$ the implementability constraint does not require each element of the vector e to be increasing in $\hat{\theta}$, but rather

$$\sum_i w_i(\theta) \frac{de_i}{d\hat{\theta}} \geq 0;$$

where $w_i \equiv C_{\theta e_i} / \sum_{j=1}^n C_{\theta e_j}$. Finally note that while it is true that $de_i/d\hat{\theta} \geq 0$ for all i is sufficient for implementability, it is not necessary, i.e. restricting attention to models where the parameters are such that $de_i/d\hat{\theta} \geq 0$ for all i is not without loss of generality.[18] □

Example 3. Consider the case where C is independent of θ and $\mu_{\theta e} = 0$. This corresponds to the case where the agent's marginal value of effort is not affected by the private information that he possesses. Equation (13) becomes

$$\mu_\theta \frac{d\beta}{d\theta} + \beta \mu_{\theta a} \frac{da}{d\theta} \geq 0.$$

If $a \in \mathbb{R}$ the above reduces to $w_1(\cdot)\beta'(\theta) + w_2(\cdot)a'(\theta) \geq 0$ for appropriate functions $w_i(\cdot)$. When the principal's marginal value of her action is independent of the agent's private information, $\mu_{\theta a} = 0$, the equation reduces to the restriction $\frac{d\beta}{d\theta} \geq 0$ (since $\mu_\theta \geq 0$), i.e. the optimal bonus coefficient must be monotonically increasing. But note that this is the only restriction that yields a one-to-one connection between implementability and monotonicity of β: in the general case if $\mu_{\theta a} > 0$ is sufficiently large and $da/d\theta > 0$ implementability is again guaranteed for *any* function β. □

With the implementability issue resolved, I turn now to characterizing the optimal mechanism. The next proposition reduces the principal's problem to a simple pointwise maximization.

Proposition 2. *The principal's problem reduces to the maximization of*

$$\max_{\beta, a} \quad \int_{\underline{\theta}}^{\bar{\theta}} \Phi(\beta(x), a(x), x) f(x) dx \tag{14}$$

where

$$\Phi(\beta, a, \theta) \equiv B(e, a, \theta) - C(e, a, \theta) - \frac{R}{2}\beta^2\sigma^2 - H(\theta)\left(\beta\mu_\theta(e, a, \theta) - c_\theta(e, \theta)\right) \tag{15}$$

and $e = e(\beta, a, \theta)$ is given by (8), such that the constraint (13) holds. □

[18] The papers Besanko and Sibley (1991) and Bernardo, Cai, and Luo (2001) guarantee that the incentive-compatibility condition holds by analyzing model specifications in which this condition holds.

The principal's problem boils down to the maximization of the "virtual surplus" function $\Phi = \Phi^{SB} - H(\theta)\left(\beta\mu_\theta(e,a,\theta) - c_\theta(e,\theta)\right)$. The last term in (14) measures the cost for the principal due to the informational rents earned by the agent: all distortions from the second-best solution are generated by this term. Given the generality of the setup thus far it is hard, if not impossible, to make general statements about the nature of the optimal contracts.

One important aspect of this role can nonetheless be obtained from inspection of (15) in the case where both e and a are one-dimensional. We argue that when the action choices by the principal and agent are complements, namely $de/da > 0$,[19] asymmetric information makes incentives, measured by β, and the principal's actions, a, substitutes for the principal. Ignoring third-order derivatives, we have that[20]

$$\frac{d^2\Phi}{dad\beta} = \frac{d^2\Phi^{SB}}{dad\beta} - H(\theta)\left[\mu_{\theta a} + \mu_{\theta e}\frac{de}{da}\right]. \tag{16}$$

The last term in (16) is unambiguously negative: asymmetric information considerations make incentives and other complementary actions into potential substitutes. Rather intuitively, raising the incentives for agents increases their informational rents, which, on the margin, makes the principal's action more costly as well.

Although Proposition 2 has significantly simplified the the problem, the constraint (13) is still potentially binding. One could further develop the model explicitly taking into account this constraint (e.g. using the ironing procedure of Mussa and Rosen, 1978). The standard approach in the literature is to ignore this constraint, and then check that it is not violated. Since the main results to be discussed in later sections are statements of (weak) generic existence of certain properties, I do not gain in generality by ignoring the cases where (13) could bind.[21] One could find conditions for the two applications presented in the next section under which we could safely ignore this constraint. Along these lines it is important to note that in all examples to be discussed later in the paper these second-order conditions are verified to hold at the optimal solutions.

4 Optimal capital budgeting schemes

In this section I consider a variation of the model in Bernardo, Cai, and Luo (2001), which are characterized by the one-dimensional nature of the agent's and principal's action, i.e. the case where $e \in \mathbb{R}$ and $a \in \mathbb{R}$, and by further structure on the functions B, μ and C. To keep the discussion concrete, and for comparison with Bernardo, Cai, and Luo (2001). I will refer to

[19] A sufficient condition for this is that $\mu_{ea} > 0$, C is convex in e, and μ is concave in e, as can be verified from total differentiating (8).

[20] Equation (16) includes, in general, two other terms multiplying the H function: (i) $\beta de/d\beta[\mu_{\theta ea} + \mu_{\theta ee}de/da]$; (ii) $-de/d\beta[C_{\theta ea} + C_{\theta ee}de/da]$. Since it seems hard to gain much insight from the second-order derivatives of the functions μ_θ and C_θ we chose to ignore them in our discussion.

[21] If anything analyzing a smaller set of the potential models makes the genericity statements harder to obtain.

the action a as the capital budget, or investment decision, set by the principal for the agent's division. It should be noted that very similar results could be generated interpreting a as effort by the principal that contributes to the profitability of the project, a description that would be better suited in a venture capital context.

In this section I assume that the output function takes on a simple linear-quadratic form

$$B(e, a, \theta) = \gamma(\theta)e + \delta(\theta)a + \eta(\theta)ea; \tag{17}$$

and that this output is contractible (i.e. $\mu = B$ in the previous notation). I further assume that the private information does not affect the costly effort of the agent, i.e. $c(e, \theta) = c(e)$, and, for simplicity, that both the cost of effort and the investment are quadratic functions, i.e. $c(e) = \frac{1}{2}e^2$ and $c(a) = \frac{1}{2}a^2$.

The specification in (17), albeit stylized, captures other potentially important aspects of the agency relationship not considered in Bernardo, Cai, and Luo (2001), which has $\gamma(\theta) = 0$, $\delta(\theta) = \delta_0 + \delta_\theta\theta$, $\eta(\theta) = \eta_0$ and $R = 0$. In their work the agent is privately informed about the marginal value of the investment decision, independently of the effort choice. The importance of this element of the private information is measured by δ_θ. Equation (17) allows for the private information of the agent to impact its own marginal productivity of effort, through $\gamma'(\theta)$, and the interaction of the marginal value of investment that depends on the effort choice, measured by $\eta'(\theta)$. As it will become clear in the following discussion, the nature of the private information of the agent has important implications for the design of the optimal contracts: it is very different to be informed about the pure marginal value of investment than about the pure marginal value of effort or about the interaction of the marginal values of effort and investment.

It is straightforward to see, using the results from the previous section, that the principal's problem, absent private information considerations, is given by

$$\max_{\beta,a} \quad \Phi^{\text{SB}} \equiv (\gamma + \eta a)^2 \left(\beta - \frac{1}{2}\beta^2\right) - \frac{R\sigma^2\beta^2}{2} + \delta a - \frac{1}{2}a^2.$$

The optimal menu (β, a) in the pure moral-hazard setting is therefore characterized by the first-order conditions

$$\beta = \frac{1}{1 + \dfrac{R\sigma^2}{(\gamma + a\eta)^2}}; \tag{18}$$

$$a = \delta + 2\eta\left(\beta - \frac{1}{2}\beta^2\right)(\gamma + \eta a). \tag{19}$$

Rather intuitively the first-order conditions show β is distorted away from 1 due to risk-sharing considerations (the standard moral hazard problem tradeoff), and the optimal capital allocation rule is simply equal to his marginal production, comprised of the pure value of

investment δ, plus the term stemming from the interaction between effort and capital (measured by η).

When the agents are privately informed, the principal solves (14), which in the setup in this section reduces to

$$\Phi = \Phi^{\text{SB}} - H\beta\left[a\delta' + \beta(\gamma + a\eta)(\gamma' + a\eta')\right]. \tag{20}$$

The distortions from the second-best outcome stem from the two new terms in (20). The first thing to note is that private information considerations add costs to the principal's objective function both through the elements of private information on the value of investment, measured by δ (and η), as well as through the effort components of output, measured by γ (and η). Moreover, there is a non-trivial interaction between capital allocation rule a and the incentives β: private information considerations make investment and incentives substitutes for the principal. As incentives are raised, the information cost of investment also goes up, which pushes optimal investment down. In general the principal can cut informational rents both by distorting the capital allocation rule, or by lowering incentives. Under the optimal contracts the principal weights these new considerations against the standard efficiency/risk-sharing (moral-hazard) trade-off captured by the maximization the second-best surplus function Φ^{SB}.

The next Proposition characterizes the optimal menu of contracts and capital allocation rules in the case where the principal faces privately informed agents.[22]

Proposition 3. *The optimal mechanism when the non-negativity constraints do not bind, is characterized by*

$$a = \delta + 2\eta\left(\beta - \frac{1}{2}\beta^2\right)(\gamma + a\eta) - H\beta\left[\delta' + \beta\eta(\gamma' + a\eta') + \beta\eta'(\gamma + a\eta)\right]; \tag{21}$$

$$\beta = \frac{1 - \dfrac{Ha\delta'}{(\gamma + a\eta)^2}}{1 + \dfrac{R\sigma^2}{(\gamma + a\eta)^2} + \dfrac{2H(\gamma' + a\eta')}{(\gamma + a\eta)}}. \tag{22}$$

A sufficient condition for the optimality of the above solutions is that η is sufficiently small, and δ' is small relative to γ and/or γ'. $\square$

The above first-order conditions cature the relevant distortions in the optimal contracts due to asymmetric information. The first thing to note, simply comparing expressions (18) and (22), is that, fixing the capital allocation rule a, the incentives created for the agent are lower in the asymmetric information case. Rather intuitively the principal lowers the incentives to cut down the informational rents earned by agents under the equilibrium contracts.

[22] It is worth noticing that in the general case it is not possible to obtain analytical solutions. Moreover, the sufficient conditions given in the proposition are far from necessary: the interested reader is urged to consult the proof of the proposition to see the precise necessary conditions for optimality.

Similarly, fixing β, the capital allocation rule goes down, due to the informational cost term $H\beta\delta'$. The interaction between investment and effort, and in particular their respective informational costs, introduce new implications on the relationship between incentives and the private information of managers. In particular, the substitability among investment and incentives introduced by private information can create new comparative statics on the relationship between capital allocation and incentives.

A list of the comparative static results of the model in full generality would require a significant number of caveats. I therefore settle on illustrating some of the more interesting features of the model not discussed previously in the literature via a set of corollaries.

Corollary 1. *When $\eta(\theta) = 0$, i.e. there is no interaction in the value function between investment and effort, the optimal investment function and contigent compensation are given by*

$$a = \frac{\delta}{1-q} - \frac{H\delta'\gamma^2}{\gamma(\gamma + 2H\gamma') - H^2(\delta')^2}; \tag{23}$$

$$\beta = \frac{1 - \dfrac{Ha\delta'}{\gamma^2}}{1 + \dfrac{R\sigma^2}{\gamma^2} + \dfrac{2H\gamma'}{\gamma}}; \tag{24}$$

where $q(\theta) = \frac{H^2(\delta')^2}{\gamma(\gamma+2H\gamma')}$. $\square$

The corollary shows that the introduction of private information creates a link between compensation and investment which was not present in a second-best world. Absent asymmetric information considerations, the optimal contracts under the corollary's conditions are characterized by $a = \delta$ and $\beta = 1/(1 + R\sigma^2/\gamma^2)$. When there are no production interactions, private information makes capital and effort (or incentives) substitutes in the principal's optimization problem. This generates novel comparative statics. For example, for γ' small the sign of $d\beta/d\theta$ is given by the sign of $-aH' - a'H$. Therefore as long as the sensitivity of investment a' and/or the hazard rate is large we can see that the optimal contracts will exhibit $d\beta/d\theta < 0$.

Corollary 2. *When $\gamma(\theta) = 0$ and $\delta'(\theta) = 0$, i.e. asymmetric information only affects the interaction term of the investment decision with the agent's effort choice, the optimal investment function and contigent compensation in the case of a risk-neutral manager, $R = 0$, are given by*

$$a = \frac{\delta}{1-p}; \qquad \beta = \frac{\eta}{(\eta + 2H\eta')};$$

where $p(\theta) = \eta^3 \frac{(\eta+4H\eta')}{(\eta+2H\eta')^2}$. $\square$

Corollary 2 considers the case where capital and effort are complementary in the output function, but the private information possessed by the agents is with respect to the marginal value of investment with respect to effort, $\eta' > 0$, rather than the case considered in Bernardo, Cai, and Luo (2001), where the agent was privately informed about the marginal value of

the investment decision irrespective of effort, $\delta' > 0$. Some algebra verifies that under the conditions of Corollary 2 incentives are positively related to private information, $\beta'(\theta) \geq 0$, but that there exists parameter values such that $a'(\theta) < 0$ for some θ. In essence, if the private information affects the complementarities between effort and capital, then the optimal contracts will exhibit a negative relationship between the investment allocated to the manager and his incentives. Since investment and effort are complementary in the firm's production function, this substitutability arises due to the existence of asymmetric information.

Bernardo, Cai, and Luo (2001) further analyzed the possibility of the non-negativity constraints on β and a binding at the optimal solution. In particular, they show that there is a strong hierarchy between capital and incentives: an agent of type θ gets incentives for effort, i.e. $\beta(\theta) > 0$, if and only if he is allocated some capital, $a(\theta) > 0$. In particular, they show how one can partition the type interval into three disjoint regions: for $\theta \in [\underline{\theta}, \theta_1]$ agents get neither contingent compesation nor capital allocated, for $\theta \in [\theta_1, \theta_2]$ agents receive capital, but no incentives (their wage is non-contingent on output), and for $\theta \in [\theta_2, \bar{\theta}]$ agents receive both incentives and capital. One can summarize the possibility of binding non-negativity constraints by defining θ_a and θ_β to be the breakpoints below which these constraints bind. The result in Bernardo, Cai, and Luo (2001) can then be tightly stated as $\theta_a < \theta_\beta$. The next example shows the hierarchy between capital allocation decision and incentives again depends on the particular way the informational asymmetry enters the production function.

Example 4. Consider again the case with no interactions between effort and investment, i.e. $\eta = 0$, and where private information does not affect the value of effort, $\gamma'(\theta) = 0$. Some simple calculations show that the breakpoints for θ at which these non-negativity constraints bind are given by $\delta' H = \delta$ for θ_a and $\delta' H = \gamma^2/\delta$ for θ_β. Therefore the relative size of the curves δ and γ^2/δ determine whether $\theta_a \geq \theta_\beta$. Moreover, as long as informational considerations, as measured by $H(\theta)$, are large, such breakpoints θ_a and θ_β will always exist. Rather intuitively, this version of the model predicts that when effort productivity γ is low, it is more likely that $\theta_\beta < \theta_a$, whereas when investment productivity δ is large, it is more likely that $\theta_a < \theta_\beta$. $\square$

5 Optimal task allocation decisions

In this section I analyze an alternative set of actions for an active principal: the restriction of the tasks to be performed by the agent.[23] Holmstrom and Milgrom (1991) were the first to point out that in a standard moral hazard model it could be optimal to restrict the number of tasks that an agent performs. This section further studies how private information affects their findings.

I will use the following version of the Holmstrom and Milgrom (1991) model. I assume that final output is given by

$$B(e, a, \theta) = e^\top (b^0 + b^\theta \theta)$$

[23] One could equivalently interpret these tasks as "projects" that an agent would get assigned to.

and the signal is of the form

$$\mu(e, a, \theta) = e^\top(\mu^0 + \mu^\theta \theta).$$

In a slight abuse of notation let $\mu(\theta) \equiv \mu^0 + \mu^\theta \theta$ and $b(\theta) \equiv b^0 + b^\theta \theta$. The effort e is costly for the agent, who incurs a loss in certainty equivalent terms of $c(e) = \frac{1}{2} e^\top C(\theta) e$, for a symmetric matrix $C(\theta) = C^0 + C^\theta \theta$.

Both output and the tasks to be performed by the agent are contractible. Therefore a mechanism now consists of a contract, characterized by a fixed wage α and a bonus coefficient β, as well as a set of actions to be performed. Define I_i to be the indicator variable of whether task i is assigned to the agent or not. Then a mechanism is given by $\{\alpha(\theta), \beta(\theta), I_1(\theta), \ldots, I_n(\theta)\}$. Define I to be the $(n \times n)$ diagonal matrix with elements $I_i(\theta)$. I will assume that there is no associated cost with the actions (task assignment) taken by the principal in this case, although this could be easily generalized. Moreover, I assume that the tasks are not exclusive, i.e. an agent can be assigned an arbitrary number of tasks.

The following proposition specializes the previous results to this setting.

Proposition 4. *At an interior solution, the optimal contract is characterized by*

$$\beta(\theta) = \frac{\ell(I, \theta)^\top b(\theta)}{R\sigma^2 + \ell(I, \theta)^\top \left(\mu(\theta) + 2H(\theta)(\mu^\theta + m(I, \theta))\right)}. \tag{25}$$

where $\ell(I, \theta) = (C_I(\theta))^{-1}\mu(\theta)$, *and* $m(I, \theta) = C^\theta \ell(I, \theta)$.

The optimal task assignments $I_i(\theta)$ *are given by the solution to*

$$\max_{I_i} \quad \frac{(\ell(I, \theta)^\top b(\theta))^2}{R\sigma^2 + \ell(I, \theta)^\top \left(\mu(\theta) + 2H(\theta)(\mu^\theta + m(I, \theta))\right)} \tag{26}$$

Action restrictions, i.e. setting $I_i(\theta) = 0$ *for some* i *and some* θ, *are optimal for an open set of parameter values.* □

The general trade-offs present in the expressions in the proposition are familiar: on one hand we have the agency costs due to the multi-dimensional moral hazard problem, now exacerbated by the existence of private information. Comparing the optimal contingent compensation given in equation (25) to the second-best solution (see equation (10)), it is immediate that holding the set of tasks constant in both problems private information lowers the bonus coefficient of the optimal contract. This is the standard result in previous agency models. But note that this statement depends crucially on the fact that the task assignments are the same in both the second- and third-best worlds. The objective function for the optimal allocation of tasks given by (26) is to be contrasted to that in a second-best world (11). The term involving the usual measure of informational costs $H(\theta)$ makes the optimal task assignments be potentially different, which indirectly affects the optimal amount of contingent compensation.

The next example gives more explicit conditions under which an action will optimally be restricted for the case $n = 2$ in order to gain some further intuition.

Example 5. Let C be diagonal. Then the optimal mechanism for $n = 2$ satisfies $I_1(\theta) = 0$ if and only if there exists θ for which

$$\frac{(\mu_2 b_2/c_{22})^2}{R\sigma^2 + (\mu_2/c_{22})(\mu_2 + k_2)} \geq \frac{(\mu_1 b_1/c_{11} + \mu_2 b_2/c_{22})^2}{R\sigma^2 + (\mu_1/c_{11})(\mu_1 + k_1) + (\mu_2/c_{22})(\mu_2 + k_2)}; \tag{27}$$

with $k_i = 2H(\mu_i^\theta + \mu_i c_{ii}^\theta / c_{ii})$. I argue next that the above condition is met in a (weakly) generic sense even when in a second-best world it is not optimal to restrict action 1. Consider the case with $C^\theta = 0$, so private information only affects the signal and output. Further assume that C is the identity matrix and that output is measurable (i.e. $\mu = b$). Recall from section 2.2 that under these assumptions it is never optimal to restrict the actions taken by the agent. It is straightforward to check that condition (27) reduces to

$$2H\left(\mu_1^\theta \mu_1 \mu_2^4 - \mu_2^\theta \mu_2 (\mu_1^4 + 2\mu_1^2 \mu_2^2)\right) \geq \mu_2^2 \mu_1^2 (\mu_2^2 + \mu_1^2) + R\sigma^2 \left(\mu_1^4 + 2\mu_2^2 \mu_1^2\right).$$

When informational rents are important, for large values of $H(\theta)$, and for values of the parameters and θ such that $\mu_1 \approx \mu_2$, the condition reduces to $\mu_1^\theta \geq 3\mu_2^\theta$, i.e. the informational sensitivity of action 1 must be at least three times that of action 2 in order for action restrictions to be optimal when both actions are approximately equally important in terms of output. This condition delimits an open set of parameters for which (27) is satisfied, even when in a second-best world action restrictions are not optimal. □

Two general statements on the model are discussed next. The first highlights the role of risk in the optimal task allocation problem. The second shows how the model can generate distortions upwards in incentives, i.e. more powerful contracts for lower types.

Corollary 3. *For large values of the risk-aversion of the agent R or of the noise in the performance measure σ^2 it becomes optimal to assign all tasks to all agents ($I_i(\theta) = 1$ for all $\theta \in \Theta$).*

The intuition for this result lies on the fact that for large values of risk-aversion and volatility the contingent compensation becomes small, since the agent is risk-averse. Now, as can be grasped from equation (33), the informational rents considerations are of order $o(\beta^2)$, whereas the benefits from production are $o(\beta)$. Therefore, as $\beta \downarrow 0$, it becomes optimal to allow all agents to perform all tasks. Note that this result yields itself to simple empirical tests, since it predicts that in environments with high uncertainty σ^2 we should see agents have more freedom in terms of the tasks assigned by the principal.

Corollary 4. *The fixed wage, contingent compensation, and effort levels on a particular task may be lower or higher in the presence of private information than the optimal contracts without asymmetric information.*

This last result highlights a significant difference between the standard model, in which private information always brings underproduction (lower levels of effort and contingent com-

pensation in our setting), and the model discussed in this section where the principal also controls the task assigned to the agents. The intuition of the standard model nevertheless applies, albeit with a caveat: now underproduction does not imply that the agent will receive a lower contingent compensation contract, underproduction is actually reflected in the potential optimality of action restrictions. When these occur, the phenomena described above can arise: with less tasks under the agent's command it may be optimal for the principal to give him a more aggressive contract (see Example 6). Note that even though the standard intuition applies, from an empirical perspective the standard comparative statics do not hold: the compensation is (highly) non-monotone with respect to private information. To the extent that task allocation decisions are hard to measure, this result shows that it would be possible to have more general relationships in the data between compensation and quality driven by private information considerations than previously considered.

I conclude this section with an illustrative example.

Example 6. Consider the case with $C^\theta = 0$, so private information only affects the signal and output. Assume the signal is uniformly distributed on $[0, 1]$, and that

$$C^0 = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}; \quad \mu^0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}; \quad \mu^\theta = \begin{pmatrix} 1 \\ 0.5 \end{pmatrix}; \quad b^0 = \begin{pmatrix} 0.2 \\ 1 \end{pmatrix}; \quad b^\theta = \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{28}$$

It is straightforward to solve the model for the second best case and note that action restrictions are *not* optimal for the above parameter values for all $\theta \in \Theta$. Figure 1 plots the optimal contracts and the principal's objective function in the case where the principal allows the agent to perform two tasks (solid line), when the agent is assigned only task 1 (dotted line), and when the agent is assigned only task 2 (dashed line). As is immediate from the figure, for sufficiently low realizations of the signal it is optimal to restrict the agent to one single action (task one). Note that this in turn implies that there is a discontinuity in the contingent compensation function $\beta(\theta)$, as shown in Figure 1. Furthermore, note how the presence of this action restriction distorts the solution significantly, even to the point of making the bonus coefficient in a third-best world higher than in a second best world (see Figure 1).

Figure 2 presents the optimal contracts for another set of parameter values, including in the graphs the second-best solutions. Note that the same features of the previous graph prevail in terms of the optimality of restricting action 1. More interestingly, note how the compensation function is dramatically non-monotone, as well as higher than in the contracting environment without private information, two features that never arise in standard models. Even more surprisingly, the compensation function in a third-best world is lower than in a second-best world for a range $[\underline{\theta}, \theta_1]$, higher for $[\theta_1, \theta_2]$, and lower again for $[\theta_2, \bar{\theta}]$. This is driven by the action restrictions: in the case of private information these start in θ_2, whereas in the pure moral hazard case action restrictions are only optimal for agents with $\theta \leq \theta_1$. $\square$

6 Extensions

Two technical aspects of the model studied in the paper deserve comment. Limited liability constraints, in particular setting $\alpha(\theta) \geq 0$, are potentially binding for θ in its upper range, since the wage function is generically decreasing in its arguments.[24] This aspect of the optimal contracts, ignored throughout the paper, could be accounted for using standard techniques (see Thomas, 2002) at the cost of analytical tractability. There would be further distortions away from the first- and second-best solutions, but none of the main qualitative aspects of the optimal contracts would be altered (see also the discussion in Bernardo, Cai, and Luo, 2001). The single-crossing properties for both the output and cost functions could be generalized without changing any of the results, as long as the agent's effective preferences did satisfy the single-crossing property (thereby small deviations from the model studied in the paper would not change the optimal contracts).

The model introduced in the paper, and in particular the results stated in section 3, can be generalized in several directions, since the potential set of actions a principal may take on an agency relationship is quite large. The next example extends the model in section 4 by allowing the risk of output to depend on the investment decision. It clearly illustrates how the standard positive relationship between incentives and risk can be reversed once the principal is active, in particular if the capital budgeting decision affects both the return and the risk of the projects under management.

Example 7. Consider the capital budgeting setting discussed in section 4 but allowing the risk of output to depend on the investment decision. To keep the discussion simple, assume that $\eta(\theta) = 1$, $\gamma(\theta) = 0$ and $\delta(\theta) = \delta_0 + \delta_\theta \theta$, i.e. private information only affects the pure marginal value of investment. Furthermore, assume that $\mu = B$ (output is contractible with noise), and that $\sigma = sa$, for some $s \in \mathbb{R}_+$. The variable s measures the impact of the investment decision on the risk of output. Note that this is a very natural generalization, since it adds an element of risk/return trade-off to the investment decision. It is straightforward to solve for the optimal menu using similar techniques as in section 4 to obtain: $a(\theta) = (\delta(\theta) - H(\theta)\delta_\theta \kappa)/(1+\kappa)$, $\beta(\theta) = \kappa(1 - H(\theta)\delta_\theta / a(\theta))$, where $\kappa \equiv 1/(1 + Rs^2)$. Consider the following parameter values: $R = 100$, $\underline{\theta} = 0$, $\bar{\theta} = 1$, $s = 0.1$, $\delta_0 = 1$ and $\delta_\theta = 0.8$. One can easily verify that in this example the bonus coefficient is positively related to the risk of the performance measure. This offers yet another theoretical explanation for the existence of a positive relationship between compensation and risk (see Prendergast, 2002, and the reference cited there for alternative models that generate this type of result). □

[24] The alternative limited liability constraint $w(\theta) \geq 0$ completely eliminates the attractiveness of the CARA/Gaussian setting, since the only linear contracts that would be in the feasible set would be those with a zero bonus coefficient.

7 Conclusions

The paper has generalized the standard principal-agent model to consider the effects on the optimal contracts of potential actions by the principal that influence the agency relationship. In particular the cases of assignment of a capital budget on top of the compensation scheme, and allocating a set of tasks among the agents have been studied in detail. The paper shows that the existence and nature of the private information plays a crucial role on the implications from models driven by private information considerations, and several standard features of the optimal contracts are altered once the principal can affect the agency relationship. For example, the standard underproduction result, when measured by the incentives received by the agent through the contracts, can disappear: an active principal may give higher incentives to lower types, as long as she compensates higher types with her actions. The paper also links the quality of measurable output with the number of tasks agents can get assigned under the optimal contracts, predicting that more noisy contracting measures will be associated with agents having more tasks at their disposal. The results illustrate the richness, in terms of implications, that models with asymmetric information can generate, and opens the door for more powerful empirical tests that can differentiate among them.

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Appendix

Proof of proposition 1.

The expected utility for the agent from an arbitrary element of the menu offered by the principal (α, β, a) is given by

$$u(\alpha, \beta, a, \theta) \equiv \alpha + \beta\mu(e(\beta, a, \theta), a, \theta) - \frac{R}{2}\beta^2\sigma^2 - c(e(\beta, a, \theta), \theta), \tag{29}$$

where for $e = e(\beta, a, \theta)$ is uniquely defined by (8).

The agent's problem, in terms of his optimal revelation of type $\hat{\theta}$, or equivalent in terms of his choice from the menu of contracts offered by the principal, is

$$\max_{\hat{\theta}} \quad u(\alpha(\hat{\theta}), \beta(\hat{\theta}), a(\hat{\theta}), \theta). \tag{30}$$

The first-order condition to (30) is

$$\begin{aligned} \frac{d\alpha}{d\theta} + \mu\frac{d\beta}{d\theta} + \beta\mu_e\left(e_\beta\frac{d\beta}{d\theta} + e_a\frac{da}{d\theta}\right) + \beta\mu_a\frac{da}{d\theta} - \beta R\sigma^2\frac{d\beta}{d\theta} - c_e\left(e_\beta\frac{d\beta}{d\theta} + e_a\frac{da}{d\theta}\right) &= \\ \frac{d\alpha}{d\theta} + \mu\frac{d\beta}{d\theta} + \beta\mu_a\frac{da}{d\theta} - \beta R\sigma^2\frac{d\beta}{d\theta} &= 0, \end{aligned}$$

where the last equality follows from equation (8), the first-order condition for the effort choice. This shows equation (12) is necessary for implementability.

Equation (12) holds as an identity in θ, so we can total differentiate to find

$$u_{\beta\beta}\frac{d^2\beta}{d\theta^2} + 2u_{\beta a}\frac{d\beta}{d\theta}\frac{da}{d\theta} + u_{\beta\theta}\frac{d\beta}{d\theta} + u_{aa}\frac{d^2a}{d\theta^2} + u_{a\theta}\frac{da}{d\theta} + \frac{d^2\alpha}{d\theta} = 0. \tag{31}$$

The second-order condition to (30) is given by

$$u_{\beta\beta}\frac{d^2\beta}{d\theta^2} + 2u_{\beta a}\frac{d\beta}{d\theta}\frac{da}{d\theta} + u_{aa}\frac{d^2a}{d\theta^2} + \frac{d^2\alpha}{d\theta} \leq 0;$$

which, using (31), reduces to (13). This proofs that (13), evaluated at $\hat{\theta} = \theta$, is a necessary for this second-order condition to be satisfied.

The same argument shows that (12), and (13) holding for all $\theta, \hat{\theta} \in \Theta$ are sufficient conditions for implementability. This completes the proof. □

Proof of proposition 2.

Using the expressions in the proof of Proposition 1, we have that

$$u_\theta(\alpha, \beta, a, \theta) = \beta\mu_\theta + (\beta\mu_e - c_e)\frac{de}{d\theta} - c_\theta = \beta\mu_\theta - c_\theta;$$

where the last equality follows from equation (8), the first-order condition for the effort choice. Integrating over θ we then have that

$$u(\alpha, \beta, a, \theta) = \bar{u} + \int_{\underline{s}}^{s} (\beta(x)\mu_\theta(e(x), a(x), x) - c_\theta(e(x), x))\, dx.$$

Equating this to the expression (29) allows us to solve for the fixed payment in wages, α, as a function of the other elements of the menus, i.e.

$$\alpha = \bar{u} + \int_{\underline{s}}^{s} (\beta(x)\mu_\theta(e(x), a(x), x) - c_\theta(e(x), x))\, dx - \beta\mu(e(\beta, a, \theta), a, \theta) + \frac{R}{2}\beta^2\sigma^2 + c(e(\beta, a, \theta), \theta).$$

This expressions allows to eliminate the fixed wage component from the principal's objective function (4), so her problem reduces to

$$\max_{\{\beta,a\}} \quad \int_{\underline{\theta}}^{\bar{\theta}} \left(B(e(x), a(x)) - C(e(x), a(x), x) - \frac{R}{2}\sigma^2\beta(x)^2 - \int_{\underline{\theta}}^{x} (\beta(u)\mu_\theta(e(u), a(u), u) - c_\theta(e(u), u))\, du \right) f(x)dx.$$

Integrating by parts this last expression yields (14) in the Proposition. This completes the proof. □

Proof of proposition 3.

The principal is maximizing

$$\max_{\beta,a} \quad \Phi = \Phi_{SB} - H\beta\left(e\gamma' + a\delta' + ea\eta'\right) \tag{32}$$

where Φ_{SB} is the surplus function when there is no asymmetric information, given by

$$\Phi_{SB} \equiv e\gamma + a\delta + ea\eta - \frac{1}{2}a^2 - \frac{1}{2}e^2 - \frac{R}{2}\beta^2\sigma^2.$$

The first-order condition for effort choice yields

$$e = \beta(\gamma + \eta a).$$

Using this expression in (32), and differentiating with respect to β and a yields the two first-order conditions. Some tedious manipulations show that these first-order conditions reduce to (21) and (22) in the Proposition.

Further differentiation yields that $d^2\Phi/d\beta^2 < 0$ and $d^2\Phi/da^2 < 0$ as long as η is small enough. Moreover, the sign of $d^2\Phi/d\beta da$ is equal to that of $\gamma(\gamma + 2H\gamma') - H\delta'$, from which the sufficient conditions in the Proposition follow. This completes the proof. □

Proof of proposition 4.

Given a menu $\{\alpha(\theta), \beta(\theta), I_1(\theta), \ldots, I_n(\theta)\}$, the agents optimal effort choice is given by the solution to

$$\alpha + \beta e^\top \mu(\theta) - \frac{1}{2} e^\top C(\theta) e - \frac{R}{2}\sigma^2\beta^2$$

such that $e_i = 0$ for those i such that $I_i = 0$, which can be expressed as

$$e = \beta(\theta) C_I(\theta)^{-1} \mu(\theta).$$

The principal's problem, plugging this action back into his objective function, is reduced to

$$\max_{\beta, I} \quad \beta \ell^\top B(\theta) - \frac{1}{2}\beta^2 \left(R\sigma^2 + \ell^\top C(\theta)\ell + 2H(\theta)\ell^\top(\mu_\theta - C_\theta \ell \right) \tag{33}$$

where ℓ is defined in the statement of the proposition. The equations in the proposition follow from the first-order conditions to the above optimization problem. The genericity statement follows immediately from Example 5. This completes the proof. □

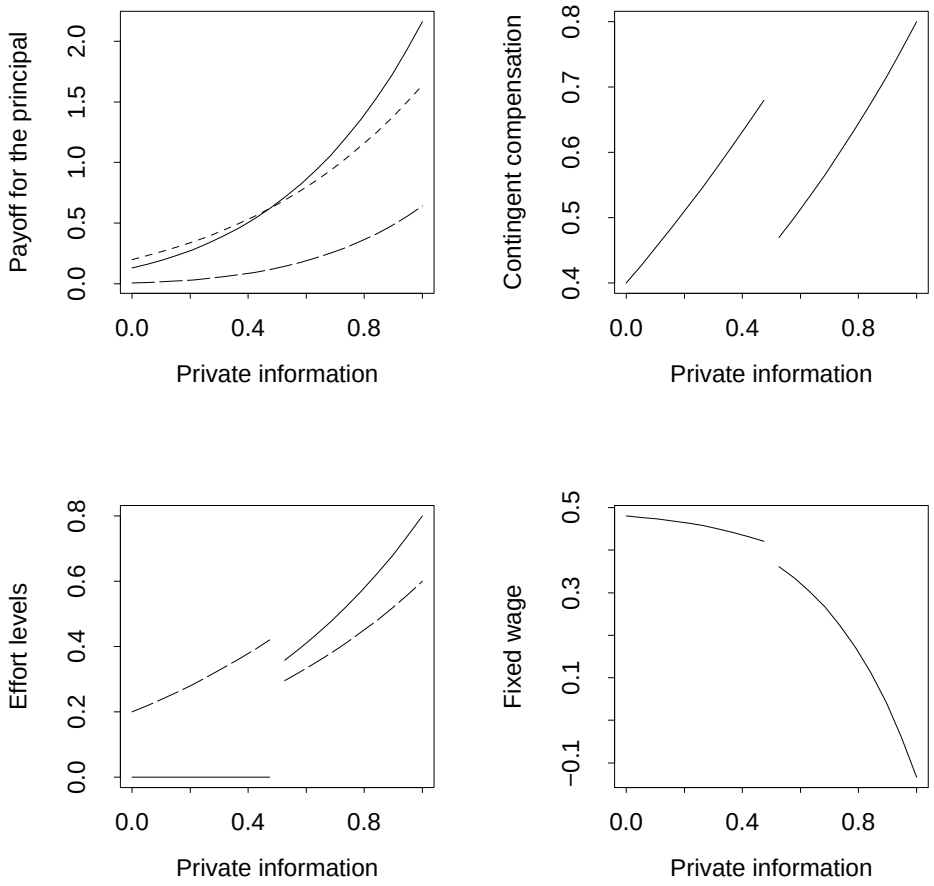

Figure 1: The top-left graph presents the principal's payoff for different action restrictions: all actions allowed (solid), only action 1 allowed (dotted), and only action 2 allowed (dashed). The bottom-left figure presents the levels of effort on the first action (dashed) and the second action (solid). The top-right graph plots the optimal bonus coefficient, whereas the bottom-right graph plots the optimal fixed wage received by the agent. The parameter values used in this example are: $R = 0$, $\sigma^2 = 0.25$, $\bar{u} = 0.5$, $\mu_0 = (1, 1)$, $b_0 = (0.2, 1)$, $\mu_1 = (1, 0.5)$, $b_1 = (1, 1)$, $C_{ij}^k = 0$ for $i \neq j$, $k = 1, 2$, $C_{ii}^0 = 2$, $C_{ii}^1 = 0$, $\underline{\theta} = 0$, $\bar{\theta} = 1$.

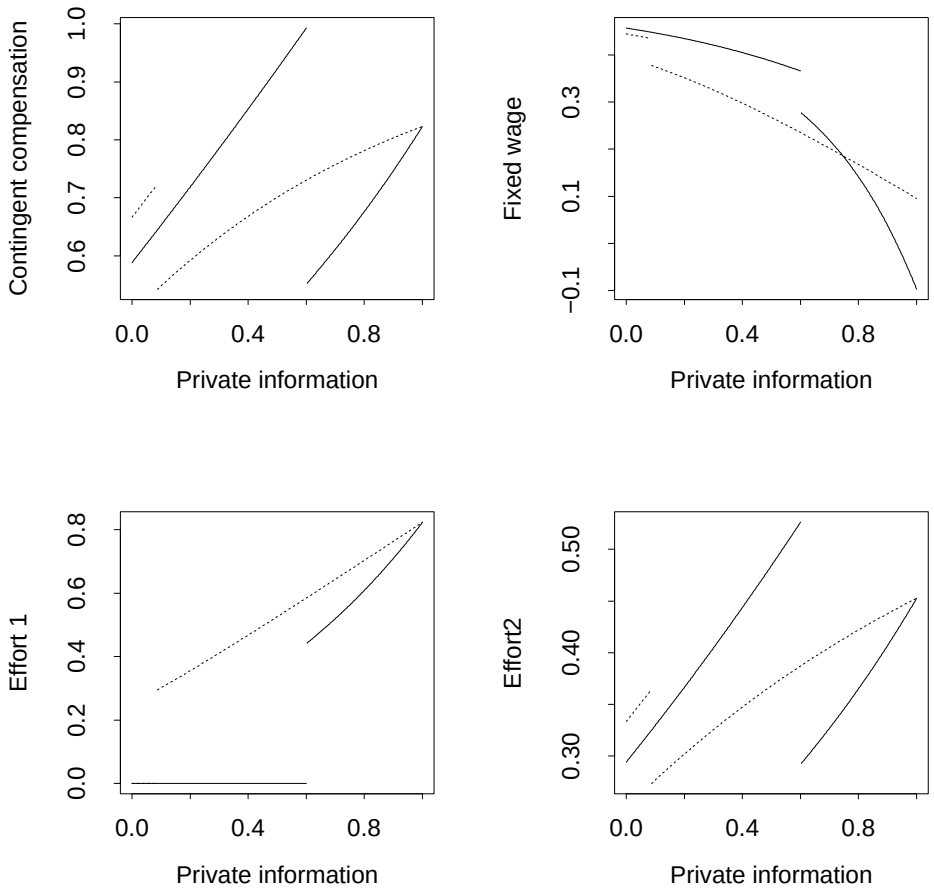

Figure 2: The graph plots the bonus coefficient and fixed wages, and the induced effort levels, under the optimal contracts under asymmetric information (solid line) and in a second-best world (dotted line). The parameter values used in this example are: $R = 0$, $\sigma^2 = 0.25$, $\bar{u} = 0.5$, $\mu_0 = (1, 1)$, $b_0 = (0.25, 1)$, $\mu_1 = (1, 0.1)$, $b_1 = (1, 1)$, $C_{ij}^k = 0$ for $i \neq j$, $k = 1, 2$, $C_{ii}^0 = 2$, $C_{ii}^1 = 0$, $\underline{\theta} = 0$, $\bar{\theta} = 1$.