

# Equilibria in an infinite horizon game with an incumbent, entry and switching costs<sup>\*</sup>

(Running head: “Equilibria with switching costs”.)

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We consider an infinite horizon game a consumer incurs a switching cost when changing supplier. There is an incumbent firm at the beginning of the game, and a new entrant in each period, all with a production cost of 0. We identify the set of possible profits of the incumbent when the common discount rate of all the agents is greater than  $1/2$ : it is the interval  $[0, \sigma/(1 - \delta)]$ : there exist equilibria in which the incumbent sells the good to the consumer in every period at any price in  $[0, \sigma]$ . The result also holds with several entrants and several consumers.

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# 1 Introduction

Consider the following very simple one-shot game. An incumbent and several potential entrants are all endowed with the same technology which enables them to produce a good at the same cost, set, for simplicity and without loss of generality, equal to zero. There is also one price taking consumer who must purchase one unit of the good. In past periods, this consumer has purchased from the incumbent, and will incur a switching cost  $\sigma$  if he purchases from one of the entrants. Assuming either a Bertrand-type competition where the firms set their price simultaneously or a Stackelberg-type competition where the incumbent “moves” first, in equilibrium the consumer will purchase from the incumbent at a price equal to  $\sigma$ ; the profit of the incumbent is therefore also equal to  $\sigma$ .

It is easy to show that the unique equilibrium of this game repeated a finite number of periods is to have the consumer purchases from the incumbent in every period, at a price equal to  $\sigma$  in the last period and equal to  $(1 - \delta)\sigma$  in previous periods, where  $\delta$  is the discount factor. The profit of the incumbent discounted to the first period is therefore equal to  $\sigma$  (see Biglaiser, Crémer, and Dobos, 2010).

On the other hand, it is a common theme in theoretical Industrial Organization that infinite horizon relationships can have properties very different from those of finite horizon relationships. The theory of oligopoly provides the canonical example. In a finitely repeated game, the per period profits of the firms are equal to their profits in the one period model (note that this is different from the switching cost model sketched above). On the other hand, as first pointed out formally in a classic paper by Friedman (1971) (see also Friedman, 1977), and explored in great detail by many others, in an infinite horizon repeated game, firms can, if the discount rate is low enough, collude so as to produce the quantities that maximize their joint profit.<sup>1</sup>

Whereas the Industrial Organization literature has focused its attention on the opportunities for collusion, the game theoretical literature has been more interested in showing that a wide variety of outcomes are possible in repeated games. This is, of course, the message of the folk theorem, which states that the per period payoff of a player can be equal to any feasible outcome of the one period game, as long as a (more precisely defined!) participation constraint can be defined.<sup>2</sup> In the case of oligopolistic markets, this implies that for high enough discount factors any average payoff between the minmax payoffs to the perfectly collusive payoff payoffs can be supported.

Turning back to the switching cost model described above, Biglaiser et al. (2010) study one focal equilibrium of the infinitely repeated game: they identify a strongly stationary outcome in which any firm which has attracted consumers will charge the same price. More precisely, they identify the equilibria in which there exists a price  $p^*$  such that

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<sup>1</sup>Friedman’s result has been generalized by Abreu (1986) by showing a larger set of discount factors may support collusion by using “stick and carrot” punishments. Other papers include Radner (1980), Porter (1983), Green and Porter (1984), Fudenberg and Maskin (1986) and Kandori and Matsushima (1998).

<sup>2</sup>See, for example, Rubinstein (1979), Abreu, Pearce, and Stacchetti (1986), Abreu (1988), Fudenberg and Maskin (1986). More recent papers on the folk theorem include Kandori and Obara (2006), Hörner and Olszewski (2006), and Fudenberg and Yamamoto (2010).

in the first period the incumbent charges  $p^*$  and in subsequent periods, the incumbent, if he still has consumers who have chosen its product in the previous period, and any entrant who has succeeded in attracting consumers in the previous period also charge  $p^*$ . They show that, in any such equilibrium, the total profit of the incumbent discounted to the first period is equal to  $\sigma$ , which is also the profit in the one period model.<sup>3</sup> Given the asymmetry of the role of the players, it would not make sense in the switching cost model to try to identify the best collusive equilibrium,<sup>4</sup> but, *mutatis mutandis*, this is the same type of exercise conducted by Friedman and his successors for the oligopoly model: identifying one focal equilibrium of the repeated game, and comparing it to a solution of the one period game.

In this paper, we take an approach closer to the ‘game theory’ approach sketched above. Assuming that the common discount factor of all the agents,  $\delta$ , is greater than  $1/2$ , we identify *all* the equilibrium profits of the incumbent in the infinitely repeated switching cost game. We show that the per period profit ranges from 0 to  $\sigma$  (*i.e.*, its total discounted profit ranges from 0 to  $\sigma/(1 - \delta)$ ). In other words, all the individually rational payoffs for the incumbent are equilibria of the infinitely repeated version of the game as long as  $\delta$  exceeds  $1/2$ . Furthermore, these equilibria are all stationary equilibria, in the sense that along the equilibrium path in every period the incumbent sells to the consumer at the same price. They are not *strongly* stationary as, after a deviation (which in equilibrium will not happen), the price would change.

In the literature on repeated oligopoly, the consumers play a passive role and have no interesting strategic role: in every period, they purchase from one of the firms who charges the lowest price. With switching costs, the consumers choose the firm from which they purchase taking into account not only the current price, but also the prices which they expect firms to charge in the future: given the ‘stickiness’ induced by switching costs, it makes no sense to switch to an entrant who proposes a lower price without taking into account both future prices and switching costs. As far as we know, this is the first paper in the Industrial Organization literature to have a folk theorem when consumers are active players and their beliefs about the future matter.

Given the complexity of the strategic interactions in our model, we develop our analysis in several steps. Section 2 introduces the model in a basic case where there is one entrant in every period and only one consumer. We first show how to obtain an average incumbent per period payoff of either 0 or the full consumer switching cost,  $\sigma$ ; this only requires that the market be in one of two possible states. Next, we show that by adding a third state, we can span the average per period incumbent for any value between 0 and  $\sigma$ . We then extend the analysis in several directions. First, we extend the model to allow for multiple entrants in each period. Next, we allow for multiple consumers along with multiple entrants, so it is feasible to have multiple incumbents. We generate the same incumbent payoff set with both of these extensions. Finally, section 4 presents some brief concluding remarks.

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<sup>3</sup>Biglaiser et al. actually consider a model with a continuum of consumers — the result would still hold with a finite number of consumers.

<sup>4</sup>Roig (2010) studies a model in which there are two incumbents, each selling to half of the consumers, and discusses the possibility of collusion.

## 2 Equilibria with one consumer and one entrant in each period

### 2.1 The model

We consider a game with an infinite number of players and an infinite number of periods, although in each period, there will be at most three “active” players.

In each period  $t = 1, \dots, +\infty$ , there will be an incumbent, whose identity is, for  $t > 1$ , determined by the past history of the game, an entrant, called entrant  $t$  and the consumer, who is active in every period. In stage 1 of period  $t$ , both the incumbent and the entrant simultaneously announce a price; in stage 2, the consumer chooses to purchase either from the incumbent or from the entrant. If he purchases from the incumbent at price  $p$ , his total cost in period  $t$  is equal to  $p$ ; if he purchases from the entrant, his total cost in period  $t$  is  $p + \sigma$ , where  $\sigma$  is the switching cost.

The firm from which the consumer has purchased in period  $t$  becomes the period  $t + 1$  incumbent, and the other firm “drops” out of the game. The identity of the incumbent in periods  $t = 2, \dots, +\infty$  is therefore determined by the moves of the players in past periods. In period 1, we simply assume that there is an entrant and an Incumbent, a firm from which the consumer purchased, for unspecified reasons, in previous periods. (Notice that we write Incumbent for the period 1 incumbent.) We assume that the consumer’s valuation for the good is  $R$  which is finite, but large (strictly greater than  $\sigma$ ).

The firms maximize the sum of their discounted profits, which, because costs are assumed to be equal to 0, is equal to the sum of their discounted revenues; the consumer maximizes the sum of his discounted surplus ( $R$  minus total cost) of each period. They all have the same discount rate,  $\delta > 1/2$ .

We consider the subgame perfect equilibria of the game.

### 2.2 Bounds on the Incumbent’s equilibrium profits

In this section, we show that the profits of the Incumbent are bounded above by  $\sigma/(1-\delta)$ . For this, we first show that the consumer’s surplus is bounded above by  $(R - \sigma)/(1 - \delta)$ .

Along any equilibrium path, let us call  $W_t$  the surplus of the consumer in period  $t$ . If in period  $t$ , the consumer purchases from the incumbent,  $W_t$  is equal to  $R$  minus the price charged by the incumbent. If the consumer purchases from the entrant, then  $W_t$  is equal to  $R$  minus the price charged by the entrant minus  $\sigma$ . Let us call  $W = \sum_{t=1}^{+\infty} \delta^t W_t$ , the consumer’s total intertemporal surplus. We prove the following lemma.

**Lemma 1.** *In all equilibria of the game, the consumer’s intertemporal surplus is greater than or equal to  $(R - \sigma)/(1 - \delta)$ .*

*Proof.* Assume that the lemma were not true and that the upper lower bound of the set of consumer’s intertemporal surplus was  $U < (R - \sigma)/(1 - \delta)$ , which implies  $U < R - \sigma + \delta U$ , and therefore that there exists an equilibrium with an associated intertemporal consumer surplus  $W \in [U, R - \sigma + \delta U)$ .

Consider two possibilities in the first period. First, the consumer is purchasing from the Incumbent. In this case the entrant makes zero profit. Assume that it deviates by offering the good for sale for a price  $p' \in (0, R - \sigma + \delta U - W)$ . If the consumer accepts this offer, his surplus is at least

$$R - p' - \sigma + \delta U > R - (R - \sigma + \delta U - W) - \sigma + \delta U = W.$$

Therefore, whatever the continuation equilibrium after he has purchased from the entrant, the consumer would be better off, and we established the contradiction.

The second possibility is that in period 1, the consumer is purchasing from the entrant. In this case, the incumbent is making zero profit. If it makes the same offer  $p'$  as above, it makes strictly positive profits and the consumer by accepting it has a surplus of

$$R - p' + \delta U > R - (R - \sigma + \delta U - W) + \delta U = W + \sigma,$$

which establishes the contradiction. □

In any period, the sum of the profits of the firms and the surplus of the consumers is either equal to  $R$  or  $R - \sigma$ , and therefore always smaller than or equal to  $R$ . Therefore, their discounted sum over all periods is smaller than or equal to  $R/(1 - \delta)$ . From Lemma 1, and from the fact that the firms can always guarantee themselves a profit of 0 by offering the good for sale at a price of 0, we obtain the following corollary.

**Corollary 1.** *In any equilibrium, the total discounted profits of the Incumbent belong to the interval  $[0, \sigma/(1 - \delta)]$ .*

## 2.3 Two extreme equilibria

In this subsection, we construct two “extreme” equilibria. Both of them lead to equilibrium outcomes where the consumer purchases from the Incumbent in every period. In the high price equilibrium, the price will be  $\sigma$  in every period; in the low price equilibrium, it will be 0 in every period.

To describe the equilibria, it is convenient to introduce the notion of state. In each state, there is a continuation move along the equilibrium path for each of the players. If the players do choose this continuation move, the game (we will sometimes speak of system) stays in that state. After a deviation, the system can change state.

We will describe our two equilibria, through the use of two states. In both of them the entrant’s equilibrium strategy is to charge a price of 0, and the consumer’s equilibrium strategy is to purchase from the incumbent. In state A, the incumbent charges  $\sigma$  and in state B, it charges 0.

The proof constructs an equilibrium such that once the system is in one of states A or B, it stays in that state. As a consequence, we describe two equilibrium outcomes, depending on the state in which the system “starts”: if the system starts in state A, along the equilibrium path in every period the incumbent sells the good to the consumer at price  $\sigma$ ; if the system starts in state B, in every period, the incumbent sells the good to the consumer at price 0.

Along the equilibrium path, the identity of the incumbent does not change, therefore we can define

$$V_S = \frac{p_S}{1 - \delta}$$

as the value of being the Incumbent when the system starts in state  $S$ , with  $p_A = \sigma$  and  $p_B = 0$ .  $V_S$  is also the discounted payment by the customer.

After a deviation, the system may change state, but, if the agents continue playing their post deviation equilibrium strategy, it will stay in that state forever. Therefore,  $V_S$  is also the payoff of the incumbent starting in any period, whether or not there has been a deviation from the equilibrium in the previous period.

The equilibria are supported by the following “punishments”:

- If the incumbent deviates by increasing its price (above  $\sigma$  in state A and above 0 in state B), the consumer purchases from the entrant, and the system does not change state. If the consumer deviates from this “punishment” by not purchasing from the entrant, the system stays in/moves to state A.
- If the incumbent deviates by decreasing<sup>5</sup> its price, the consumer purchases from the incumbent, and the system does not change state. If the consumer decided to purchase from the entrant, then the system stays in/moves to state A.
- If the entrant offers a price strictly greater than 0, then the consumer purchases from the incumbent, and the system stays in the same state. If the consumer decided to purchase from the entrant, then the system stays in/moves to state A.
- We must distinguish three cases if the entrant charges a negative price:
  - if the system is in state A, then the consumer purchases from the entrant and the system moves to state B. If the consumer purchases from the incumbent, the system stays in state A.
  - if the system is in state B and the entrant offers a price strictly smaller than

$$\bar{p} = -\delta \frac{\sigma}{1 - \delta} - \sigma = -\delta V_A - \sigma,$$

then the consumer purchases from the entrant and the system moves to state A. If the consumer does not purchase from the entrant, then the system remains in state B.

- If the system is in state B and the entrant offers a price in  $[\bar{p}, 0)$ , then the consumer purchases from the incumbent, and the system stays in state B. If the consumer purchases from the entrant, then the system moves to state A.

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<sup>5</sup>We assume that firms can charge a negative price.

- Finally, if the consumer deviates by purchasing from the entrant, when neither the incumbent nor the entrant have deviated, the system stays in/moves to state A (of course, the entrant becomes the new incumbent in the subsequent period).<sup>6</sup>

Easy computations show that there exists no profitable single deviation for any of the players. For instance, if the system is in state B and the entrant offers a price strictly smaller than  $\bar{p}$ , then the consumer is better off purchasing from the entrant (his total cost is strictly smaller than  $\sigma + \bar{p} + \delta V_A = 0$ ), but then, the entrant's profit are at most  $\bar{p} + \delta V_A < 0$  — the entrant has no incentive to decrease the price.

The assumption  $\delta > 1/2$  is used to show that the consumer has incentives to “punish” an incumbent who would charge a strictly positive price when in state B. By purchasing from the entrant (and following the equilibrium strategy thereafter), the consumer would have total discounted costs of  $\sigma$ ; if he deviated from the punishment and purchased from the incumbent, his total cost would be strictly greater than  $0 + \delta V_A = \delta\sigma/(1 - \delta)$ , which is greater than  $\sigma$  if  $\delta > 1/2$ .

We have shown just shown that a single deviation from the equilibrium path that is never profitable for any of the players. By the one-stage deviation principle, no sequence of deviations by a player can be profitable (see Fudenberg and Tirole, 1991, pages 108–110 in particular Theorem 4.2, for details).

## 2.4 Intermediate equilibria

In the previous section, we showed that there exists an equilibrium outcome in which the incumbent charges 0 in every period, and another one in which it charges  $\sigma$ . We now show that there exists “intermediate” equilibria in which, in equilibrium, the incumbent sells to the consumer in every period at any price  $p_C \in (0, \sigma)$ , while in every period the entrant charges 0.

For this, we add a third state to the equilibrium description of the preceding section, state C. If the system is in state A or B, the equilibrium description is as above. If it is in state C, along the equilibrium path, the incumbent announces a price equal to  $p_C$ , the entrant announces a price equal to 0, and the consumer chooses to purchase from the incumbent. If there is a deviation, things unfold as follows:

- If the entrant proposes a strictly positive price or the incumbent proposes a price strictly smaller than  $p_C$ , the consumer purchases from the incumbent, and the system stays in state C no matter which firm the consumer purchases from.

*Clearly, the consumer's response is incentive compatible and the deviations by the firms unprofitable.*

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<sup>6</sup>Although, this does not affect the analysis, we should also describe the “punishment” strategies when several players deviate from the equilibrium. In our context, this can only happen when both the incumbent and the entrant deviate from the equilibrium strategy: it is sufficient to assume, for instance, that in this case, the consumer purchase from the firm which offers him the lowest cost in the current period (and from the incumbent in case of ties) and that system switches to state A whatever the consumer does. If the consumer does not follow this strategy, the system also moves to state A.

- If the entrant proposes a strictly negative price, and the consumer purchases from the entrant, the system moves to state B. If the consumer purchases from the incumbent, the system stays in state C.

*This ensures that any deviation by the entrant is unprofitable.*

- If the incumbent increases its price, then the customer purchases from then entrant and the system goes to state B. If the customer purchases from the incumbent, then the system goes to state A.

*With  $\delta > 1/2$ , the response by the consumer is incentive compatible (by the same reasoning we made for state B above) and this implies that the deviation by the incumbent is not profitable.*

- If the consumer purchases from the entrant, the system moves to state A with the entrant being the new incumbent.

*This makes the deviation by the consumer unprofitable.*

### 3 Extensions to the basic model

We now show that the same results proved in section 2 hold when there are several entrants in each period or several consumers. As we will see, the case of several consumers is the more complicated as we can find ourselves, off the equilibrium path, with several incumbents. We therefore begin with the case of several entrants.

#### 3.1 Several entrants in each period

We modify the game presented in 2.1 to allow for  $m$  entrants in each period (the number of entrants could vary with the period without changing anything).

The equilibria presented in 2.3 and 2.4 are still equilibria with the following modification:

- If following a deviation by the incumbent, the definition of equilibrium calls for the consumer to purchase from “the” entrant, the consumer purchases from, let us say, entrant 1. If the consumer either purchases from another entrant or from the incumbent, then the system moves to state A.
- Following a deviation by an entrant, if, for a similar deviation, the equilibrium of sections 2.3 and 2.4 calls for the consumer to purchase from the incumbent, the consumer still purchases from the incumbent. If the equilibrium calls for the consumer to purchase from “the” entrant, then the consumer purchases from the entrant who has deviated.

The consumer can deviate from this “punishment” strategy in three ways

1. he can purchase from the incumbent when he should have purchased from the entrant; then the continuation equilibrium is as in 2.3 or 2.4;

2. he can purchase from the entrant who has deviated rather than from the incumbent; there again the continuation equilibrium is as in 2.3 or 2.4;
3. he can purchase from one of the entrants who has not deviated; then, the continuation equilibrium is the same as when the consumer deviated in 2.3 or 2.4 by purchasing from the entrant when neither the incumbent nor the entrant had deviated: the system stays in/moves to state A.

*It is easy to check that point 3 ensures that the deviation from the punishment path is not profitable for the consumer. Therefore, the incentives of the incumbent and of the entrants are the same as in 2.3 or 2.4 — deviations are not profitable.*

- Following a deviation by the consumer, when the consumer purchases from an entrant, the continuation equilibrium is as in 2.3 or 2.4, where “the entrant” is taken to mean “the entrant from which from consumer purchased”.

## 3.2 Several consumers

We now assume that there are  $n > 1$  consumers, along with several entrants, in each period. Along the equilibrium path, the consumers all purchase from the Incumbent in every period. We will also assume that when the Incumbent or one entrant deviates, along the punishment paths all the consumers will choose to purchase from the same firm. If we assume that these punishment paths are the same as in 3.1 with “the consumer” replaced by “consumers”, it is clear that the firms have no incentive to deviate. On the other hand, we face a new situation following a deviation by one consumer who purchases the good from an entrant rather than from the Incumbent: there would be several incumbents in the next period. Therefore, to prove that consumers have no incentives to deviate, we must describe what happens along the deviation paths where we have several incumbents.

To describe the strategies of the agents along these deviation paths, we use the same technique as above: we assume that the “system” is in one of three states. When the system is in state S, the equilibrium strategy is for every incumbent to charge  $p_S$ , for the entrants to charge 0, and for the consumers to purchase from the incumbent from which they purchased in the preceding period.

First, to ensure that no individual customer has incentives to purchase from an entrant either in period 1 or in period  $t > 1$  when in periods  $\tau < t$  all consumers purchased from the Incumbent, we assume that following such a deviation, the entrant from which the consumer purchased in period  $t$  becomes a period  $t + 1$  incumbent, and the system moves to the same state as it would have if the single consumer of 2.4 had purchased from an entrant. Obviously, the incentives to deviate of a single consumer are therefore the same as in 3.1, and therefore negative.

Now, we need to ensure that after one or several deviations which have led to the presence of several incumbents, it is indeed a continuation equilibrium for the incumbents to charge  $p_S$  in state S, for the entrants to charge 0 and for the consumers to purchase from the same firm from which they purchased in the preceding period. In order to do

so, we describe the consequences of deviations by the different agents starting from a situation with several incumbents.

**Deviation by an incumbent** If an incumbent deviates, the system moves to the same state (which depends on the current state and the type of deviation) as it would have moved in 3.1; if the equilibrium of 3.1 calls for “the” consumer to purchase from an entrant, all the consumers of the deviating incumbent purchase from the same entrant, let us say entrant 1. With these modifications, the benefits that an incumbent derives from deviating are equal to the number of its customers in the previous period multiplied by the benefits of deviating for the incumbent of 3.1, and therefore negative.

**Deviation by an entrant** If the equilibrium of 3.1 calls for “the” consumer not to purchase from the entrant no consumer purchases from the entrant, and, as in 3.1, the system stays in the same state. If the equilibrium of 3.1 calls for the consumer to purchase from the entrant, all the consumers, whatever the incumbent they purchased from in the previous period, purchase from the deviating entrant. With these modifications, the benefits of deviating for an entrant are equal to  $n$  times the benefits from deviating in 3.1, and therefore negative.

**Deviation by a consumer** If a consumer purchases from an entrant or from an incumbent other than from the firm from which it purchased in the previous period, the system moves to the same state as it would have moved in 3.1. Clearly, this implies that the consumers have no incentives to purchase from an entrant. It also implies that they have no incentives to purchase from another incumbent, as all incumbents charge prices which are at least as large as the price charged by the entrants.

In order to complete the description of the equilibrium, we would also need to describe the continuation strategies after simultaneous deviations by several agents, but, as this does not affect the analysis in any way, we will spare the reader!

It may be of some interest to note that the preceding analysis carries over with minor modifications if there are some consumers with a switching cost equal to  $\sigma > 0$  and the rest a switching cost equal to 0. Assume that along the equilibrium path, the 0 switching cost consumers always purchase from an entrant, and that the strategies of the other players are independent of their behavior. Along the equilibrium path, it is clearly optimal for them to purchase for entrants. The only change in the incentives of the other players with their presence induces is to reduce the incentives for entrants to charge negative prices. Hence, the above analysis carries over.

## 4 Conclusion

In recent years, there has been a renewed interest in the folk theorem. Important extensions, in particular for stochastic games, have been provided by, for example, Fudenberg and Yamamoto (2010) and the references within. The game which we consider has an

infinite number of players and allows for consumers to be strategic players, but also, in many ways a employ a simpler structure than the stochastic game literature. We leave the exploration of the links to this literature in future work.

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