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HOMEWORK 9

$$\max_{c_{1A}, c_{1B}} \frac{1}{2}\sqrt{c_{1A}} + \frac{1}{2}\sqrt{c_{1B}}$$

$-\lambda$ s.t $p_A \cdot c_{1A} + p_B \cdot c_{1B} = 2p_A + 4p_B$

x_{1A}: $\frac{1}{4} \cdot c_{1A}^{-1/2} = \lambda p_A$

$\frac{1}{4} c_{1B}^{-1/2} = \lambda p_B$

$$\left(\frac{c_{1B}}{c_{1A}}\right)^{1/2} = \frac{p_A}{p_B}$$

$$c_{1B} = \frac{p_A^2}{p_B^2} \cdot c_{1A}$$

~~For 2nd consumer~~ Substitute into BC of cons. 1

$$p_A \cdot c_{1A} + \cancel{p_B} \cdot \frac{p_A^2}{p_B^{\cancel{2}}} c_{1A} = 2p_A + 4p_B$$

$$c_{1A} = \frac{2p_A + 4p_B}{p_A + \frac{p_A^2}{p_B}}$$

For the second consumer

$$\max_{c_{2A}, c_{2B}} \frac{1}{2}\sqrt{c_{2A}} + \frac{1}{2}\sqrt{c_{2B}}$$

s.t. (λ) $\quad p_A \cdot c_{2A} + p_B \cdot c_{2B} = p_A + 10p_B$

FOCs c_{2A}: $\quad \frac{1}{4} c_{2A}^{-1/2} = \lambda p_A$

$\quad \frac{1}{4} c_{2B}^{-1/2} = \lambda p_B$

$$\Rightarrow \left(\frac{c_{2B}}{c_{2A}}\right)^{1/2} = \frac{p_A}{p_B}$$

$$\Rightarrow c_{2B} = \frac{p_A^2}{p_B^2} \cdot c_{2A}$$

Substitute into BC:

$$p_A \cdot c_{2A} + \cancel{p_B} \cdot \frac{p_A^2}{p_B^{\cancel{2}}} \cdot c_{2A} = p_A + 10p_B$$

$$c_{2A} = \frac{p_A + 10p_B}{p_A + \frac{p_A^2}{p_B}}$$

Normalize $p_B = 1$. Use resource constraint for state A

$$c_{1A} + c_{2A} = 5 \Rightarrow \frac{2p_A + 4}{p_A(1+p_A)} + \frac{p_A + 10}{p_A(1+p_A)} = 3$$

$$\Rightarrow \cancel{3p_A} + 14 = \cancel{3p_A} + 3p_A^2 \Rightarrow p_A^2 = \frac{14}{3} \Rightarrow p_A = \sqrt{\frac{14}{3}}$$

Therefore eq. price ratio $p_A/p_B = \sqrt{\frac{14}{3}}$

Substitute these prices into demand functions, to get equilibrium allocations.

$$c_{1A}^{*} = \frac{2\cdot\sqrt{\frac{14}{3}} + 4}{\sqrt{\frac{14}{3}} + \frac{14}{3}} = 1.2188 \qquad c_{1B}^{*} = \frac{14}{3}\cdot\frac{\left(2\cdot\sqrt{\frac{14}{3}} + 4\right)}{\left(\sqrt{\frac{14}{3}} + \frac{14}{3}\right)} = 5.6876$$

$$c_{2A}^{*} = \frac{\sqrt{\frac{14}{3}} + 10}{\sqrt{\frac{14}{3}} + \frac{14}{3}} = 1.7812 \qquad c_{2B}^{*} = \frac{14}{3}\cdot\frac{\left(\sqrt{\frac{14}{3}} + 10\right)}{\left(\sqrt{\frac{14}{3}} + \frac{14}{3}\right)} = 8.3124$$

HW.4/2

- Note we can first compute A-D eq. in the usual way then normalize prices to sum to 1.

Consumer A solves:

$$\max \; u_A(x) = -\frac{1}{3} e^{-x_a^A} - \frac{2}{3} e^{-x_b^A}$$

s.t

$$p_a \cdot x_a^A + p_b \cdot x_b^A = p_a$$

FOC

$$\left.\begin{aligned} \frac{1}{3} e^{-x_a^A} &= \lambda p_a \\ \frac{2}{3} e^{-x_b^A} &= \lambda p_b \end{aligned}\right\} \Rightarrow \frac{e^{x_b^A}}{2 e^{x_a^A}} = \frac{p_a}{p_b}$$

$$e^{x_b^A} = 2 \cdot \frac{p_a}{p_b} \cdot e^{x_a^A}$$

$$x_b^a = \ln(2p_a/p_b) + x_a^A$$

Substitute into BC^A

$$p_a \cdot x_a^A + p_b \cdot \left(\ln(2p_a/p_b) + x_a^A\right) = p_a$$

$$x_a^A = \frac{p_a - p_b \cdot \ln(2p_a/p_b)}{p_a + p_b}$$

Consumer B solves

$$\max_{X_a^B,\, X_b^B} \quad u_B(x) = \frac{1}{3} X_a^B + \frac{2}{3} X_b^B$$

$$\text{s.t} \quad p_a \cdot X_a^B + p_b \cdot X_b^B = 2 \cdot p_b$$

$$MRS_B = \frac{1/3}{2/3} = \frac{1}{2}$$

Since the ~~demand function~~ preference are liner demand function will be

$$X_a^B(p_a, p_b) = \begin{cases} \left(\frac{2p_b}{p_a}, 0\right) & \text{if} \quad \frac{p_a}{p_b} < \frac{1}{2} \\ \text{any } p_a \cdot X_a^B + p_b \cdot X_b^B = 2p_b & \text{if} \quad \frac{p_a}{p_b} = \frac{1}{2} \\ \left(0, \frac{2p_b}{p_b}\right) & \text{if} \quad \frac{p_a}{p_b} > \frac{1}{2} \end{cases}$$

We see that consumer B always want to consume in the corners unless the price ratio $p_a/p_b = 1/2$.

Consumer A wants to consume in interior points.

Therefore we'll have an equilibrium only if $\frac{p_a}{p_b} = \frac{1}{2}$

When $\frac{p_a}{p_b} = \frac{1}{2}$, in the equilibrium A will consume her optimum amount, B will consume the remaining amount, since he is indifferent btw. any bundles satisfying his budget constraint.

Normalize $p_a = 1 \Rightarrow p_b = 2$

Substitute into demand of A

$$x_a^{*A} = \frac{1 - 2 \cdot \ln\left(2 \cdot \frac{1}{2}\right)}{1+2} = \frac{1}{3}$$

$$x_b^{*A} = \underbrace{\ln\left(2 \cdot \frac{1}{2}\right)}_{= 0} + x_a^{A} = \frac{1}{3}$$

Then, eq. consumption of B is

$$x^{*B} = e^A + e^B - x^A = (1, 2) - \left(\frac{1}{3}, \frac{1}{3}\right) = \left(\frac{2}{3}, \frac{5}{3}\right)$$

HW.9/3

We have eight possible states:

S_1: $(e_A^1, e_B^1, e_C^1) = (1, 0, 0)$: S_1

S_2: $= (1, 0, 1)$: S_2

$= (1, 1, 0)$: S_3

$= (1, 1, 1)$: S_4

$= (0, 1, 1)$: S_5

$= (0, 1, 0)$: S_6

$= (0, 0, 1)$: S_7

$= (0, 0, 0)$: S_8

We can treat these states as different goods.

$$\max \sum_{s=1}^{8} \frac{1}{8} \ln(1 + X_s^{A=i}) \quad \text{s.t.} \quad \lambda^{A} \quad \sum_{s=1}^{8} p_s \cdot X_s^A = p_1 + p_2 + p_3 + p_4 \quad \omega'$$

$$\text{FCC} \quad \frac{1}{8} \frac{1}{1 + X_s^A} = \lambda^{A=i} \cdot p_s \qquad i = A, B, C$$

$$\frac{1}{8} \frac{1}{1 + X_s^B} = \lambda^B \cdot p_s$$

$$\Rightarrow \frac{1 + X_s^B}{1 + X_s^A} = \frac{\lambda^A}{\lambda^B} \Rightarrow X_s^B = \frac{\lambda^A}{\lambda^B} \cdot (1 + X_s^A) - 1$$

similarly $\quad X_s^C = \dfrac{\lambda^A}{\lambda^C}(1 + X_s^A) - 1$

$X_s^A + X_s^B + X_s^C = \bar{e}_s$ (total endowment in state s)

$$X_s^A + \frac{\lambda^A}{\lambda^B}(1 + X_1^A) - 1 + \frac{\lambda^A}{\lambda^C}(1 + X_1^A) - 1 = \bar{e}_s$$

Guess $\quad \lambda_1 = \lambda_2 = \lambda_3 = 1$

$$\Rightarrow 3X_s^A = e_s \Rightarrow X_s^A = \frac{\bar{e}_s}{3}$$

Prices then computed from first order cond. of one of the consumers, say A.

$$\frac{1}{8} \cdot \frac{1}{1+x_s^A} = \lambda^A \cdot p_s \quad \Rightarrow \quad p_s = \frac{1}{8(1+x_s^A)}$$

$$\cancel{1+x_s^A = \frac{1}{8p_s\lambda^A}}$$

Therefore for $s = 1, 6$ and 7 we have: ($x_s = \frac{1}{3}$ for these states)

$$p_1 = p_6 = p_7 = \frac{1}{8\left(1+\frac{1}{3}\right)} = \frac{1}{32/3} = \frac{3}{32}$$

for states $s = 2, 3, 5$ we have

$$p_2 = p_3 = p_5 = \frac{1}{8\left(1+\frac{2}{3}\right)} = \frac{3}{40}$$

For state $s = 1$ we have

$$p_1 = \frac{1}{8(1+1)} = \frac{1}{16}$$

And note that for state $s = 8$, the price is not well defined, since total endowments in that state is 0.

Now check if BCs of all consumers are satisfied under these price.

Since all consumers are identical it will be enough to check for one consumer, say A:

$$BC_A : \sum_{s=1}^{8} p_s \cdot X_s^A = p_1 + p_2 + p_3 + p_4$$

$$\Rightarrow \underbrace{3\left(\frac{\cancel{3}}{32}\cdot\frac{1}{\cancel{3}}\right)}_{\text{states } 1,6,7} + \underbrace{3\cdot\left(\frac{\cancel{3}}{40}\cdot\frac{2}{\cancel{3}}\right)}_{\text{states } 2,3,5} + \underbrace{\frac{1}{16}\cdot 1}_{\text{state } 4} \stackrel{?}{=} \frac{3}{32} + \frac{2\cdot 3}{40} + \frac{1}{16}$$

$$\frac{3}{32} + \frac{6}{40} = \frac{3}{32} + \frac{6}{40} \quad \checkmark$$

Hence, budget constraints are satisfied.

The Arrow-Debreu eq. is

$$X^{i*} = \left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}, 1, \frac{2}{3}, \frac{1}{3}, \frac{1}{3}, 0\right) \text{ for } i = A, B, C$$

$$p^* = \left(\frac{3}{32}, \frac{3}{40}, \frac{3}{40}, \frac{1}{16}, \frac{3}{40}, \frac{3}{32}, \frac{3}{32}, 0\right)$$

p_8 can be anything!

Ex $\quad t = 0, 1 \quad S = \{a, b\} \quad \Gamma = \{(0,s)\ (1,a)\ (1,b)\}$

$$e = (e_{0s}, e_{1a}, e_{1b}) = (1, 0, 0)$$

Robinson-Crusoe Example in the class

$$MRT_{0,a} = \frac{1}{2\sqrt{-y_{0s}}} \qquad MRT_{0,b} =$$

$\boxed{\text{let } y = -y_{0s}}$

$x_{1a} =$

$$\max_{y_{0s}} \ln(1 + y_{0s}) + \frac{1}{2}\ln\left(\sqrt{-y_{0s}}\right) + \frac{1}{2}\ln(-y_{0s})$$

$$\max_{y} \ln(1-y) + \frac{1}{2}\ln y^{1/2} + \frac{1}{2}\ln y \qquad \text{s.t } 0 \le y \le 1$$

FOC

$$-\frac{1}{1-y} + \frac{1}{2y^{1/2}} \cdot \frac{1}{2} \cdot y^{-1/2} + \frac{1}{2y} = 0$$

$$\frac{1}{4y} + \underset{(2)}{\frac{1}{2y}} = \frac{1}{1-y}$$

$$\frac{3}{4y} = \frac{1}{1-y} \Rightarrow$$

$$\Downarrow$$

$$\Rightarrow 4y = 3 - 3y$$

$$\Leftarrow \boxed{y = \frac{3}{7} = -y_{0s}}$$

$x_{0s} = 1 + y_{0s} = \frac{4}{7}$

$x_{1a} = y_{1a} = \sqrt{3/7}$

$x_{1b} = y_{1b} = 3/7$