Consider the Edgeworth box economy described by following preferences and initial endowments:

$$u_A(x_{1A}, x_{2A}) = \min\{x_{1A}, x_{2A}\} \qquad u_B(x_{1B}, x_{2B}) = 2x_{1B} + x_{2B}$$

$$e_A = (3, 2) \qquad e_B = (1, 2)$$

Find the Walrasian equilibrium.

- Since consumer A has Leontieff preferences we know that he will consume $x_{1A} = x_{2A}$ at an optimal bundle. Here, by his budget constraint we have

$$p_1 \cdot x_{1A} + p_2 \cdot x_{1A} = 3p_1 + 2p_2$$

$$\Rightarrow x_A(p_1, p_2) = \left(\frac{3p_1 + 2p_2}{p_1 + p_2}, \frac{3p_1 + 2p_2}{p_1 + p_2} \right)$$

To find Marshalian demand function for consumer B, note that two goods are perfect substitutes with $MRS = 2$.

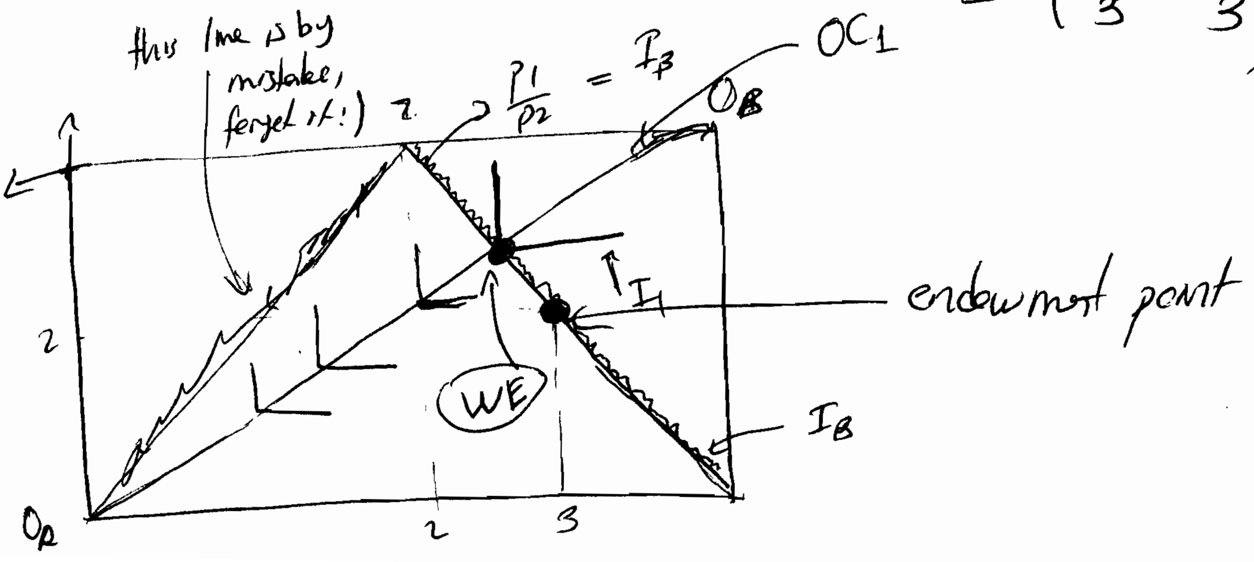
Then:

$$x_B(p_1, p_2) = \begin{cases} \left(\frac{p_1 + 2p_2}{p_1}, 0 \right) & \text{if } \frac{p_1}{p_2} < 2 \\ \text{any } x_{1B}, x_{2B} \text{ s.t. } p_1 x_{1B} + p_2 x_{2B} = p_1 + 2p_2 & \text{if } \frac{p_1}{p_2} = 2 \\ \left(0, \frac{p_1 + p_2}{p_2} \right) & \text{if } \frac{p_1}{p_2} > 2 \end{cases}$$

To find the market clearing prices note that consumer A never ~~f~~ wants to consume at the boundary, but consumer B always wants to consume at boundary, unless $\frac{p_1}{p_2} = 2$. Therefore, the only possible candidate for equilibrium price is $p_1/p_2 = 2$.

To see if we have ~~an~~ an eq. when $\frac{p_1}{p_2} = 2$, note that cons. B is fine with consuming any point on her budget line. So ~~if~~ if we ~~ga~~ let A to consume his desired consumption level, and give rest of the goods to B, we can have an equilibrium. Thus Walrasian equilibrium prices ~~are~~ are $(p_1, p_2) = (2, 1)$ (after normalizing $p_2 = 1$) and allocations in the eq. are

$$X_A = X_A(2,1) = \left(\frac{8}{3}, \frac{8}{3}\right) \quad \text{and} \quad X_B = e_A + e_B - X_A = \left(\frac{4}{3}, \frac{4}{3}\right)$$



Ex 1. Find the competitive eq. for the following edgeworth b[ox]
economy.

$u_A(x_1, x_2) = x_{1A}\, x_{2A}^2$ and $u_B(x_1, x_2) = x_{1B}^2\, x_{2B}$

$e_A = (1, 0)$ and $e_B = (0, 1)$

Equate MRS's to price ratios for both consumers:

$$|MRS_{12}^A| = \frac{x_{2A}^{\cancel{2}}}{2x_{1A} \cdot \cancel{x_{2A}}} = \frac{x_{2A}}{2x_{1A}} = \frac{p_1}{p_2} \qquad (1)$$

$$|MRS_{12}^B| = \frac{2\cancel{x_{1B}} \cdot x_{2B}}{x_{1B}^{\cancel{2}}} = \frac{2x_{2B}}{x_{1B}} = \frac{p_1}{p_2} \qquad (2)$$

From (1) we have

$x_{2A} = 2\frac{p_1}{p_2} x_{1A}$, substitute this into BC for consumer A:

$$p_1 \cdot x_{1A} + p_2 \cdot x_{2A} = p_1 \cdot 1 + p_2 \cdot 0$$

$$p_1 \cdot x_{1A} + \cancel{p_2} \cdot 2 \cdot \frac{p_1}{\cancel{p_2}} \cdot x_{1A} = p_1 \qquad (\text{normalize } p_1 = 1)$$

$$3x_{1A} = 1 \longrightarrow x_{1A} = \frac{1}{3}$$

$$\Rightarrow x_{2A} = 2 \cdot \frac{p_1}{p_2} \cdot x_{1A} = 2 \cdot \frac{1}{p_2} \cdot \frac{1}{3} = \frac{2}{3p_2}$$

From eq. (2) we have that

$$X_{2B} = \frac{1}{2} \cdot \frac{p_1}{p_2} \cdot X_{1B}$$

substitute this for the BC for cons. B:

$$p_1 \cdot X_{1B} + \cancel{p_2} \cdot \frac{1}{2} \cdot \frac{p_1}{\cancel{p_2}} \cdot X_{1B} = p_1 \cdot 0 + p_2 \cdot 1$$

(since we normalized $p_1 = 1$)

$$X_{1B} + \frac{X_{1B}}{2} = p_2$$

$$\Rightarrow \frac{3X_{1B}}{2} = p_2 \Rightarrow X_{1B} = \frac{2}{3} p_2 \Rightarrow X_{2B} = \frac{1}{3}$$

use feasibility constraints for good 1 (you can do with feasibility constraint for good 2 as well.)

$$X_{1A} + X_{1B} = 1$$

$$\frac{1}{3} + \frac{2}{3} p_2 = 1 \Rightarrow \frac{2}{3} p_2 = \frac{2}{3} \Rightarrow \underline{p_2 = 1}$$

so the ~~p~~ eq. price ratio $p_1/p_2 = 1$. Then

$$\Rightarrow X_{1A}^* = \frac{1}{3} \quad X_{2A}^* = \frac{2}{3} \quad \& \quad X_{1B} = \frac{2}{3}, \quad X_{2B} = \frac{1}{3}$$

in the Walrasian competitive eq.

Ex2 Consider the same Edgeworth box economy described in the previous question. Can the allocation $(x_A^*, x_B^*) = ((1,1),(3,3))$ be supported as a price equilibrium? If yes, find the supporting prices and transfers. If no, explain.

- Solution By Second Welfare Theorem, we can support some ~~equilibrium~~ allocation as price equilibrium with transfers only if it is feasible and Pareto optimal. Clearly, the allocation above is feasible. So, let's check Pareto optimality: Remember:

$MRS_A(x^*) = x_{2A}^* = 1$

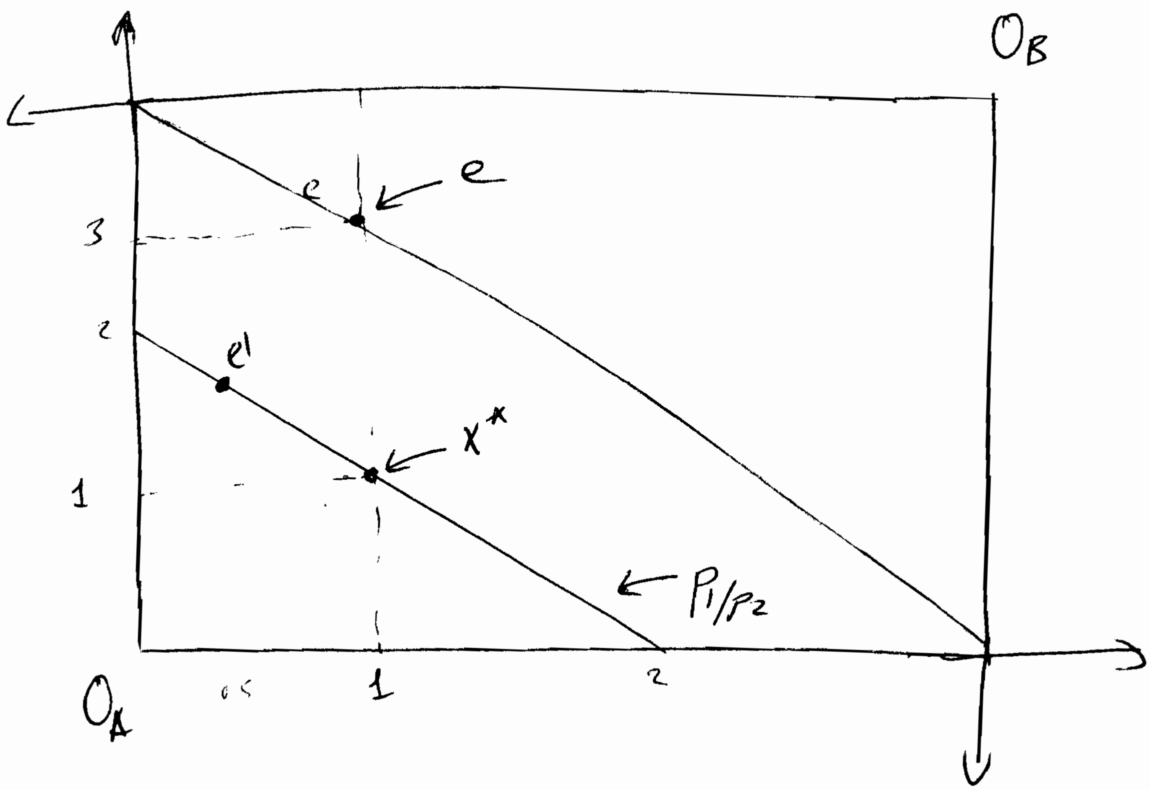
$MRS_B(x^*) = \dfrac{x_{2B}^*}{x_{1B}^*} = \dfrac{3}{3} = 1 \Rightarrow MRS_A(x^*) = MRS_B(x^*)$

Hence, this point is Pareto Optimal.

So, we can support this allocation as an equilibrium with transfers. Supporting prices are determined by MRS of consumers at this allocation. So $p_1/p_2 = 1$. Since, we normalize $p_2 = 1 \Rightarrow p_1 = 1$, So, transfers are

$T_A = \langle x_A, p \rangle - \langle e_A, p \rangle = (1.1 + 1.1) - (1.1 + 3.1) = -2$

$T_B = -T_A = 2.$



Note that if we move endowments from e to any point on the price line passing through x^*, they will end up consuming x^* in the equilibrium.

Check $e' = ((1/2, 3/2), (7/2, 5/2))$

You'll see that competitive eq. for this endowments is exactly x^*!