Revisiting the Delegation Problem in a Sticky Price and Wage Economy

Gregory E. Givens*
UNC-Chapel Hill
(Job Market Paper)

October 28, 2004

Abstract

In a stylized Neo-Keynesian model with Calvo-type price and wage stickiness, this paper evaluates the usefulness of delegating price level and nominal wage targets to a discretionary central bank when the monetary policy objectives are summarized by a utility-based loss function. Despite its ability to engender policy inertia, price level targeting is often dominated by inflation targeting when defined over a combination of goods-price and nominal wage inflation. Alternatively, a suitably designed wage target has desirable stabilization properties that reduce the cost of policy tradeoffs. For a number of different parameter configurations, wage targeting strictly dominates price level and inflation targeting. On a final note, a dual policy mandating separate price and wage targets nearly replicates the optimal equilibrium dynamics induced by minimization of the welfare-relevant loss function under commitment from a timeless perspective.

Keywords: Optimal Delegation, Price Level Targeting, Wage Targeting, Timeless Perspective

JEL Classification: E42, E50, E52, E58

* *Correspondence*: Department of Economics, 107 Gardner Hall CB 3305, University of North Carolina, Chapel Hill, NC 27599-3305, givens@email.unc.edu. I would like to thank Richard Froyen and Michael Salemi for invaluable guidance and advice. I have also benefited from helpful discussions with and comments from Catalin Stefanescu and participants at the UNC Macro-Money Seminar. All remaining errors and omissions are my own.

1 Introduction

Following the seminal contributions of Kydland and Prescott [24] and Barro and Gordon [3], monetary economists began to identify the inefficiencies associated with discretionary policymaking, or alternatively, the advantages of policy commitment. The particular inefficiency addressed here concerns the suboptimal response to random shocks when expectations of future policy decisions impact the current equilibrium outcome, the so-called "stabilization bias" coined by Woodford [39]. Generally speaking, a discretionary policy is purely forward-looking. The central bank observes the economy's current position, selects an optimal interest rate conditional on that observation, and makes no commitment regarding future policy behavior aside from the fact that it will conduct policy in the same manner in subsequent decision periods. An optimal policy, on the other hand, is not purely forward-looking. Instead, it requires an advanced commitment to administer an inertial response to economic disturbances that would otherwise be suboptimal under discretion. The promise to conduct policy in such a way, and the correct anticipation by the private sector, enables the policymaker to harness expectations in a way that improves the stabilization outcome.

In this paper, we take the position that the central bank is unable to commit to an optimal policy, so the prevailing monetary regime is one of discretion. In the spirit of Rogoff [30], we invoke the concept of optimal delegation to circumvent the ensuing stabilization bias. Specifically, the policymaker delegates monetary conduct to an independent central bank with an obligation to minimize an assigned quadratic loss function whose policy weight coefficients are preselected to insure maximum equilibrium welfare. In an attempt to devise a mechanism capable of mitigating the stabilization bias, a number of recent papers, including Kiley [20], Svensson [36], Dittmar and Gavin [9], and Vestin [37], suggest that assigning a target for the price level actually delivers a more favorable tradeoff between output gap and inflation volatility than the assignment of an inflation target. Additionally, Woodford [39] demonstrates that mandating an objective designed to smooth quarterly changes in the interest rate induces policy inertia, enabling a discretionary central banker to approximate the stabilization tradeoff attainable under commitment. Jensen [19], Walsh [38], Nessén and Vestin [26], and Söderström [35] argue that similar behavior can be manufactured by assigning any one of a litany of alternative targets, among them, the growth rate of nominal income, the first-difference of the output gap, a multi-period average rate of inflation, or the growth rate of the nominal money stock.

Our analysis departs from the current delegation literature along two dimensions. One, each set of authors referenced above rely on models that implicitly assume only a single source of nominal stickiness. In light of the recent theoretical contributions by Chari, Kehoe, and McGratten [6] and Huang and Lui [17] and the empirical study conducted by Christiano, Eichenbaum, and Evans [7] that collectively call into question the ability of sticky price models to generate a persistent response of real output to monetary shocks, we use a version of the model developed by Erceg, Henderson, and Levin [10] that incorporates two sources of nominal stickiness in the form of random duration staggered price and wage contracts. Two, alternative targeting regimes are ranked on a welfare-basis according to a social loss function that is derived by taking a quadratic approximation to the representative agent's lifetime utility. As illustrated by Rotemberg and Woodford [31] and Erceg *et al.* [10], the use of a utility-based criterion for social loss permits us to endogenously determine the stabilization objectives consistent with household optimization and the policy weights measuring the relative strength in which each objective is pursued. The ensuing measure of social loss contains an additional target variable disregarded in the delegation literature until now, namely, nominal wage inflation.

The aim of this paper is to conduct a normative evaluation of a set of alternative delegation schemes with an emphasis on examining the desirability of assigning price level and nominal wage targets to a discretionary central bank. Foreshadowing a number of our principal results, we find that a price level targeting regime is often welfare-dominated by an inflation targeting regime defined over a combination of goods-price and nominal wage inflation, despite the ability of the former to impart inertial policy behavior. Conversely, an optimally designed target for the aggregate nominal wage has more desirable stabilization properties that reduce the cost of achieving a given degree of price and wage inflation volatility. For numerous empirically relevant parameter configurations, wage targeting strictly dominates price level and inflation targeting. We also find that the advantages of a wage target do not preclude any gains from implementation of a suitably designed price level target. In fact, a dual stabilization policy involving separate price and wage targets nearly replicates the optimal equilibrium dynamics obtainable through minimization of the social loss function under commitment. The dominance of the combination price and wage targeting regime is robust to any conceivable variation in the structural parameters and consistently outperforms each of the alternative targeting procedures alluded to earlier.

The remainder of this paper is structured as follows. Section 2 presents the structural equations of the sticky price and wage model and the corresponding social loss function. Section 3 characterizes the optimal equilibrium dynamics using the timeless perspective concept of commitment. Section 4 presents the various discretionary targeting regimes considered and illustrates how to nest each one into a generalized loss function. Section 5 records the performance of each regime and examines the sensitivity of the results to variations in the structural parameters. Section 6 evaluates the comparative performance of several alternative targets examined previously in the literature within the sticky price and wage framework. Section 7 concludes.

2 A Sticky Price and Wage Model

The economic model is of the Neo-Keynesian variety expounded by Yun [41], Rotemberg and Woodford [31], Goodfriend and King [16], and Galí [12]. The distinguishing features of the prototype Neo-Keynesian model include utility-maximizing households and monopolistically competitive firms that set price contracts in a staggered fashion. Because of its ability to capture a reasonable degree of nominal rigidity while maintaining consistency with the underlying behavior of optimizing agents, the Neo-Keynesian environment is an attractive framework for the evaluation of alternative monetary policy strategies. As the title suggests, however, we examine the implications of using a variant of the conventional model that incorporates an additional source of nominal stickiness. In the spirit of Erceg *et al.* [10], our model accounts for sluggish adjustment in nominal wages and prices.

2.1 The Economy

The aggregate demand component is derived from first principles by taking a log-linear approximation of the intertemporal Euler equation characterizing the representative household's optimal consumption path.

Denote x_t the output gap, or the log deviation of real output from potential (a hypothetical level that would prevail in a perfectly flexible price and wage economy), and π_t the inflation rate (log difference in the price level between periods $t-1$ and t). The output gap is determined by the familiar equilibrium condition

$$x_t = E_t x_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1} - r_t^n) \tag{1}$$

where i_t is the single-period nominal interest rate, and E_t is the expectations operator conditional on information available through time t. The parameter $\sigma > 0$ represents the inverse of the intertemporal elasticity of substitution, and r_t^n is a stochastic disturbance summarizing exogenous variation in the Wicksellian natural rate of interest, the equilibrium real interest rate obtained under flexible prices and wages.[1]

The structural equations comprising aggregate supply are log-linear approximations to the first-order-conditions of a dynamic general equilibrium problem in which monopolistically competitive firms and households stagger price and wage contracts in the manner pioneered by Calvo [5]. Denote π_t^w the rate of wage inflation (log difference in the aggregate nominal wage between periods $t-1$ and t) and w_t the log of the real wage. Goods-price and nominal wage inflation and the aggregate real wage are determined by the following equilibrium conditions:

$$\pi_t = \beta E_t \pi_{t+1} + \xi_p \left(\frac{\alpha}{1-\alpha}\right) x_t + \xi_p (w_t - w_t^n) + e_{\pi t} \tag{2}$$

$$\pi_t^w = \beta E_t \pi_{t+1}^w + \xi_w \left(\frac{\phi}{1-\alpha} + \sigma\right) x_t + \xi_w (w_t^n - w_t) + e_{wt} \tag{3}$$

in conjunction with the identity

$$\pi_t^w = w_t - w_{t-1} + \pi_t \tag{4}$$

where w_t^n is the natural real wage and $\beta \in (0,1)$ measures the subjective discount factor.

Amato and Laubach [2], Sbordone [32], and Giannoni and Woodford [14] demonstrate that ξ_p and ξ_w are functions of the deep parameters governing the tastes and technologies of households and firms. Specifically,

$$\xi_p = \frac{(1-\varepsilon_p)(1-\beta\varepsilon_p)}{\varepsilon_p(1+\frac{\alpha}{1-\alpha}\theta_p)} \quad \text{and} \quad \xi_w = \frac{(1-\varepsilon_w)(1-\beta\varepsilon_w)}{\varepsilon_w(1+\phi\theta_w)}$$

where $\varepsilon_p \in (0,1)$ and $\varepsilon_w \in (0,1)$ carry information regarding the frequency of price and wage adjustments, and $\theta_p, \theta_w > 1$ are the elasticities of demand for differentiated consumption goods and labor types.[2] The parameter $\phi > 0$ measures the inverse of the elasticity of labor supply with respect to the real wage, while $\alpha \in (0,1)$ is the capital share of income.

Equation (2) is a generalization of the standard "New-Keynesian Phillips curve," coined by Roberts [28], that accounts for additional stickiness in wages. The common belief is that the output gap appears in the supply equation because firms select prices as a constant mark-up over a discounted stream of marginal costs, which are theoretically proportional to x_t. The addition of $w_t - w_t^n$ reflects the fact that marginal costs are no longer proportional to x_t in an economy with sticky wages. As the frequency of wage adjustments increases (i.e. wages become more flexible), ε_w approaches zero, ξ_w becomes unboundedly large, and the

[1] Details concerning the derivation of model are relegated to the appendix.

[2] In "Calvo" terminology, ε_p and ε_w are the fixed probabilities that firms and households will be unable to optimally reset price and wage contracts in any given period.

block given by (2) and (3) reduces to a single equation recognizable as the conventional "New-Keynesian Phillips curve" referred to above.

In order to account for the reality that stabilization goals are not often mutually attainable in the short run, we follow Clarida, Galí, and Gertler [8] by amending the supply equations with two additive disturbances.[3] The term $e_{\pi t}$ represents a "cost-push" shock, summarizing all exogenous variation in price inflation not attributed to fluctuations in marginal costs, while e_{wt} collects all exogenous shifts in the structural relationship between wage inflation and the output and wage gaps.[4] Moreover, the empirical studies by Fuhrer and Moore [11], Roberts [29], and Galí and Gertler [13] suggests that inflation is substantially inertial, so we allow $e_{\pi t}$ and e_{wt} to follow first-order autoregressive processes:

$$e_{\pi t} = \rho_\pi e_{\pi t-1} + u_{\pi t} \qquad \rho_\pi \in [0,1) \tag{5}$$

$$e_{wt} = \rho_w e_{wt-1} + u_{wt} \qquad \rho_w \in [0,1) \tag{6}$$

where $u_{\pi t}$ and u_{wt} and independent, mean-zero innovations with standard deviations σ_π and σ_w.

When prices and wages are flexible, monetary policy has no impact on the competitive equilibrium allocations; therefore, w_t^n and r_t^n depend entirely on the fundamental shocks to preferences and technologies. To simplify the following exposition, we assume that the only shock generating exogenous variation in the flexible price and wage equilibrium is a standard productivity disturbance. The variable a_t represents the productivity shock, and is assumed to follow a first-order autoregressive process

$$a_t = \rho_a a_{t-1} + u_{at} \qquad \rho_a \in [0,1) \tag{7}$$

where u_{at} is a mean-zero innovation with standard deviation σ_a. In the appendix we show that w_t^n and r_t^n can be expressed in terms of a_t and the underlying structural coefficients in the following way:

$$w_t^n = \left(\frac{1 - \frac{\alpha}{1-\alpha}\frac{\alpha+\phi}{1-\alpha}}{\frac{\alpha+\phi}{1-\alpha} + \sigma} \right) a_t \qquad \text{and} \qquad r_t^n = \left(\frac{-\sigma(1 + \frac{\alpha+\phi}{1-\alpha})(1-\rho_a)}{\frac{\alpha+\phi}{1-\alpha} + \sigma} \right) a_t.$$

Consequently, w_t^n and r_t^n inherit the same basic stochastic properties as a_t.

2.2 The Social Loss Function

The singular duty of monetary policy is to minimize the microeconomic distortions arising from the inability of firms and households to freely adjust prices and wages. A natural metric for evaluating the magnitude of these distortions is the expected utility of the representative agent. Following the recent contributions of Rotemberg and Woodford [31], Erceg *et al.* [10], Amato and Laubach [2], and Giannoni and Woodford [14], we assume that the policymaker seeks to minimize the following loss function formed by taking a quadratic approximation of the unconditional expectation of household utility around a nondistorted steady state equilibrium:

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \{\pi_t^2 + \lambda_w \pi_t^{w^2} + \lambda_x x_t^2\} \qquad \lambda_w, \lambda_x > 0. \tag{8}$$

[3] Erceg *et al.* [10] discover that incorporating sticky wages actually creates an endogenous stabilization tradeoff. Nevertheless, we include the supply shocks because, without them, our results indicated that the responses under commitment and discretion were not significantly different.

[4] The incidence of supply-side shocks does not lead to any cyclical variation in the flexible price and wage equilibrium.

Equation (8) indicates a social desire to stabilize output around potential and price and wage inflation around target rates of zero.[5]

It turns out that a policymaker can avoid the distortions caused by nominal rigidities provided that it eliminate individual price and wage dispersion, and a sufficient condition for achieving such an objective is maintaining stability of the aggregate price and wage.[6] By manufacturing an atmosphere in which suppliers who have an opportunity to select a new price choose not to deviate from an existing average of prices, the policymaker can affectively keep the aggregate price and wage constant and eliminate costly dispersion.

The nonnegative coefficients λ_w and λ_x are preference weights that measure the strength in which the policymaker pursues wage inflation and output gap stability relative to price inflation stability. One advantage of using a utility-based loss function is that it provides an endogenous determination of the size of the policy weights. In terms of the structural parameters,

$$\lambda_w = \frac{\theta_w \xi_p}{\left(\frac{1}{1-\alpha}\right)\theta_p \xi_w} \qquad \text{and} \qquad \lambda_x = \frac{\left(\frac{\alpha+\phi}{1-\alpha} + \sigma\right)\xi_p}{\theta_p}.$$

A common criticism of much of the recent work by Jensen [19], Walsh [38], Vestin [37], and Söderström [35] concerns the failure to adequately confront the relationship between the loss function and the model of the economy. These authors stipulate an "ad hoc" objective function and then adjust the policy weights while holding the structural parameters fixed, ignoring the additional cross-equation restrictions implied by utility maximization. By making the objectives of households, firms, and the government mutually compatible, we impose an unambiguous relationship between the policy weight coefficients and the structural parameters. For example, when the frequency of price changes falls, ε_p rises and ξ_p falls, causing a reduction in λ_w and λ_x. Thus, increases in the average duration of price contracts weakens the policymaker's preference for stabilizing wage inflation and the output gap relative to price inflation. Similarly, when the frequency of wage adjustments falls, ε_w rises and ξ_w falls, causing λ_w to rise. So in contrast, increasing the mean duration of wage contracts strengthens the policymaker's resolve for stabilizing wage inflation relative to the output gap and price inflation.

3 Optimal Policy From a Timeless Perspective

As a benchmark for the comparison of alternative targeting regimes under discretion, we first characterize the optimal equilibrium dynamics for $\{\pi_t, \pi_t^w, x_t, w_t\}$ that minimize (8) under commitment subject to the

[5]The assumption of monopoly power implies that potential output is inefficiently low, and so (8) should include a positive target value for the output gap. We assume that it is the duty of fiscal policy to mitigate the distortions caused by imperfect competition. Accordingly, the optimal output gap target is zero. For a detailed discussion concerning the derivation of (8), see Woodford [40].

[6]Erceg *et al.* [10] demonstrate that the addition of sticky wages makes it impossible to simultaneously achieve price and wage stability. Likewise, the inclusion of $e_{\pi t}$ and e_{wt} to (2) and (3) produce similar tradeoffs. Naturally, an optimal policy will exhibit a nonzero variance of price and wage inflation.

behavioral constraints given by (2) – (4).[7] We follow Woodford [39] and formulate the Lagrangian

$$\mathcal{L} = \min_{\{x_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \{[\pi_t^2 + \lambda_w \pi_t^{w^2} + \lambda_x x_t^2]$$
$$+2\varphi_{pt}[\pi_t - \beta\pi_{t+1} - \xi_p \left(\frac{\alpha}{1-\alpha}\right) x_t - \xi_p(w_t - w_t^n) - e_{\pi t}]$$
$$+2\varphi_{wt}[\pi_t^w - \beta\pi_{t+1}^w - \xi_w \left(\frac{\phi}{1-\alpha} + \sigma\right) x_t - \xi_w(w_t^n - w_t) - e_{wt}]$$
$$+2\upsilon_t[w_t - w_{t-1} + \pi_t - \pi_t^w]\} \tag{9}$$

where φ_{pt}, φ_{wt}, and υ_t are the Lagrange multipliers associated with constraints (2), (3), and (4), respectively.[8]

Differentiating (9) gives us a system of first-order conditions

$$\pi_t + \varphi_{pt} - \varphi_{pt-1} + \upsilon_t = 0 \tag{10}$$

$$\lambda_w \pi_t^w + \varphi_{wt} - \varphi_{wt-1} - \upsilon_t = 0 \tag{11}$$

$$\lambda_x x_t - \xi_p \left(\frac{\alpha}{1-\alpha}\right) \varphi_{pt} - \xi_w \left(\frac{\phi}{1-\alpha} + \sigma\right) \varphi_{wt} = 0 \tag{12}$$

$$\upsilon_t = \xi_p \varphi_{pt} - \xi_w \varphi_{wt} + \beta E_t \upsilon_{t+1} \tag{13}$$

for any $t \geq 0$. The optimality requirements (10) – (13), together with (2) – (4) and the initial conditions $\varphi_{p(-1)} = \varphi_{w(-1)} = 0$, fully characterize the optimal state-contingent paths for $\{\pi_t, \pi_t^w, x_t, w_t, \varphi_{pt}, \varphi_{wt}, \upsilon_t\}$. From an operational standpoint, however, the problem with this equilibrium is that it is not time consistent. Woodford [39] demonstrates that there is an alternative concept of commitment generating the optimal equilibrium response to the exogenous shocks that also satisfies the principle of time consistency. Instead of imposing the boundary conditions $\varphi_{p(-1)} = \varphi_{w(-1)} = 0$, imagine that (10) – (13) hold for any $-\infty < t < \infty$. Woodford describes this notion of equilibrium as optimal from a "timeless perspective" because it forbids the policymaker from exploiting the existing stance of private sector expectations in the initial period.

To find such a policy, eliminate the Lagrange multipliers from (10) – (13). All of the information contained in the first-order conditions collapses to the following time-invariant criterion that involves only leads and lags of the variables in the loss function:

$$\kappa(\xi_p \pi_t - \lambda_w \xi_w \pi_t^w) + (\xi_p + \xi_w) q_t + [q_t - q_{t-1} - \beta E_t q_{t+1} + \beta E_{t-1} q_t] = 0 \tag{14}$$

where the variable q_t satisfies

$$q_t = \xi_p \left(\frac{\alpha}{1-\alpha}\right) \pi_t + \lambda_w \xi_w \left(\frac{\phi}{1-\alpha} + \sigma\right) \pi_t^w + \lambda_x (x_t - x_{t-1}) \tag{15}$$

and $\kappa = \xi_w \left(\frac{\phi}{1-\alpha} + \sigma\right) - \xi_p \left(\frac{\alpha}{1-\alpha}\right)$. Because the optimal target criterion is consistent with the first-order conditions, the joint equilibrium dynamics of $\{\pi_t, \pi_t^w, x_t, w_t, q_t\}$ implied by (2) – (4) and (14) – (15) produces the desired state-contingent evolution characterized above.

[7]We treat the output gap as the policy instrument, and then subsequently use the aggregate demand schedule to find the interest rate path that is consistent with the optimal path of the output gap.

[8]The law of iterated expectations allows us to drop the conditional expectations operators from (2) and (3).

To examine the model's dynamic properties under the optimal rule, we first solve the resulting system of expectational difference equations. Denote $Z_{1t} = [w_t^n \ e_{\pi t} \ e_{wt} \ w_{t-1} \ x_{t-1} \ q_{t-1} \ E_{t-1}q_t]'$ the column vector of predetermined state variables, $Z_{2t} = [\pi_t \ \pi_t^w \ w_t \ x_t \ q_t \ E_t q_{t+1}]'$ the vector of forward-looking variables, and $\eta_t = [\psi u_{at} \ u_{\pi t} \ u_{wt} \ 0 \ 0 \ 0 \ 0]'$ the vector of innovations to Z_{1t}.[9] In compact notation, the system can be written as

$$\Gamma \begin{bmatrix} Z_{1t+1} \\ E_t Z_{2t+1} \end{bmatrix} = \Lambda \begin{bmatrix} Z_{1t} \\ Z_{2t} \end{bmatrix} + \begin{bmatrix} \eta_{t+1} \\ 0_{6\times 1} \end{bmatrix} \qquad \eta_t \sim (0, \Sigma_\eta) \tag{16}$$

where Γ and Λ are matrices containing the various structural parameters and policy weights, and Σ_η is a covariance matrix.[10] We seek a unique bounded solution to the system given by (16) of the form

$$Z_{2t} = \Phi Z_{1t} \tag{17}$$

where Φ is a matrix characterizing the linear mapping of the state vector into the decision variables. Because Γ is singular by construction, we follow the technique expounded in Klein [23] which uses the generalized Schur form to separate (16) into stable and unstable blocks of equations. Paralleling the conditions in Blanchard and Kahn [4], a unique bounded solution exists of the form (17) provided that the number of stable eigenvalues (i.e. the number equations comprising the stable block) equals the number of predetermined variables. We check numerically that the determinacy condition is satisfied for the various parameter constellations used in this paper.

Having solved for the rational expectations equilibrium, we now describe the quintessential features of the model's optimal response to stochastic shocks. Figure 1 illustrates the response of the output gap to a simultaneous, one standard deviation shock to $e_{\pi t}$ and e_{wt}. By setting $\rho_\pi = \rho_w = 0$, we make the disturbances purely transitory; nonetheless, the optimal response to upward pressure on price and wage inflation is an immediate contraction of the output gap followed by a measured ascent back to its long run target. In other words, monetary policy remains tight for several periods after the realization of the shock. Woodford [39] labels this characteristic of commitment "optimal monetary policy inertia." Due to the length of time in which output is held below potential, π_t and π_t^w, while initially positive, overshoot their respective long-run target values and remain negative for a number of periods. The tendency to overshoot has implications for the behavior of the actual price level and the nominal wage. Indeed, Figure 1 illustrates that prolonged episodes of deflation cause prices and wages to descend back to the same paths anticipated prior to the realization of the shocks. An optimal plan, therefore, generates stationary fluctuations in prices and wages.

4 Targeting Regimes under Discretion

We now shift focus to the central goal of this paper, the design of alternative stabilization targets when the central bank operates with discretion. According to Jensen [19], a targeting regime is an "institutional

[9] The coefficient $\psi = \left(\frac{1 - \frac{\alpha}{1-\alpha}\frac{\alpha+\phi}{1-\alpha}}{\frac{\alpha+\phi}{1-\alpha}+\sigma} \right)$.

[10] The elements of Γ, Λ, and Σ_η are presented in the appendix.

set-up" compelling the central bank to minimize an assigned loss function whose policy weight coefficients are preselected to insure the lowest possible social loss as measured by (8).

The family of regimes considered are defined by a set of target variables that include π_t, π_t^w, x_t, the price level, p_t, and the nominal wage, n_t. We nest each regime in a general loss function of the form

$$L^d = E_0 \sum_{t=0}^{\infty} \beta^t \{(1+f_\pi)\pi_t^2 + (\lambda_w + f_w)\pi_t^{w^2} + \lambda_x x_t^2 + g_p p_t^2 + g_w n_t^2\} \tag{18}$$

where λ_w and λ_x are the same weights appearing in (8). The justification for designating p_t and n_t as potential target variables is based on the fact that prices and wages are stationary in an optimal equilibrium. The policymaker can induce stationarity in a discretionary environment by making p_t and n_t explicit stabilization objectives. The policy weight coefficients, $\{f_\pi, f_w, g_p, g_w\}$, are chosen optimally to minimize the asymptotic value of (8) prior to the delegation of monetary authority to the central bank. Each regime is demarcated by certain constraints placed on the values of the chosen weights.

The first regime considered is *pure discretion* (PD) in which case $f_\pi = f_w = g_p = g_w = 0$. PD amounts to discretionary optimization of the welfare-theoretic loss function because the target variables and policy weights are identical to those of society. It provides a natural reference point for quantifying the gains from designing alternative targets.

The second regime, *inflation targeting* (IT), has been the topic of a large body of recent literature. While it is now widely accepted that the aim of an IT policy is to stabilize some measure of price inflation, we generalize this popular concept by expanding the set of objectives to include wage inflation. Specifically, IT is the case where $f_\pi \in [-1, \infty)$, $f_w \in [-\lambda_w, \infty)$, and $g_p = g_w = 0$. Notice that while the target variables coincide with the ones in (8), the weights assigned to these objectives may differ from their socially optimal counterparts. Values of $f_\pi, f_w > 0$, for instance, correspond to the appointment of a "conservative central banker" (using the terminology of Rogoff [30]) because additional emphasis is placed on attaining inflation stability relative to output gap stability.

Vestin [37] argues convincingly that under certain conditions, a suitably designed price level target is equivalent to inflation targeting with commitment. In light of this finding, the third regime, *price level targeting* (PT), requires that $g_p \in [0, \infty)$, $f_\pi = -1$, $f_w = -\lambda_w$, and $g_w = 0$. Under PT, the central bank directly pursues stabilization of only the price level and the output gap.

Due to the apparent success of PT in some forward-looking models, we also explore the possible benefits of implementing an explicit wage target. The fourth regime, *nominal wage targeting* (WT), is defined as the case where $g_w \in [0, \infty)$, $f_\pi = -1$, $f_w = -\lambda_w$, and $g_p = 0$. Under WT, the central bank is only concerned with stabilizing the nominal wage and the output gap.

The fifth and final regime considered is called *price and wage targeting* (PWT), a combination policy where $g_p \in [0, \infty)$, $g_w \in [0, \infty)$, $f_\pi = -1$, and $f_w = -\lambda_w$. A PWT strategy seeks an optimum balance in price, nominal wage, and output gap stability. Clearly, PWT encompasses PT and WT as special cases. If the optimal value of g_w turns out to be zero, for instance, then PWT is equivalent to PT. The combination policy serves primarily to illustrate the added gain of implementing separate price and wage targets. Each of the targeting regimes considered is "flexible" in the usual sense of the word because the policymaker values a certain degree of real stability as measured by variation in the output gap.

To solve for the equilibrium dynamics implied by discretionary optimization, we cast the model into state-space form. First, as a matter of convenience, we rewrite the aggregate supply equations in terms of the price level and the nominal wage using the identities $\pi_t = p_t - p_{t-1}$, $\pi_t^w = n_t - n_{t-1}$, and $w_t = n_t - p_t$. Denote $X_{1t} = [w_t^n \; e_{\pi t} \; e_{wt} \; p_{t-1} \; n_{t-1}]'$ the vector of exogenous and endogenous predetermined variables, $X_{2t} = [p_t \; n_t]'$ the vector of forward-looking variables, and $\varepsilon_t = [\psi u_{at} \; u_{\pi t} \; u_{wt} \; 0 \; 0]'$ the vector of innovations to X_{1t}. Again, we treat the output gap, x_t, as the policy instrument to simplify the ensuing exercise. Next, stack the policy constraints in the following way:

$$\begin{bmatrix} X_{1t+1} \\ \Omega E_t X_{2t+1} \end{bmatrix} = A \begin{bmatrix} X_{1t} \\ X_{2t} \end{bmatrix} + B x_t + \begin{bmatrix} \varepsilon_{t+1} \\ 0_{2\times 1} \end{bmatrix} \qquad \varepsilon_t \sim (0, \Sigma_\varepsilon) \tag{19}$$

where Ω, A, and B are matrices of structural parameters, and Σ_ε is a covariance matrix.[11] Similarly, denote $G_t = [\pi_t \; \pi_t^w \; x_t \; p_t \; n_t]'$ the vector containing each of the potential target variables. G_t is related to the state vector and the policy instrument by the following linear relationship:

$$G_t = C \begin{bmatrix} X_{1t} \\ X_{2t} \end{bmatrix} + D x_t \tag{20}$$

where

$$C = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}.$$

Formulating the modified loss function in terms of G_t, the central bank's control problem entails minimizing

$$L^d = E_0 \sum_{t=0}^{\infty} \beta^t G_t' Q G_t \qquad Q = \begin{bmatrix} 1+f_\pi & 0 & 0 & 0 & 0 \\ 0 & \lambda_w + f_w & 0 & 0 & 0 \\ 0 & 0 & \lambda_x & 0 & 0 \\ 0 & 0 & 0 & g_p & 0 \\ 0 & 0 & 0 & 0 & g_w \end{bmatrix}$$

subject to (19), where Q is a diagonal matrix whose nonzero elements are the policy weight coefficients. The discretionary outcome corresponds to a Markov-perfect equilibrium in which the central bank reoptimizes every period taking the expectations of households and firms as exogenous.[12]

To determine the policy weights characterizing each regime, we perform a numerical search over the acceptable parameter space of $\{f_\pi, f_w, g_p, g_w\}$. For a given set of weights corresponding to a particular regime, use the reduced-form solution to the model under discretion to calculate the asymptotic value of the social loss function (8). Continue searching over the allowable parameter space until social loss reaches a minimum.

[11] Details on the construction of Ω, A, B, and Σ_ε are found in the appendix.

[12] Söderlind [34] demonstrates how to perform the optimal control exercise when the constraints are inherently forward-looking.

Parameter	Value	Description
β	0.99	Subjective discount factor
α	1/3	Capital share of income
σ	1.5	Inverse of the intertemporal elasticity of substitution
ϕ	1	Inverse of the real wage elasticity of labor supply
θ_p	6	Elasticity of demand for a specific consumption good
θ_w	6	Elasticity of demand for a specific labor input
$1-\varepsilon_p$	0.5	Fraction of firms resetting prices optimally
$1-\varepsilon_w$	0.5	Fraction of households resetting wages optimally
σ_a	0.01	Standard deviation of technology shock
σ_π	0.05	Standard deviation of price inflation shock
σ_w	0.05	Standard deviation of wage inflation shock
ρ_a	0.95	Serial correlation of the technology shock
ρ_π	0	Serial correlation of the price inflation shock
ρ_w	0	Serial correlation of the wage inflation shock

Table 1: Calibrated parameter values

5 Comparative Regime Perfomance

The aim of this section is to assess the relative performance of the targeting regimes introduced above. As the conclusions depend heavily on the chosen parameterization, we first discuss model calibration, using values consistent with numerous empirical studies and then repeat the exercise for a broad range of values in an attempt to demonstrate robustness.

5.1 Model Calibration

Of critical importance are the parameters governing the distributional properties of the structural shocks. Recent empirical contributions by Kim [22] and Ireland [18] reinforce the longstanding belief that productivity shocks are small but highly persistent. Accordingly, we fix $\sigma_a = 0.01$ and $\rho_a = 0.95$. There is, unfortunately, little consensus regarding the size and persistence of supply shocks. To avoid biasing our results, we set $\sigma_\pi = \sigma_w = 0.05$, imposing equal volatility of shocks to the price and wage sectors. Initially, we make the supply shocks purely transitory by fixing $\rho_\pi = \rho_w = 0$, but later relax this assumption to examine whether additional persistence alters the relative performance of competing regimes.

Concerning the time rate of preference, we fix $\beta = 0.99$, matching closely the estimates reported in Kim [22], Amato and Laubach [2], and Ireland [18] and identical to the calibrated value used in most of the policy literature. We set α, the Cobb-Douglas parameter representing the capital share of income, equal to $1/3$. As for the parameters describing household preferences, we fix $\sigma = 1.5$ and $\phi = 1$, values close to those reported in Smets and Wouters [33] and Laforte [25].[13] Concerning the elasticity terms, we set $\theta_p = \theta_w = 6$,

[13]Each set of authors use Bayesian techniques to estimate a stochastic dynamic general equilibrium model with the same

implying a 20% steady state markup of prices over marginal cost and the real wage over the marginal rate of substitution, within reasonable proximity to the point estimates provided by Rotemberg and Woodford [31], Amato and Laubach [2], and Christiano *et al.* [7].

Finally, the empirical literature contains mixed opinions concerning the absolute and relative degrees of price and wage rigidity. Smets and Wouters [33], for instance, conclude that ε_p and ε_w, the probabilities that firms and households are unable to renegotiate current prices, are close to 0.9, meaning that the average lifespan of a contract is 10 quarters. Christiano *et al.* [7] report values in the vicinity of 0.6, implying that contract lengths are roughly 2.5 quarters instead. The estimates provided by Givens [15] and Kim [21], on the other hand, indicate that while the mean duration of price contracts are approximately 2 quarters, wage contracts last up to 4 quarters on average. For the numerical exercise that follows, we initially set $\varepsilon_p = \varepsilon_w = 0.5$, imposing an equal degree of nominal rigidity in prices and wages and then later vary both along the entire unit interval.

5.2 Policy Evaluation

The following tables display the value of social loss, the optimal policy weights (when relevant), the standard deviations of $\{\pi_t, \pi_t^w, x_t\}$, and the welfare cost of deviating from the timeless perspective expressed as a permanent output gap.[14] The leftmost column of numbers are those associated with the optimal timeless perspective (TP) policy.

5.2.1 Baseline Configuration

Inspection of Table 2 reveals the principal inefficiencies of PD and IT for the baseline parameterization. Not only are the standard deviations of $\{\pi_t, \pi_t^w, x_t\}$ uniformly larger than their counterparts under TP, the welfare cost associated with moving from TP to PD or IT is equivalent to a permanent output gap of more than 7%.[15] Furthermore, the optimal weights assigned to π_t and π_t^w under IT indicate little incentive to appoint a "Rogoff" conservative central banker. Figure 2 plots the responses of the output gap to positive, simultaneous supply shocks under the TP and PD policies.[16] In contrast to the inertial property of commitment, PD is characterized by a large single-period monetary contraction. The absence of inertia prevents π_t and π_t^w from mimicking the optimal "overshooting" behavior and generates nonstationary fluctuations in p_t and n_t. The excessive volatility in π_t, π_t^w, and x_t observed under PD and IT is undoubtedly a direct result of the inability to induce a persistent monetary response to random shocks.

The fourth column of Table 2 highlights the first basic conclusion of this paper, that PT is dominated by IT for a plausible calibration of the model. The welfare cost of moving from TP to PT is equivalent to a sustained output gap in excess of 8.5%. Moreover, the standard deviations indicate that while PT achieves a more desirable price inflation volatility than IT, it permits an immoderate level of wage inflation volatility

utility structure as the one employed here.

[14] Social loss is multiplied by 100, while the standard deviations and output costs are given in percent.

[15] To compute the welfare cost of regime i, we follow Jensen [19] by equilibrating the loss differential with the loss produced by a constant output gap for $t = 0, \ldots, \infty$: $Loss^i - Loss^{\mathrm{TP}} = \Delta L = (1-\beta)^{-1}\lambda_x\delta^2$, where δ is the permanent value of the output gap.

[16] We do not display the impulse response function associated with IT because it is essentially identical to PD.

	TP	**PD**	**IT**	**PT**	**WT**	**PWT**
Social Loss	25.054	29.042	29.041	30.469	25.809	25.089
Output Cost	–	7.358	7.358	8.575	3.200	0.687
Optimal f_p	–	–	-0.022	–	–	–
Optimal f_w	–	–	0.008	–	–	–
Optimal g_p	–	–	–	0.715	–	1.025
Optimal g_w	–	–	–	–	1.345	1.353
S.d. of π	3.578	3.735	3.739	3.477	3.788	3.585
S.d. of π^w	2.571	2.713	2.707	3.826	2.506	2.570
S.d. of x	8.131	9.631	9.639	4.836	7.795	8.121

Table 2: Results for baseline parameterization

and over-stabilizes the output gap. This particular result contrasts somewhat with the findings reported by Vestin [37], who demonstrates that in a purely forward-looking model with a traditional Calvo-style Phillips curve, commitment to an inflation target is equivalent to price level targeting under discretion.[17] Walsh [38] later provides corroboratory evidence by showing that price level targeting unambiguously yields the best discretionary outcome when the inflation process features little or no endogenous persistence.

In any model with forward-looking elements, the success of a price level target is predicated on the inherent ability of expectations to prevent undesirable volatility. Because the price level is persistent, the central bank's optimal response to supply shocks involves a sustained adjustment of the output gap. The private sector anticipates this pattern of conduct, causing inflation expectations to shift in a way that improves the current stabilization outcome. Figure 3 shows that the same mechanism is at work in our model, specifically, PT generates a persistent contraction of the output gap in the face of supply shocks, enabling π_t and π_t^w to overshoot their target values and imposing stationarity in p_t and n_t. Yet, PT is still inferior to the non-inertial IT policy because of its failure to minimize inefficient wage dispersion as indicated by the suboptimal standard deviation of π_t^w. Evidently, this shortcoming outweighs any advantages PT may have as a result of policy inertia.

The fifth column exemplifies the second major result of this paper, that a policy designed to stabilize the nominal wage delivers a more favorable outcome than either a price level or inflation targeting policy. In contrast to PT and IT, the welfare cost of moving from TP to WT is identical to a permanent output gap of a more modest 3.2%. Additionally, the standard deviations demonstrate that while WT generates a mildly inefficient level of price inflation volatility, the implied variance of wage inflation is nearly optimal, and unlike PT, does not impose over-stability of the output gap.

There is nothing intrinsically more distorting about wage stickiness than price stickiness, yet the sizeable advantages of WT prevail for two reasons. One, for the baseline configuration, $\lambda_w \approx 1.17$, implying that the variance of π_t^w should be somewhat smaller than the variance of π_t under an optimal policy. For a given

[17] It is somewhat difficult to make a direct comparison here between our results and Vestin's because his definition of inflation targeting does not include a measure of wage inflation.

	TP	PD	IT	PT	WT	PWT
Social Loss	95.155	135.966	113.456	128.050	105.153	96.134
Output Cost	–	23.541	15.764	21.135	11.652	3.647
Optimal f_p	–	–	1.982	–	–	–
Optimal f_w	–	–	3.249	–	–	–
Optimal g_p	–	–	–	0.395	–	0.595
Optimal g_w	–	–	–	–	0.779	0.672
S.d. of π	6.544	8.018	6.833	5.888	7.844	6.626
S.d. of π^w	3.907	5.643	2.959	7.711	3.126	3.867
S.d. of x	22.292	22.463	28.243	19.139	21.673	22.392

Table 3: Persistence in supply shocks ($\rho_\pi = \rho_w = 0.7$)

variance of π_t^w, a policy designed to target the nominal wage is more desirable because it requires a lower output gap volatility than one focused on sustaining a price level target. The second reason is that WT entails a larger contemporaneous response to supply shocks than PT, curtailing the rise of inflationary expectations and thus improving the stabilization tradeoffs. Recall that a discretionary central banker assigned the goals of stabilizing p_t or n_t on the one hand and x_t on the other will pursue a "lean against the wind" policy. The vigor with which the central bank leans depends positively on the benefit from an incremental reduction of prices or wages per unit of output loss, or equivalently, on the magnitude of the output gap elasticity of inflation (i.e. the slope of the Phillips curve).[18] For the baseline parameterization, the output gap elasticity of π_t^w is greater than the corresponding elasticity of π_t, implying that the policymaker should, under WT, pursue wage stability with more intensity than price stability under PT. This property is reflected in the fact that the optimal value of g_w is nearly twice that of g_p.

Figure 3 reinforces the concept graphically. The aggressive output gap contraction in response to simultaneous supply shocks under WT comes closer to approximating the TP response. As a consequence of the more temperate reaction prescribed by PT, the rise in wage inflation is too severe, elevating the standard deviation of π_t^w and depressing the standard deviation of x_t. The advantages of WT come from the ability to impart the proper degree of output gap inertia. Because the model does exhibit endogenous persistence in the nominal wage, disturbances will have enduring effects. A promise to uphold a wage target is desirable since it requires a sustained adjustment of the output gap for as long as the actual wage is misaligned with the chosen target path. Designing a target for the nominal wage is simply a clever wage of engineering policy inertia for a discretionary central bank that would otherwise only care about the volatility of $\{\pi_t, \pi_t^w, x_t\}$.

The sixth column illustrates our last principal result of this paper, that the combination PWT regime outperforms all others considered and nearly replicates the equilibrium attainable under the TP policy. To illustrate the magnitude of the welfare cost associated with deviating from an optimal policy, the loss of moving from TP to PWT is tantamount to a permanent output gap of only 0.687%, and the standard

[18] As indicated by the aggregate supply block (2) - (3), the coefficient, $\xi_p\left(\frac{\alpha}{1-\alpha}\right)$, measures the output gap elasticity of price inflation, and the coefficient, $\xi_w\left(\frac{\phi}{1-\alpha}+\sigma\right)$, represents the output gap elasticity of wage inflation.

	TP	**PD**	**IT**	**PT**	**WT**	**PWT**
Social Loss	56.881	62.454	60.489	58.348	59.684	56.998
Output Cost	–	8.699	6.999	4.462	6.170	1.257
Optimal f_p	–	–	-0.341	–	–	–
Optimal f_w	–	–	1.326	–	–	–
Optimal g_p	–	–	–	0.396	–	0.954
Optimal g_w	–	–	–	–	2.329	1.155
S.d. of π	7.149	7.331	7.397	7.137	7.544	7.162
S.d. of π^w	1.535	1.806	1.138	2.135	1.088	1.531
S.d. of x	7.148	8.750	8.235	6.262	5.429	7.115

Table 4: Larger variance of price inflation shock ($\sigma_\pi = 0.10$ and $\sigma_w = 0.025$)

deviations of $\{\pi_t, \pi_t^w, x_t\}$ are practically identical in the two cases. It is our contention that PWT is a pragmatic and transparent way of reaping the benefits of commitment in a situation where the central bank must act in a time consistent manner. Instead of having to convey to the public a complicated targeting criterion like equation (14), the policymaker can perform the simpler task of assigning the optimal price and wage targets to an otherwise independent central bank.

5.2.2 Persistent Supply Shocks

Before testing the sensitivity of our results to variations in the structural parameters, we repeat the analysis for alternative assumptions about the distributional properties of the supply shocks. First, we document the implications of adding serial correlation by setting $\rho_\pi = \rho_w = 0.7$. In an environment with persistent shocks, the difference between IT and PD is no longer trivial. Table 3 shows that the welfare cost of IT is equivalent to a permanent output gap of approximately 15.7%, while the same calculation for PD exceeds 23.5%. When supply shocks have prolonged effects on π_t and π_t^w, the prospect of greater stability is enhanced by delegating authority to a central bank with a larger priority on stabilizing inflation, a "Rogoff" conservative.

Although the addition of serial correlation undermines the performance of PT and WT compared to the baseline configuration, it has little impact on their comparative ranking among alternatives for the same reasons discussed above. PT remains inferior to IT, generating a welfare cost equivalent to a permanent output gap of over 21%. WT, on the other hand, dominates PT and IT, inducing a similar cost comparable to a lasting output gap change of about 11.6%. When supply shocks are persistent, however, the relevant policy weights, g_p and g_w, are smaller than in the baseline case, indicating a more relaxed position on price and wage stability. Figure 4 plots the impulse response functions for TP, PT, and WT to simultaneous supply shocks. Notice that while π_t and π_t^w continue to overshoot their target paths, the process is gradual. The delayed pace of disinflation enables p_t and n_t to linger above target for several periods. Consequently, PT and WT emulate the response patterns corresponding to TP only if the policy weights warrant a mild pursuit of the price and wage targets. Finally, in the event that it is feasible to pursue simultaneous targets for prices and wages, the joint PWT regime is again remarkably efficient, manufacturing a welfare cost

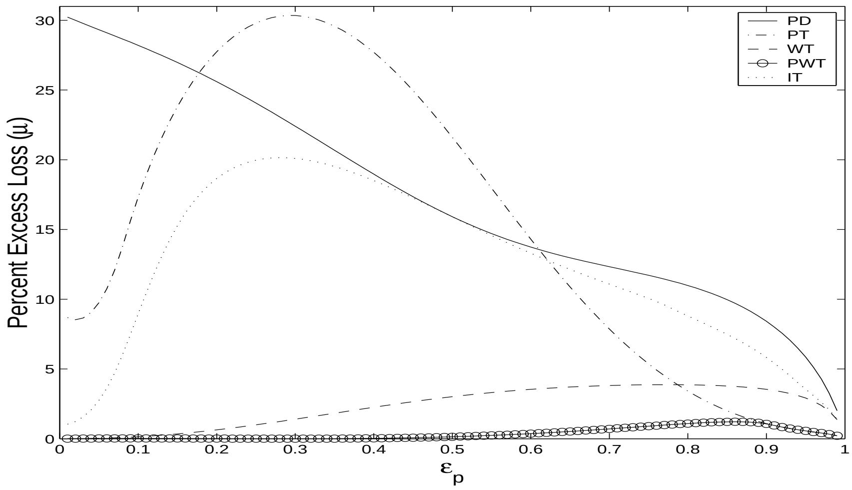

Figure 5: Excess Loss (μ) for $\varepsilon_p \in (0, 1)$

equivalent to a permanent output gap of roughly 3.6%.

5.2.3 Higher Variance of Price Level Shocks

We now investigate the consequences of changing the relative magnitudes of the two supply shocks. The figures presented in Table 4 are calculated under the assumption that disturbances to the price sector are four times the size of shocks to the wage sector by setting $\sigma_\pi = 0.10$ and $\sigma_w = 0.025$. Without serial correlation, it is not surprising that IT only moderately outperforms PD. In contrast to our earlier findings, however, it appears that PT dominates WT as a means of stabilizing $\{\pi_t, \pi_t^w, x_t\}$. To achieve a given variance of π_t, WT requires greater output gap volatility than PT. Now that the variance of π_t represents the largest contributor to social loss, PT is naturally the preferred regime. Strikingly, the differences are marginal at best, indicating that shocks to the price sector would have to be unusually large relative to wage shocks for PT to measurably outperform WT. Lastly, the dominance of PWT seems robust to changes in the relative magnitude of supply shocks, as the welfare departure from TP is trivial.

5.3 Sensitivity Analysis

To insure that our conclusions are not overly sensitive to the chosen calibration, we repeat the analysis for alternative values of the structural parameters. In the illustrations that follow, we plot the amount of excess social loss for each regime expressed as a fraction of the loss under the timeless perspective by varying $\{\varepsilon_p, \varepsilon_w, \sigma, \phi, \alpha\}$ in a continuous fashion within a neighborhood encompassing their respective baseline values. In other words, for regime i, we plot the function

$$\mu^i(\varepsilon_p, \varepsilon_w, \sigma, \phi, \alpha) = \frac{Loss^i - Loss^{\text{TP}}}{Loss^{\text{TP}}} \times 100 \tag{21}$$

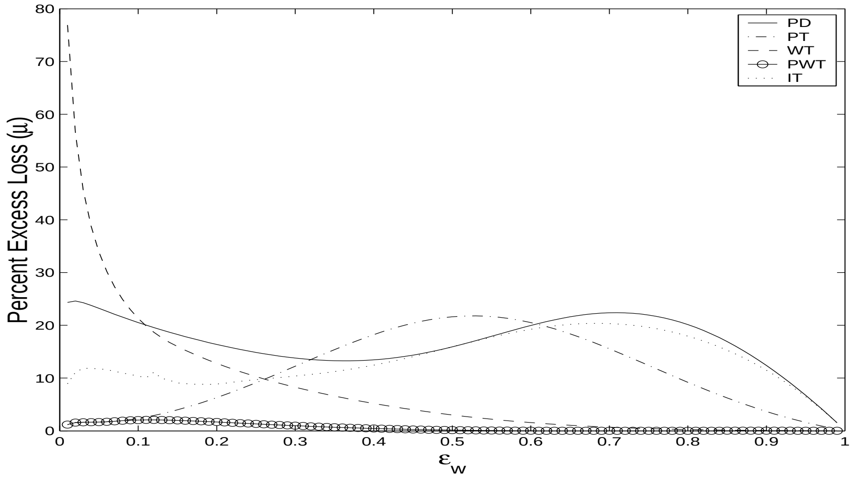

Figure 6: Excess Loss (μ) for $\varepsilon_w \in (0, 1)$

where i is a member of the family $\Psi = \{\text{PD}, \text{IT}, \text{PT}, \text{WT}, \text{PWT}\}$. For all $i, j \in \Psi$, regime i dominates regime j if and only if $\mu^i < \mu^j$.[19]

Figure 5 shows how excess social loss varies for each regime when altering ε_p across the entire unit interval. Due to the inability of a policy focused on sustaining a price level target to adequately address the problem of wage inflation volatility, IT and even PD outperform PT for a wide range of plausible values. For $\varepsilon_p > 0.63$, however, the output gap elasticity of price inflation, $\xi_p \left(\frac{\alpha}{1-\alpha}\right)$, is small enough to cause a reversal in the relative performance of PT to IT. As this elasticity term shrinks, a given level of inflation stability requires a more aggressive response of x_t under IT because that policy prescribes only a one-period contraction in the aftermath of a transitory supply shock. A PT regime, on the other hand, requires a multi-period contraction because the same shocks have an inertial effect on the price level but only a transitory effect on the inflation rate. Consequently, PT distributes "policy medicine" in smaller doses over a longer horizon, reducing the variability of both inflation and the output gap through its effect on expectations.

Figure 5 also reaffirms our central argument concerning the superiority of WT relative to PT. WT strictly dominates for $\varepsilon_p < 0.79$, but as a consequence of the inverse relationship between ε_p and λ_w, values above this threshold indicate a sharp decline in the social desire to stabilize π_t^w. Only when the stability of price inflation becomes the principal focus of monetary policy, will PT dominate WT, but surprisingly, even for large ε_p, the difference amounts to only a few percentage points. Interestingly, the figure suggests that if the mean duration of wage contracts is 2 quarters, price contracts must be fixed for a term of at least 5 quarters before PT outperforms WT. To date, we are unaware of any studies suggesting that renegotiation of wage contracts occur with much greater frequency than price contracts.

Lastly, Figure 5 reveals that the optimality of PWT is robust to any conceivable variation in ε_p. For $\varepsilon_p \in [0.15, 0.87]$, it is optimal to mandate separate price and wage targets; however, PWT is equivalent to

[19] In each of the following exercises, we vary one of the structural parameter while keeping the others fixed at their baseline values.

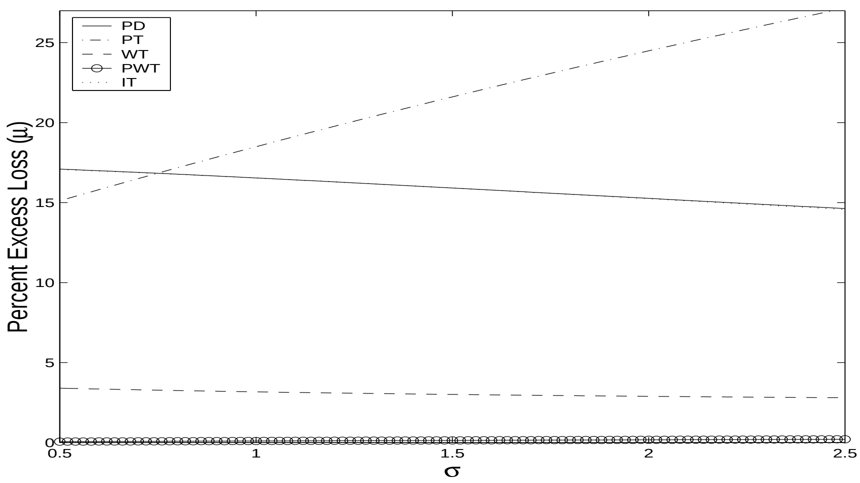

Figure 8: Excess Loss (μ) for $\sigma \in (0.5, 2.5)$

PT when $\varepsilon_p > 0.87$ and equivalent to WT when $\varepsilon_p < 0.15$.

Figure 6 plots excess loss for each regime by varying ε_w along the unit interval. For $\varepsilon_w \in [0.27, 0.62]$, IT dominates PT, but like the previous illustration, PT outperforms IT for values beyond the upper bound of this interval because the coefficient in (3) measuring the output gap elasticity of wage inflation falls as ε_w rises, handicapping the non-inertial policies. In contrast to variations in ε_p, however, IT does not uniformly outperform PT for small values of ε_w. As ε_w falls, the social penalty for wage inflation volatility shrinks, enabling PT to better approximate the optimal equilibrium outcome given by TP.

The dominance of WT over PT is remarkably robust to variations in the mean duration of wage contracts. Only when $\varepsilon_w < 0.27$, signalling a relatively weak social desire for wage inflation stability, is a price level target more desirable than a wage target. In order to demonstrate the preponderance of situations in which WT is the preferred regime, we compare asymptotic social loss under both for all possible combinations of ε_p and ε_w. Figure 7 is the contour version of a 3-dimensional graph plotting deviations of WT loss from PT loss, expressed as a percentage of the loss under WT. To be precise, we plot a contour map of the function

$$\nu(\varepsilon_p, \varepsilon_w) = \frac{Loss^{\mathrm{PT}} - Loss^{\mathrm{WT}}}{Loss^{\mathrm{WT}}} \times 100 \tag{22}$$

in which case positive entries on the map represent those $(\varepsilon_p, \varepsilon_w)$ combinations where WT outperforms PT.[20] A number of conclusions can be drawn from the figure. First, for every point below the 45° line (points where $\varepsilon_w > \varepsilon_p$), WT unambiguously dominates PT, indicating that a suitably designed wage target is more successful at minimizing distortions engendered by sticky prices and wages provided the duration of wage contracts are at least as long as price contracts. In fact, for numerous estimates of the Calvo price and wage parameters reported in the current empirical literature, we find that loss under PT is anywhere from 3% to 20% larger than WT in our model. Second, for those combinations implying that the frequency of wage

[20]We divide each axis into 99 equally spaced grid points ranging from .01 to .99. For every $(\varepsilon_p, \varepsilon_w)$ pair out of a possible 10,000, we compute asymptotic social loss for PT and WT and record the corresponding value of ν.

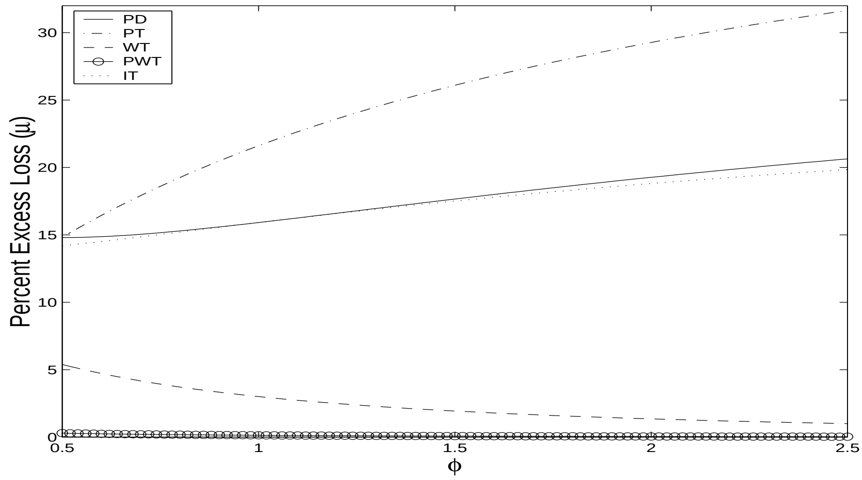

Figure 9: Excess Loss (μ) for $\phi \in (0.5, 2.5)$

adjustments is moderately higher than the frequency of price adjustments (points slightly north of the 45° line), WT continues to perform just as well or better than PT. Third, PT leads to sizeable gains relative to WT, but only in the unlikely event that wages are almost fully flexible.

Returning to Figure 6, we see that regardless of the average duration of wage contracts, the combination PWT regime effectively eliminates any bias resulting from discretionary optimization. For all possible values of ε_w, the difference between PWT and the first-best outcome associated with the TP regime is trivial.

Figure 8 depicts the sensitivity of excess social loss to alternative assumptions regarding σ, the intertemporal elasticity of substitution parameter, and it shows that a number of our main conclusions still hold. First, despite its inherent ability to impart policy inertia, PT is dominated by PD and IT for a broad range of acceptable values.[21] In fact, the relative performance of PT deteriorates sharply as households become increasingly unwilling to alter consumption plans in response to adjustments in the real interest rate. Second, WT unambiguously outperforms PT and IT and resides within a few percentage points of TP for all values of σ considered. The uniform strength of WT (and also the accelerated decline in PT) is due to the precise way in which σ enters the structural model. Recall from (2) - (3) that while σ is positively related to the output gap elasticity of π_t^w, it has no direct influence on the corresponding elasticity of π_t. In light of our previous discussion, increasing σ serves only to strengthen the case for WT. On this point, recent contributions by Rotemberg and Woodford [31], Kim [22], and Ireland [18], estimate a range of values for σ spanning the interval $[5, 50]$. Our results indicate that expanding the set of allowable values for σ along these dimensions would amplify the already sizeable advantages of WT. Third, changing the value of σ does not inhibit the ability of PWT to approximate the optimal outcome under TP.

Figure 9 illustrates the functional relationship between excess social loss and values of ϕ along the interval [0.5,2.5]. Shadowing the conclusions drawn for variations in σ, our results suggest that the inferiority of PT to IT is unaffected by changes in labor supply elasticity term. In addition, increases in ϕ aggravate the

[21] IT is difficult to distinguish on Figure 8 because it is practically equivalent to PD for $\sigma \in [0.5, 2.5]$.

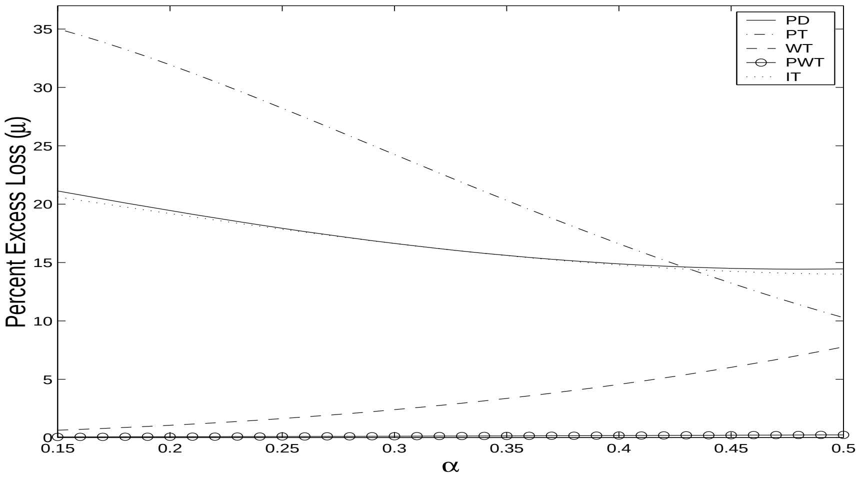

Figure 10: Excess Loss (μ) for $\alpha \in (0.15, 0.5)$

already weak performance of PT while closing the relatively small gap between WT and TP. Unlike the results depicted in Figure 8, however, the growing dominance of WT reflects the impact of ϕ on λ_w. All else constant, larger values of ϕ actually diminish the size of the output gap elasticity of wage inflation, strengthening the relative success of PT as a means of stabilizing $\{\pi_t, \pi_t^w, x_t\}$. Nevertheless, the use of an endogenously determined set of monetary policy objectives renders the ceteris paribus assumption invalid because, at the same time, a growth in ϕ generates a corresponding increase in λ_w, elevating the social desire to stabilize wage inflation and magnifying the benefits of WT. Evidently, the latter effect outweighs the former. Lastly, previous conclusions concerning the robustness of PWT remain unchanged to alternative assumptions regarding the size of ϕ.

We conclude this section by examining the implications of adjusting α, the fraction measuring the share of income allocated to owners of capital, along the closed interval $[0.15, 0.5]$. The results depicted in Figure 10 share a number of similarities with Figures 8 and 9, and so we choose not to comment on them any further. Instead, we focus our remarks on one key difference concerning the relationship between PT and WT. As illustrated in the figure, the superiority of WT to PT diminishes sharply as α increases. Although the output gap elasticities of wage and price inflation are both positively related to α, a larger capital share causes a reduction in λ_w, and consequently, an expansion in the social desire to stabilize price inflation. Provided that α is large enough (i.e. that λ_w is small enough), PT is the preferred regime. For this to occur, however, Figure 10 indicates that $\alpha > 0.5$, a value well in excess of any plausible estimate. Accordingly, we contend that the relative dominance of WT is robust to any reasonable variation in α.

6 Alternative Targeting Procedures

The success of regimes focused on achieving price and wage stability is a consequence of the inertial policy behavior they engender. Recognizing the benefits of policy inertia for a discretionary central bank has inspired other researchers to devise alternative institutional arrangements capable of delivering such persistence. In this section, we compare the stabilization properties of the aforementioned regimes to a number of delegation schemes that have received considerable attention in the literature.

Walsh [38] argues that a "speed limit" policy, designed to balance the stability of inflation and the one-period change in the output gap, generates a substantial degree of inertia, improving upon the outcome resulting from discretionary optimization of the social loss function.[22] We investigate the stabilization properties of implementing a *speed limit* policy (SL) by delegating the period loss function

$$L_t^{SL} = \pi_t^2 + \lambda_w \pi_t^{w^2} + \lambda_{\Delta x}(x_t - x_{t-1})^2 \tag{23}$$

to the monetary authority. The value of $\lambda_{\Delta x}$ is chosen optimally according to the same procedure outlined above.

For a central bank expected to set policy in a discretionary fashion, Woodford [39] demonstrates that modifying the loss function to include a specific objective designed for smoothing interest rate changes engenders an inertial policy response that resembles the kind observed under optimal commitment.[23] We analyze the desirability of assigning an *interest rate smoothing* (IS) objective by constructing the period loss function

$$L_t^{IS} = \pi_t^2 + \lambda_w \pi_t^{w^2} + \lambda_{\Delta i}(i_t - i_{t-1})^2 \tag{24}$$

where $\lambda_{\Delta i}$ is selected to minimize the social loss function (8).

Jensen [19] explores the possibility of targeting the growth rate of nominal income as a means of imparting policy inertia. Denoting y_t the log deviation of real output from its steady state level, we characterize a *nominal income growth* (NIG1) targeting regime with the period loss function

$$L_t^{NIG1} = \pi_t^2 + \lambda_w \pi_t^{w^2} + \lambda_{NI}(y_t - y_{t-1} + \pi_t)^2 \tag{25}$$

where λ_{NI} measures the optimized relative weight attached to the goal of stabilizing nominal income growth.[24] Instead of minimizing a loss function like (25), however, some suggest incorporating a nominal income growth target as a substitute for the inflation targets while maintaining a separate goal of stabilizing the output gap. Accordingly, we examine this alternative notion of nominal income growth targeting (NIG2) using a period loss function

$$L_t^{NIG2} = \lambda_x x_t^2 + \lambda_{NI}(y_t - y_{t-1} + \pi_t)^2 \tag{26}$$

where λ_x is the true social weight on the output gap.

Nessén and Vestin [26] advocate the targeting of a smoothed average of inflation instead of the standard one-period inflation rate. Even in a purely forward-looking environment, stabilizing average inflation is enough to ensure that outcomes will depend on lagged inflation, and thus impose a modicum of persistence

[22] Walsh demonstrates this result using a model that incorporates only a single source of nominal rigidity but exhibits a degree of endogenous persistence in the inflation process. Consequently, the supply side is fully characterized by a hybrid "New-Keynesian" Phillips curve.

[23] The principal features of Woodford's model include Calvo price-setting and an explicit monetary policy goal of minimizing fluctuations in the level of the nominal interest rate.

[24] Equilibrium real output and the output gap are related according to the following identity: $x_t = y_t - y_t^n$, where y_t^n represents the log deviation of potential output from steady state. Consult the appendix for further details.

	B.P.	$\rho_\pi = .7$ $\rho_w = .7$	$\sigma_\pi = .10$ $\sigma_w = .025$	$\varepsilon_p = .8$	$\varepsilon_w = .8$	$\sigma = 5$	$\phi = 5$
PD	15.92	42.89	9.80	10.99	20.13	11.93	24.88
IT	15.91	19.23	6.34	8.80	17.95	11.59	22.86
PT	21.61	34.57	**2.58**	**3.41**	9.20	38.10	38.01
WT	**3.01**	10.51	4.93	3.86	**0.25**	**2.58**	**0.35**
PWT	*0.14*	*1.03*	*0.21*	*1.09*	*0.01*	*0.34*	*0.01*
SL	3.29	3.09	3.31	5.93	4.83	3.32	4.20
IS	3.47	4.07	3.17	5.96	4.91	3.39	4.51
NIG1	3.80	**3.00**	2.90	4.09	4.49	4.94	4.61
NIG2	28.57	51.92	4.08	4.09	14.63	47.85	49.42
AIT	7.16	13.91	3.73	6.09	11.44	5.10	10.38

Table 5: Excess Loss for alternative delegation schemes

on policy actions. We test the implications of mandating *average inflation* targets (AIT) by assigning the period loss function

$$L_t^{AIT} = \overline{f}_p \overline{\pi}_t^2 + \overline{f}_w \overline{\pi}_t^{w^2} + \lambda_x x_t^2 \tag{27}$$

where $\overline{\pi}_t = \frac{1}{2}(\pi_t + \pi_{t-1})$ and $\overline{\pi}_t^w = \frac{1}{2}(\pi_t^w + \pi_{t-1}^w)$, the two-period average rates of price and wage inflation.[25] The policy weights, $\overline{f}_p$ and $\overline{f}_w$, are again chosen to minimize the asymptotic value of social loss.

Table 5 records excess social loss (the function μ introduced in the previous section) for each delegation scheme under a few different parameter configurations. For the baseline parameterization, the outcomes under SL, IS, and NIG1 are competitive with WT, each generating a loss roughly 3% to 4% in excess of TP. In addition, AIT yields a reasonably efficient stabilization outcome, garnering a loss little more than 7% above the TP policy. Interestingly, the results indicate that inflation targeting is more effective when the actual targets are inflation averages rather than levels. In contrast, Jensen's proposed form of nominal income growth targeting, NIG2, performs worst, as it permits suboptimal variations in wage inflation.

Columns two and three are computed under alternative assumptions regarding the distributional properties of supply shocks. We have already documented that adding serial correlation ($\rho_\pi = \rho_w = 0.7$) causes a deterioration in the performance of PT and WT. At the same time, however, it does not appear to generate any measurable reduction in the performance of SL, IS, or NIG1, each yielding loss again between 3% and 4% larger than TP, with NIG1 the lowest of the three. Evidently, these three regimes are more robust to uncertainty in the persistence of supply shocks. Somewhat surprisingly, targeting the price level and nominal wage jointly eliminates much of the inefficiency associated with either PT or WT alone. In fact, among all regimes considered, PWT remains the first-bets option.

When price shocks are amplified ($\sigma_\pi = 0.10$ and $\sigma_w = 0.025$) the equilibrium induced by NIG2 more closely approximates the relatively efficient outcomes associated with SL, IS, NIG1, and AIT. Recall that

[25] While Nessén and Vestin inspect the properties of delegating multi-period average inflation targets, we report on the performance of simple two-period averages.

NIG2 applies no direct penalty to fluctuations in wage inflation. Contracting the relative magnitude of wage shocks reduces the contribution of variations in wage inflation to the overall level of social loss, enabling NIG2 to avoid highly suboptimal outcomes.

Columns four and five show, in turn, the consequences of increasing the average duration of price contracts ($\varepsilon_p = 0.8$) and wage contracts ($\varepsilon_w = 0.8$). Depending on the relative contract length, PT or WT is always the second-best option among all delegation schemes, although WT is clearly more robust because it performs nearly as well as PT even when prices are the dominant source of nominal rigidity. SL, IS, and NIG1, on the other hand, undergo marginal reductions in performance relative to the baseline parameterization, each yielding losses only 4% to 6% larger than TP. Nevertheless, these three regimes appear more robust to uncertainty in relative contract length than PT because of the poor performance of a price level target when wages are the dominant source of rigidity.

The final two columns are computed for larger values of the elasticity parameters characterizing household utility with respect to consumption ($\sigma = 5$) and to labor ($\phi = 5$). While the relative performance of SL, IS, and NIG1 remain largely unchanged, the basic inefficiencies of PT are exacerbated for large σ or ϕ, with the natural implication that the WT outcome approaches the one attainable under PWT.

Collectively, the results of this exercise support the following general conclusions. One, the dominance of PWT is not only robust to uncertainty concerning the structural parameters, it also consistently outperforms each of the alternative delegation schemes considered. Two, the second-best policy is almost always WT. Only when the relative length of price contracts or the relative magnitude of price sector shocks are large, does PT become the second-best option. Three, SL, IS, and NIG1 are the only regimes (aside from PWT) that continue to perform at a high level when supply shocks are persistent. Four, AIT is uniformly superior to IT. Five, Jensen's proposed form of nominal income growth targeting, NIG2, leads to poor stabilization outcomes for several different configurations.

7 Closing Remarks

The central monetary policy objective in this paper is the minimization of a quadratic, utility-based welfare function reflecting the microeconomic distortions brought about by the incidence of sticky prices and wages. Although the optimal way to achieve such a goal requires commitment, we restrict the central bank to pursue its objective in a discretionary fashion. Accordingly, the task facing a policymaker involves the design of a mandatory loss function (i.e. a targeting regime) that when minimized under discretion, nearly replicates the welfare-maximizing equilibrium outcome.

A number of recent studies have concluded that the discretionary pursuit of a price level target, as opposed to an inflation target, leads to a more desirable outcome in predominately forward-looking models with conventional inflation and output gap stabilization objectives. Our work indicates that the same conclusion is not necessarily supported in a version of the standard model that incorporates sticky wages. For several plausible configurations, a policy aimed at stabilizing the price level cannot deliver a level of social loss lower than one designed to target price and wage inflation despite the former's ability to manufacture an inertial response to transitory disturbances. It is possible, however, to witness a reversal in their comparative performance for large enough values of ε_p or ε_w, or equivalently, for a sufficient increase in mean contract duration.

Interestingly, our analysis also suggests that the implementation of a suitably designed target for the nominal wage is quite capable of attenuating much of the distortion caused by price and wage rigidities. In fact, wage targeting consistently outperforms price level targeting and inflation targeting for a number of empirically relevant variations in the structural parameters. Evidently, its success depends primarily on the relative social desire to stabilize wage inflation and on the sensitivity of wage inflation to adjustments in the output gap. Only in the unlikely event that price shocks are disproportionately larger than wage shocks, or that the average lifespan of a price contract far exceeds a wage contract, is a price level target more desirable than a wage target.

After evaluating a number of alternatives, we also conclude that the dominance of a dual policy mandating separate price and wage targets is robust to any possible variation in the structural parameters. The combination policy exhibits stabilization properties that are superior to many other delegation schemes that have been the subject of considerable discussion in the literature, namely, speed limit policies, interest rate smoothing, nominal income growth, and average inflation targeting.

Our results leave open several questions deserving of further attention. First, in light of their declining performance in the face of serially correlated supply shocks, it would be interesting to explore the usefulness of price and wage targets in an economic environment characterized by endogenous persistence in the inflation process. Second, the model used here disregards several popular features that help explain important aspects of the business cycle, among them, endogenous investment dynamics, variable capital utilization, and habit persistence in consumption. Incorporating these into a dynamic general equilibrium model would naturally alter the makeup of the welfare function and perhaps influence the relative performance of the targeting regimes examined in this paper. Finally, it would be especially useful to repeat the analysis under a more realistic assumption concerning the observability of certain macroeconomic variables. The output gap, for instance, is a theoretical construct that is not known to the central bank at the time of the policy decision. Changing the information structure to more accurately reflect reality, therefore, would be especially useful to an actual policymaker.

Appendix

Aggregate Demand

The economy is populated by a continuum of households indexed by $i \in [0,1]$ whose preferences are defined over a composite-based consumption good, C_t, and work hours, $h_t(i)$.[26] Household i is a monopolistic supplier of labor type i and maximizes a utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \{u(C_t;\xi_t) - \upsilon(h_t(i);\xi_t)\} \tag{28}$$

subject to a period-by-period budget constraint

$$P_t C_t + Q_{t,t+1} A_{t+1}(i) = (1+\tau_w) W_t(i) h_t(i) + A_t(i) + Div_t - T_t \tag{29}$$

where $u(C_t;\xi_t)$ measures the utility from consumption and $\upsilon(h_t(i);\xi_t)$ represents the disutility of supplying labor. For any value of ξ, $u(\cdot;\xi)$ is concave and strictly increasing, while $\upsilon(\cdot;\xi)$ is convex and strictly increasing. P_t is the price of a unit of the consumption good, $A_t(i)$ is a scalar denoting the nominal value of the household's contingent claims at the beginning of period t, $A_{t+1}(i)$ is a vector representing the entire portfolio of state-contingent nominal assets that pay one unit of currency in a particular state of nature in the next period, and $Q_{t,t+1}$ is a stochastic nominal discount factor.[27] The term $W_t(i)h_t(i)$ represents labor income and is subsidized at a fixed rate τ_w. Denote Div_t the share of profits received from ownership of firms and T_t a lump-sum tax payment used to finance the subsidies that offset the distortions arising from imperfect competition. Finally, ξ_t is a vector collecting all of the exogenous disturbances to the period utility function.

Combining the first-order conditions for consumption and contingent claims produces the intertemporal Euler equation

$$u_c(C_t;\xi_t) = \beta E_t \left[u_c(C_{t+1};\xi_{t+1}) R_t \frac{P_t}{P_{t+1}} \right] \tag{30}$$

where R_t is the single-period, riskless gross nominal interest rate. Log-linearizing equation (30) around a steady state consistent with zero inflation yields the aggregate demand component of our model

$$\hat{Y}_t - \hat{Y}_t^n = E_t(\hat{Y}_{t+1} - \hat{Y}_{t+1}^n) - \sigma^{-1}(i_t - E_t\pi_{t+1} - r_t^n) \tag{31}$$

where $\hat{Y}_t = \log\left(\frac{Y_t}{\overline{Y}}\right)$, $i_t = \log\left(\frac{R_t}{\overline{R}}\right)$, $\pi_t = \log\left(\frac{P_t}{P_{t-1}}\right)$, and $\sigma = \frac{-u_{cc}(\overline{C};0)\overline{C}}{u_c(\overline{C};0)\overline{C}}$.[28] In (31) we have made use of the aggregate resource constraint $Y_t = C_t$.

To find potential output, $\hat{Y}_t^n$, and the Wicksellian real interest rate, r_t^n, we compute the equilibrium under flexible prices and wages. The production sector is composed of a continuum of monopolistic firms

[26] C_t is assembled with a continuum of differentiated goods, $c_t(j)$ indexed by $j \in [0,1]$ with the aggregator: $C_t = \left[\int_0^1 c_t(j)^{\frac{\theta_p - 1}{\theta_p}} dj\right]^{\frac{\theta_p}{\theta_p - 1}}$.

[27] Although labor income various across households due to the incidence of sticky wages, the assumption of complete contingent-claims markets guarantees that consumption is identical for all agents.

[28] $\overline{Y}$, $\overline{R}$, and $\overline{C}$ are the nonstochastic steady state values of output, the gross nominal interest rate, and consumption.

indexed by $j \in [0, 1]$, each responsible for the sale of one of the differentiated goods that comprise C_t. Firm j maximizes the profit function

$$(1+\tau_p)p_t(j)y_t(j) - W_t h_t(j) \tag{32}$$

subject to the economy-wide demand for good j, $y_t(j) = \left(\frac{p_t(j)}{P_t}\right)^{-\theta_p} Y_t$, and a production technology, $y_t(j) = \mathrm{e}^{a_t} f(\overline{K}, h_t(j)) = \mathrm{e}^{a_t} f(h_t(j))$.[29] The terms $p_t(j)$ and $y_t(j)$ represent the price and quantity of good j, while $h_t(j)$ and W_t are the quantity of labor hired and the nominal wage paid for a unit of that labor.[30] The function f is increasing and concave, and a_t is a productivity shock. The firm's profit-maximization condition yields a supply curve for good j

$$p_t(j) = \left(\frac{1}{1+\tau_p}\right)\left(\frac{\theta_p}{\theta_p - 1}\right)\frac{W_t}{f'\left[f^{-1}\left(\frac{y_t(j)}{\mathrm{e}^{a_t}}\right)\right]\mathrm{e}^{a_t}}. \tag{33}$$

where $\tau_p = \left(\frac{1}{\theta_p - 1}\right)$ to eliminate the distortions arising from market power in product markets. Equilibrating supply and demand and recognizing that $y_t(j) = Y_t^n$ and $h_t(i) = f^{-1}\left(\frac{Y_t^n}{\mathrm{e}^{a_t}}\right)$ $\forall i, j$ when prices and wages are flexible delivers the following equation:

$$(1+\tau_p)(1+\tau_w)\left(\frac{\theta_p - 1}{\theta_p}\right)\left(\frac{\theta_w - 1}{\theta_w}\right) = \frac{\upsilon_h\left[f^{-1}\left(\frac{Y_t^n}{\mathrm{e}^{a_t}}\right); \xi_t\right]}{u_c(Y_t^n; \xi_t) f'\left[f^{-1}\left(\frac{Y_t^n}{\mathrm{e}^{a_t}}\right)\right]\mathrm{e}^{a_t}} \tag{34}$$

which implicitly defines Y_t^n as a function of the exogenous shocks.[31] To simplify the exposition, we assume that a_t is the only stochastic disturbance causing fluctuations in the flexible price and wage equilibrium. Denoting $\hat{Y}_t^n = \log\left(\frac{Y_t^n}{\overline{Y}}\right)$, log-linearizing (34) yields

$$\hat{Y}_t^n = \left(\frac{\frac{1+\phi}{1-\alpha}}{\frac{\alpha+\phi}{1-\alpha} + \sigma}\right) a_t \tag{35}$$

where $\phi = \frac{\upsilon_{hh}(\overline{h}; 0)\overline{h}}{\upsilon_h(\overline{h}; 0)}$ and $\frac{\alpha}{1-\alpha} = \frac{-f''(\overline{h})\overline{Y}}{\left[f'(\overline{h})\right]^2}$. Combining (35) with (30) and the household's optimality condition for labor supply produces the equilibrium expressions for w_t^n and r_t^n found in the body of the paper.

Aggregate Supply

Firms set prices using Calvo-style staggered contracts. Each period, a fraction $1 - \varepsilon_p$ of producers are able to reoptimize existing prices, whereas the remaining firms adjust their price by the indexation rule:

[29] Each firm employs a fixed quantity of capital $\overline{K}$.

[30] Firms employ a composite measure of labor from every household aggregated using the following technology: $h_t(j) = \left[\int_0^1 h_t(i,j)^{\frac{\theta_w - 1}{\theta_w}} di\right]^{\frac{\theta_w}{\theta_w - 1}}$, where $h_t(i,j)$ is the quantity of labor type i supplied to firm j. Furthermore, household i faces an economy-wide demand for its labor type, $h_t(i) = \left(\frac{W_t(i)}{W_t}\right)^{-\theta_w} h_t$, where $h_t = \int_0^1 h_t(j)dj$ and $W_t(i)$ is related to W_t via the aggregator: $W_t = \left[\int_0^1 W_t(i)^{1-\theta_w} di\right]^{\frac{1}{1-\theta_w}}$.

[31] We use the first-order-condition for the optimal wage selection to eliminate W_t from (33) and set $\tau_w = \frac{1}{\theta_w - 1}$ in order to eliminate market power in labor markets.

$p_t(j) = \Pi \times p_{t-1}(j)$.[32] Accordingly, any firm choosing a new price in period t maximizes the expected present discounted value of profits

$$E_t \sum_{T=0}^{\infty} \varepsilon_p^T Q_{t,t+T}\{(1+\tau_p)\Pi^T \overline{p}_t y_{t+T}(j) - W_{t+T}h_{t+T}(j)\} \tag{36}$$

where $\overline{p}_t$ is the chosen price. The first-order condition can be expressed as

$$E_t \sum_{T=0}^{\infty} (\varepsilon_p \beta)^T \frac{\lambda_{t+T}}{\lambda_t} y_{t+T}(j) \left[\tilde{p}_t \left(\prod_{k=1}^{T} \Pi_{t+k} \right)^{-1} - mc_{t+T}(j) \right] = 0 \tag{37}$$

where $\tilde{p}_t = \frac{\overline{p}_t}{P_t}$, $\Pi_t = \frac{P_t}{P_{t-1}}$, $\lambda_t = u_c(Y_t;\xi_t)$, and $mc_{t+T}(j)$ refers to the real marginal cost function for firm j at time $t+T$.[33] Log-linearizing (37) around a zero-inflation steady state yields

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\varepsilon_p)(1-\varepsilon_p\beta)}{\varepsilon_p \left(1 + \frac{\alpha}{1-\alpha}\theta_p\right)} \left(w_t + \frac{\alpha}{1-\alpha}\hat{Y}_t - \frac{1}{1-\alpha}a_t \right) \tag{38}$$

where $w_t + \frac{\alpha}{1-\alpha}\hat{Y}_t - \frac{1}{1-\alpha}a_t = \frac{\alpha}{1-\alpha}(\hat{Y}_t - \hat{Y}_t^n) + (w_t - w_t^n)$ when using the expressions for $\hat{Y}_t^n$ and w_t^n derived in the previous section.

The wage-setting and price-setting problems are isomorphic. Accordingly, households reoptimize their contract wage with a fixed probability $1-\varepsilon_w$ each period, and the remaining fraction of households adjust their wage with an indexation rule: $W_t(i) = \Pi \times W_{t-1}(i)$. All households that reoptimize in period t select a wage, $\overline{W}_t$, that maximizes the expected present discounted value of utility. The first-order condition for such a problem can be written as

$$E_t \sum_{T=0}^{\infty} (\varepsilon_w \beta)^T \lambda_{t+T} h_{t+T}(i) \left[\tilde{W}_t \frac{W_{t+T}}{P_{t+T}} \left(\prod_{k=1}^{T} \Pi_{t+k}^w \right)^{-1} - mrs_{t+T}(i) \right] = 0 \tag{39}$$

where $\tilde{W}_t = \frac{\overline{W}_t}{W_t}$, $\Pi_t^w = \frac{W_t}{W_{t-1}}$, and $mrs_{t+T}(i)$ refers to the marginal rate of substitution between leisure and consumption for household i at time $t+T$.[34] Log-linearizing (39) around a zero-inflation steady state yields an expression for the wage inflation equation

$$\pi_t^w = \beta E_t \pi_{t+1}^w + \frac{(1-\varepsilon_w)(1-\varepsilon_w\beta)}{\varepsilon_w(1+\phi\theta_w)} (\hat{mrs}_t - w_t) \tag{40}$$

where $\hat{mrs}_t - w_t = \left(\frac{\phi}{1-\alpha} + \sigma\right)(\hat{Y}_t - \hat{Y}_t^n) + (w_t^n - w_t)$ after substituting the expressions for $\hat{Y}_t^n$ and w_t^n.[35]

Timeless Perspective

To solve for the equilibrium dynamics under the timeless perspective, collect equations (2) - (7), (14) - (15), and a few trivial identities into the system given by (16). Denote e_j, $j = 1, \ldots, m$, a $1 \times m$ row vector, where

[32] Π is the long-run steady state level of gross inflation and is assumed equal to unity.

[33] In terms of the production technology, $mc_{t+T}(j) = \frac{W_{t+T}}{P_{t+T} f'\left[f^{-1}\left(\frac{y_{t+T}(j)}{e^{a_{t+T}}}\right)\right] e^{a_{t+T}}}$.

[34] In terms of household preferences, $mrs_{t+T}(i) = \frac{v_h(h_{t+T}(i);\xi_{t+T})}{u_c(Y_{t+T};\xi_{t+T})}$.

[35] $mrs_t = \int_0^1 mrs_t(i)di$.

each element is zero if $j = 0$, and each element is zero except for the j^{th} element, which is unity if $j \neq 0$. For $m = 13$, the coefficient matrices Γ and Λ are

$$\Gamma = \begin{bmatrix} e_1 \; e_2 \; e_3 \; e_4 \; e_5 \; e_6 \; e_7 \; \beta e_8 \; \beta e_9 \; e_0 \; \beta e_{12} \; e_0 \; e_{12} \end{bmatrix}' \qquad \text{and}$$

$$\Lambda = \begin{bmatrix} \rho_a e_1 \\ \rho_\pi e_2 \\ \rho_w e_3 \\ e_{10} \\ e_{11} \\ e_{12} \\ e_{13} \\ e_8 - \xi_p \left(\frac{\alpha}{1-\alpha}\right) e_{11} - \xi_p(e_{10} - e_1) - e_2 \\ e_9 - \xi_w \left(\frac{\phi}{1-\alpha} + \sigma\right) e_{11} - \xi_w(e_1 - e_{10}) - e_3 \\ e_9 - e_{10} + e_4 - e_8 \\ k(\xi_p e_8 - \lambda_w \xi_w e_9) + (\xi_p + \xi_w) e_{12} + e_{12} - e_6 + \beta e_7 \\ e_{12} - \xi_p \left(\frac{\alpha}{1-\alpha}\right) e_8 - \lambda_w \xi_w \left(\frac{\phi}{1-\alpha} + \sigma\right) e_9 - \lambda_x(e_{11} - e_5) \end{bmatrix}.$$

The covariance matrix Σ_η is given by

$$\Sigma_\eta = \eta \times \eta' \quad \text{where} \quad \eta = [\psi\sigma_a \; \sigma_\pi \; \sigma_w \; 0 \; 0 \; 0 \; 0]'.$$

Discretion

To construct the coefficient matrices in (19) used to compute the discretionary outcome, we make use of the previously defined row vectors e_j with $m = 7$. The matrices Ω, A, and B are

$$\Omega = \begin{bmatrix} \beta & 0 \\ 0 & \beta \end{bmatrix}$$

$$A = \begin{bmatrix} \rho_a e_1 \\ \rho_\pi e_2 \\ \rho_w e_3 \\ e_6 \\ e_7 \\ \xi_p e_1 - e_2 - e_4 + (1 + \beta + \xi_p) e_6 - \xi_p e_7 \\ -\xi_w e_1 - e_3 - e_5 + (1 + \beta + \xi_w) e_7 - \xi_w e_6 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\xi_p \left(\frac{\alpha}{1-\alpha}\right) \\ -\xi_w \left(\frac{\phi}{1-\alpha} + \sigma\right) \end{bmatrix}.$$

The covariance matrix Σ_ε is given by

$$\Sigma_\varepsilon = \varepsilon \times \varepsilon' \quad \text{where} \quad \varepsilon = [\psi\sigma_a \; \sigma_\pi \; \sigma_w \; 0 \; 0]'.$$

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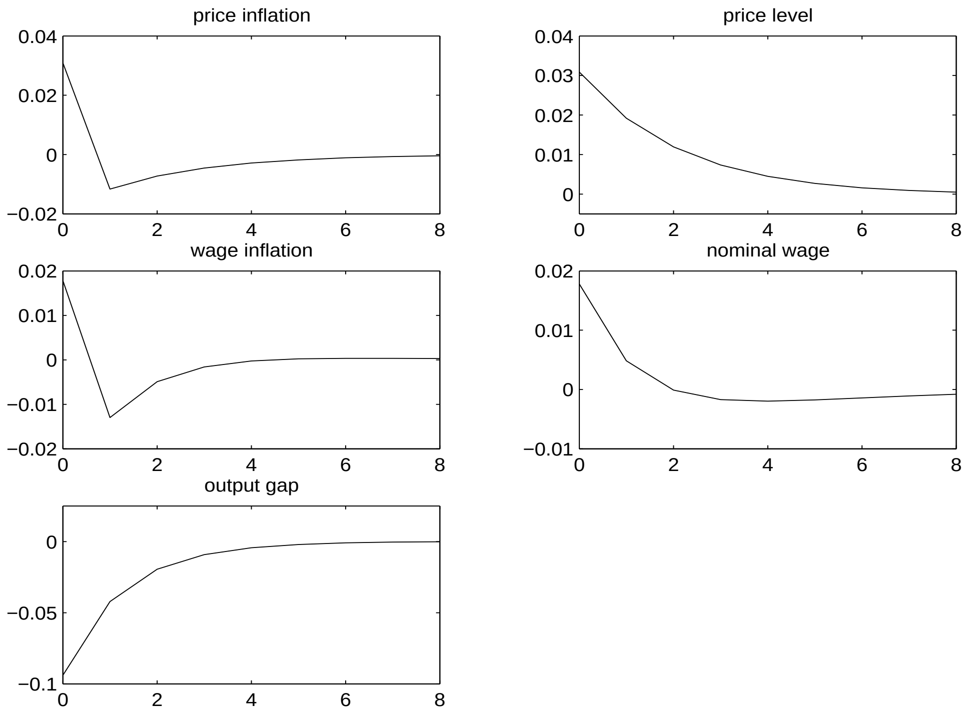

Figure 1: TP response to transitory supply shocks

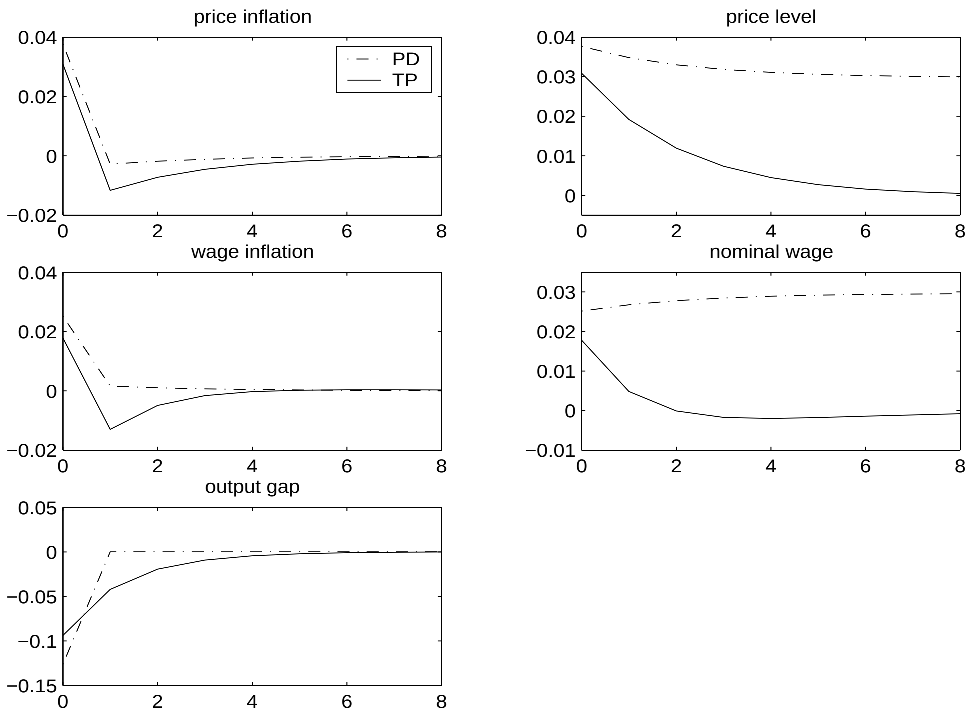

Figure 2: TP and PD responses to transitory supply shocks

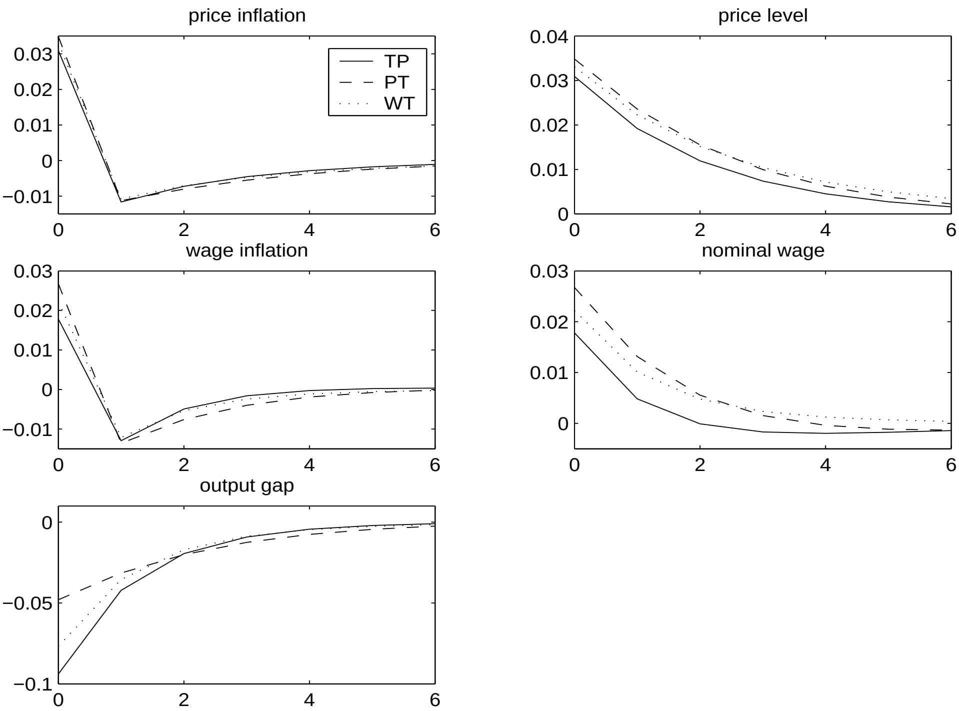

Figure 3: TP, PT, and WT responses to transitory supply shocks

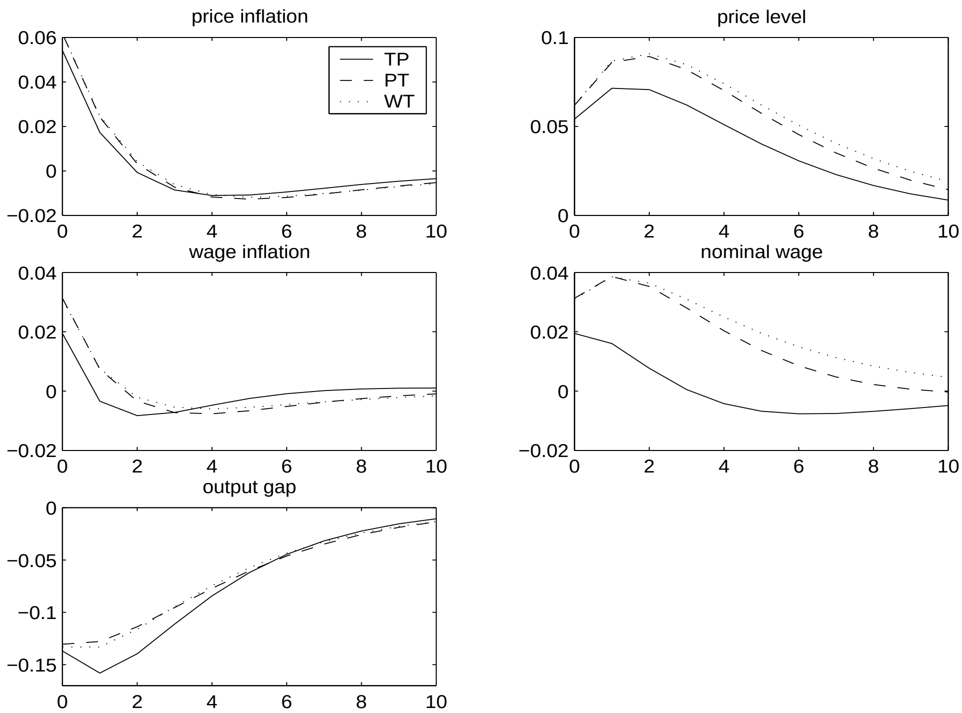

Figure 4: TP, PT, and WT responses to serially correlated supply shocks

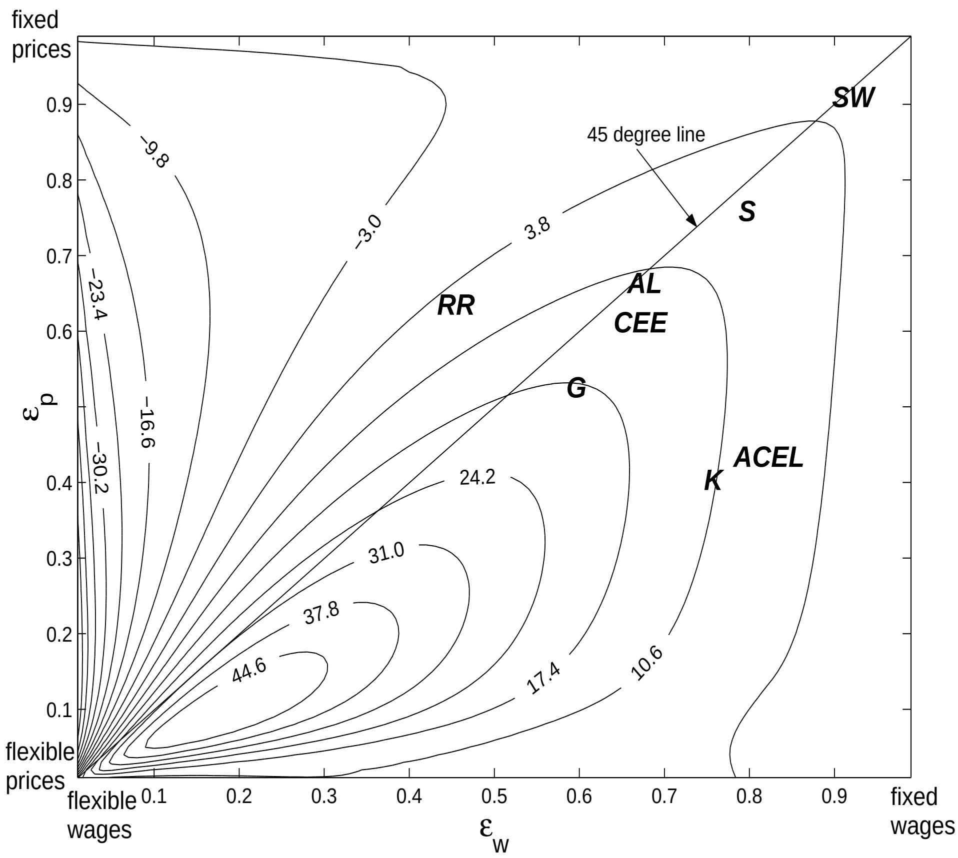

Figure 7: Deviation of WT from PT (ν) for $\varepsilon_p, \varepsilon_w \in (0, 1)$

CEE	–	Christiano, Eichenbaum, and Evans [7]
AL	–	Amato and Laubach [2]
K	–	Kim [21]
G	–	Givens [15]
ACEL	–	Altig, Christiano, Eichenbaum, and Linde [1]
RR	–	Rabanal and Rubio-Ramirez [27]
S	–	Sbordone [32]
SW	–	Smets and Wouters [33]