Single good

Let the demand function be $q = D(p)$, where q is the output and p is the price

Let the inverse demand be $p = Q(q)$

Elasticity of demand is $\varepsilon = -(\partial D/D)/(\partial p/p)$

Costs are $C(q)$, with $\partial C/\partial q \geq 0$

Assume that the second order conditions are satisfied, so that first order conditions characterize a maximum

The firm's problem can be written as either

$$Max_p \; \Pi = pD(p) - C(D(p))$$

or

$$Max_q \; \Pi = qP(q) - C(q)$$

The first order condition is

$$\frac{\partial \pi}{\partial p} = D(p) + p\frac{\partial D(p)}{\partial p} - \frac{\partial C(q)}{\partial q}\frac{\partial D(p)}{\partial p} = 0$$

Or,

$$\frac{\partial \pi}{\partial q} = P(q) + q\frac{\partial P(q)}{\partial q} - \frac{\partial C(q)}{\partial q} = 0$$

Interpretation of first order condition

- Marginal Revenue = Marginal Costs
- Lerner Index

$$\frac{p - C'(q)}{p} = \frac{1}{\varepsilon}$$

Multiple Products
Independent Demands and Costs

$$\pi_1 + \pi_2 = p_1 D_1(p_1) - C_1(D_1(p_1)) + p_2 D_2(p_2) - C_2(D_2(p_2))$$

The solution is the same as before. Namely,

$$\frac{p_i - C_i'(q_i)}{p_i} = \frac{1}{\varepsilon_i} \text{ for } i = 1, 2$$

Interdependent Demands

- Price of one good, effects demand for the other good
- Examples of goods

Assume the following demand functions

$$q_i = a - bp_i + gp_j \text{ with } i = 1, 2 \text{ and } i \neq j$$

Interpretation of g
Assume $|g| < b$ own effect dominates, $a > c(b - g)$; positive output
Assume Costs $C(q_1, q_2) = c(q_1 + q_2)$

The monopolist's total profit is

$$\pi = (a - bp_1 + gp_2)(p_1 - c) + (a - bp_2 + gp_1)(p_2 - c)$$

The first order conditions have

$$\frac{\partial \pi}{\partial p_i} = a - 2bp_i + 2gp_j + c(b - g) = 0, \text{ with } i = 1, 2 \text{ and } i \neq j$$

At the symmetric solution, $p_1 = p_2 = p_M$

$$p_M = \frac{a + c(b-g)}{2(b-g)} \quad q_M = \frac{a - c(b-g)}{2}$$

Interpretation

$$\frac{\partial p_M}{\partial g} = \frac{a}{2(b-g)^2} > 0$$

Why?

Intepretations

Dynamic goods-If you buy a good, then changes how much you want to buy goods in future. Ex. Buy car, buy maintanence in future.

$$q_2 = a - bp_2 + \lambda q_1$$

Profits for firm

$$\pi = (a - bp_1)(p_1 - c) + (a - bp_2 + \lambda(a - bp_1))(p_2 - c)$$

First order conditions

$$\frac{\partial \pi}{\partial p_1} = a - 2bp_1 + bc - \lambda b(p_2 - c) = 0$$

$$\frac{\partial \pi}{\partial p_2} = a(1 + \lambda) - 2bp_2 + bc - \lambda bp_1 = 0$$

From this we get

$$p_1 = \frac{a(1 - \lambda) + cb}{b(2 - \lambda)} \quad p_2 = \frac{a + cb(1 - \lambda)}{b(2 - \lambda)}$$

Interdependent Costs- Economies and Diseconomies of Scope
$C(q_1, q_2) = cq_1 + cq_2 + \mu q_1 q_2$
Economies of scope $\mu < 0$, Diseconomies of scope $\mu > 0$
Let $q_i = a - bp_i$, profits are

$$\pi = (a - bp_1)p_1 + (a - bp_2)p_2 - c(a - bp_1 + a - bp_2) - \mu(a - bp_1)(a -$$

From the first order conditions we get equal price of

$$p_M = \frac{a(1 + b\mu) + cb}{b(2 + b\mu)}; \frac{\partial p_M}{\partial \mu} > 0$$

Learning by doing

$C_1' = c,\ C_2' = c - \lambda q_1;\ p_t = 1 - q_t$

$$\pi = (1 - q_1 - c)q_1 + (1 - q_2 - c + \lambda q_1)q_2$$

Equilibrium outputs satisfy

$$q_1 = \frac{1 - C_1' + \lambda q_2}{2}\ q_2 = \frac{1 - C_2'}{2}$$

Interpretation

Solving for outputs

$$q_1 = q_2 = \frac{1 - c}{2 - \lambda}$$

Interpretation- internalize externality, produces more than without externality $\frac{1-c}{2}$