Impact of Productivity on Optimal Cross-Training Decisions

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Abstract

One of the important resources in a hospital that has been most difficult to supply in recent years is nursing staff. To cost effectively meet uncertain demand, hospitals must either hire more contract nurses from external agencies or train more flexible nurses. In this paper, we address the following questions: What is the appropriate amount of cross-training (flexibility) required to minimize costs? What is the effect of productivity of flexible nurses on the amount of cross-training? Does more demand variability necessarily mean more flexibility?

Considering the contract nurse cost, cross-training cost and productivity, we optimize the amount of cross-training for nurses in two hospital units. The model is formulated as a two-stage stochastic programming problem with recourse in the second stage. We derive a closed form expression for the optimal amount of cross-training in two units when demand follows a general, continuous distribution.

When the cost of cross-training is high, an increase in productivity leads to an increase in the amount of cross-training. When the cost of cross-training is relatively low, however, there is a productivity level beyond which further increases in productivity reduce the amount of cross-training. Above this threshold, the productivity of cross-trained nurses allow the benefits of flexibility to be obtained with fewer cross-trained nurses.

At low productivity, more demand variability leads to less cross-training. For high productivity, the effect of demand variability depends on cross-training cost. When cross-training cost is high, more variability continues to cause less cross-training. For low cross-training cost, however, increasing demand variability leads to more cross-training.

Keywords: Service Operations, Flexibility, Productivity, Stochastic Programming

1 Introduction

Due to their inability to hold inventory, service institutions face an especially difficult challenge in matching supply and demand. In the health care sector, hospital administrators can provide high quality of care only when the resources required by the patients, both equipment and staff, are supplied at the right time and at the right quantity. Providing high quality of care to all patients, however, results in huge costs for the hospital. Hospital administrators are trying to reduce operating costs, especially labor cost, without compromising on the service level for patients. A study in Healthcare Financial Management (Gaughan 2005) shows that, for US hospitals, on average, total labor expense accounts for about half of hospital operating costs.

One of the most important hospital resources, and one resource that has been among the most difficult to supply in recent years, is nursing staff. Typically, shortages of regular nurses were supplemented by hiring contract (travel) nurses for a shift or two until the demand stabilizes. The contract nurses are more expensive than the regular nurses, but are flexible to float to multiple units. American Hospital Directory indicates that contract labor as a percentage of total operating cost increased steadily from 1.4% to 3.8% over a five-year period (Shoemaker and Howell 2005).

In recent years, hospital administrators are under tremendous pressure to cut costs and have resorted to cross-training programs (Lyons 1992, Siferd and Benton 1992) for nurses, enabling them to float between units in the same specialization. These units have varying acuity levels, but are similar enough to cross-train nurses. Cross-training (floating / flexibility) of nurses helps to meet heavy demand in one unit by using nursing hours from another unit where the demand is lean. Many hospitals have reaped financial benefits from successfully implementing cross-training programs (Altimier and Sanders 1999, Snyder and Nethersole-Chong 1999).

To cost effectively meet uncertain demand, hospitals must either hire more contract nurses from external agencies incurring higher contract cost, or train more flexible nurses incurring higher training cost. There is a trade-off between hiring contract nurses and training flexible nurses. This observation leads to the question: What is the appropriate amount of cross-training (flexibility) required so that the service level is maintained and costs are minimized? Though cross-training nurses seems to be a cost effective solution, the nursing director of a tertiary hospital reported that cross-trained (flexible) nurses are not as productive as the regular nurses in the floated unit because of infrequent use of the skills required in the floated unit. The aspect of productivity plays a vital role when analyzing worker flexibility in both manufacturing and service settings. Yield of flexible machines is analogous to the productivity of flexible workers, but papers concerned with product-machine flexibility assume 100% yield in the allocated machines. In this paper, we explicitly model the productivity of flexible nurses in the floated unit and determine the effect of productivity of flexible nurses on flexibility. The other research questions addressed in this paper include: How does flexibility change with changes in demand variability in the units? What is the impact of costs on the amount of flexibility?

In this paper, we optimize the amount of cross-training in multiple units. We formulate a two-stage stochastic programming model with recourse in the second stage to solve this problem. The first stage decision is the amount of cross-training in each unit. When demand is realized in the second stage, nurses are allocated optimally to meet demand such that the resulting costs are minimized. We consider general, continuous demand distributions, and derive a closed form expression for the optimal amount of cross-training.

The analysis shows that at a given level of productivity for flexible nurses, the optimal amount of cross-training in unit i decreases with an increase in cross-training cost in unit i, and increases with an increase in shortage cost in unit j as expected.

When the cost of cross-training is high, an increase in productivity leads to an increase in the amount of cross-training. When the cost of cross-training is relatively low, however, there is a productivity level beyond which further increases in productivity reduce the amount of cross-training. Above this threshold, the productivity of cross-trained nurses allow the benefits of flexibility to be obtained with fewer cross-trained nurses.

For low cross-training productivity, more demand variability leads to less cross-training. For high cross-training productivity, the effect of demand variability on the amount of cross-training depends on cross-training cost. When cross-training cost is high, more variability continues to cause less cross-training. For low cross-training cost, however, increasing demand variability leads to more cross-training (with high cross-training productivity).

This paper has integrated the planning and scheduling (allocation) phases of nurse staffing associated with utilizing flexible nurses in a hospital. To date, the literature has focussed independently on the nurse planning problem and the scheduling (allocation) of flexible nurses problem. In our model, we try to integrate both planning and scheduling phases of the nurse staffing problem across a planning horizon.

This paper is organized as follows. §2 gives the literature review and discusses how our model differs from the existing literature. §3 introduces and formulates our model of cross-training and scheduling decisions. §4 analyzes the model as a two-stage stochastic program with recourse and §5 shows numerical analysis and highlights its implications for management. §6 concludes the paper and discusses possible extensions.

2 Literature Review

This section consists of literature related to implementation of cross-training in hospitals, as well as more general papers on multi-skill sharing and serial cross-training. We also review the literature on manufacturing flexibility, and discuss how it differs from our model. Cross-training of nurses in hospitals can lead to both financial and non-financial benefits. Wheaton (1996) and Lyons (1992) list additional benefits such as increased job-satisfaction, decreased job stress, and increased marketability of nurses. Li and King (1999) develops a goal programming approach for optimizing the cross-trained staff for sub-divided tasks in health care. Inman, Blumenfeld, and Ko (2005) use number of shifts where contract nurses are used as their objective to compare different types cross-training policies for nurses. They simulate cross-training policies considering patient census as poisson arrivals and absenteeism rate of nurses following a binomial distribution.

Quite a few papers have been published in work-force cross-training. Agnihothri, Mishra, and Simmons (2003) balances the trade-off between customer delay cost and premium for flexibility and models a queueing system to determine the mix of dedicated and cross-trained servers for two job types using simulation, and extend this work to three job types in their 2004 paper (Agnihothri and Mishra 2004). In our paper, we determine the optimal amount of cross-training in two units when demand follows general distribution. Agnihothri, Mishra, and Simmons (2003) use discrete values for mix of dedicated and cross-trained servers to study the effect of parameters on the mix ratio. Brusco and Johns (1998) present an ILP to evaluate cross-training configurations for a multi-skilled work force. Brusco, Johns, and Reed (1998), minimizes the total number of labor for two skill class considering productivity. They do not optimize on the level of flexibility but assume that all laborers are totally flexible. McClain, Schultz, and Thomas (2000), show that work-in-process inventory has a significant effect on the productivity of workers when there is work sharing in a serial system. Also, Schultz, McClain, and Thomas (2003) in their paper experimentally show the negative effects in worker productivity because of work interruptions when using flexible workers. These papers in work-force cross-training have not considered the impact of productivity and cross-training costs on the optimal amount of cross-training particularly when demand follows a general distributions.

Cross-training in manufacturing has widely been used to balance the work load in an assembly system in order to maximize throughput. A recent paper by Hopp, Tekin, and Oyen (2004) analyzes two different cross-training structures, skill chaining and cherry-picking, for a serial production system. They find that when capacity is fairly imbalanced but variability is low, the cherry-picking approach can be used. Jordan, Inman, and Blumenfeld (2004) evaluate the performance of three cross-training configurations in parallel systems using queueing theory and simulation. They conclude that complete chaining gives the maximum benefit and is also robust. Pinker and Shumsky (2000) analyzes a system with specialist and flexible servers when there is a trade off between the efficiency of the specialist and the quality of the flexible servers. Iravani, Oyen, and Sims (2005) finds a structural flexibility index that chooses the best pattern among all the alternative patterns of flexibility in parallel systems without having to evaluate all the patterns. Vairaktarakis and Winch (1999) develops heuristics for scheduling work orders through assembly systems so that the cross-training costs are minimized when multi-skilled workers are used. The above papers focus on finding the best configuration or flexibility structure (level of cross-training) in different scenarios and do not optimize the amount of cross-training.

Van Mieghem (1998), studies the effect of cost and price differentials on the flexibility of two-product, three-plant system. He maximizes the total revenue by optimizing the capacities for two dedicated-capacity plants and one flexible-capacity plant. The differences between Van Mieghem's model and the model in this paper are given below:

1. Unlike the single flexible resource in Van Mieghem's model, nurses on both units become flexible with cross-training

2. In our paper, the impact of productivity of floated nurses on the amount of cross-training is analyzed.

3. Flexible resources are associated with a home unit, and only floated if they are not needed at their home unit.

4. All demand must be met. An additional, more costly resource (contract nurses) is available to overcome capacity deficiencies.

Campbell (1999) develops a model for allocating cross-trained workers to a multi-department service environment. He determines the benefit of cross-training using a simulation study. His model maximizes utility (weighted sum of squared requirement that is satisfied) of meeting the requirements, considering the capabilities of the workers subject to their allocation in different departments and assuming different levels of cross-training (training breadth). Jordan and Graves (1995) expands the model to M products and N plants and analyzes the level of flexibility required to reap significant benefits. They conclude that small additional flexibility is sufficient to get results of total flexibility. Graves and Tomlin (2003), has extended the idea of Jordan and Graves (1995) to multi-stage supply chains.

In this paper, we analyze the effects of cross-training cost, contract cost, productivity and demand variability on the optimal amount of cross-training for two units in a health care setting. This analysis can be applied to other service settings, such as hotels, restaurants and call centers.

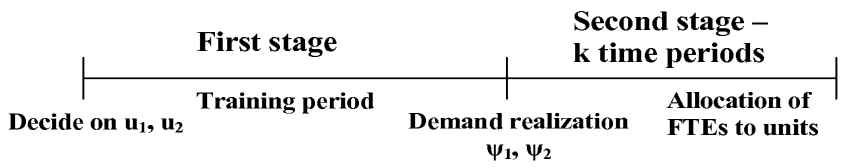

Figure 1: Time Line for the Model

3 Problem Formulation

This section gives the notation used in the model and formulates the model. We model two hospital units, each of which has a pool of nursing resources and a (stochastic) demand for those resources. In practice, supply and demand for each unit is measured as the number of full-time-equivalent (FTE) nurses required each day. The full-time-equivalents can also be represented as nursing hour requirements for each unit. In our model, all allocations and demand are measured on continuous scale indicating that the unit of measurement if the full-time-equivalents.

The sequence of events is as follows :

Stage 1 - A proportion of each unit's nurses are cross-trained, enabling them to float to the other unit as needed.

Stage 2 - Demand is realized and nurses are allocated to units. Excess demand is met by hiring contract nurses.

The time line for our model is shown in figure 1. There are two nurse pools consisting of dedicated and flexible nurses to meet demand across both units. The dedicated nurses in unit 1 and unit 2 can serve their home unit with 100% productivity. Flexible nurses can also serve their home units at 100% productivity, but are generally less productive when required to float. Abernathy, Baloff, Hershey, and Wandel (1973) defines a person's efficiency as a fraction of FTE. Campbell (1999) uses a capability measure between 0 and 1 to measure the capability of workers based on their training. In our model, e_{ij} is the measure of productivity for flexible nurses who are allocated from unit i to unit j. This measure varies between 0 and 1.

Allocation strategy :

Dedicated nurses who are not cross-trained can be allocated only to their home unit. Consistent

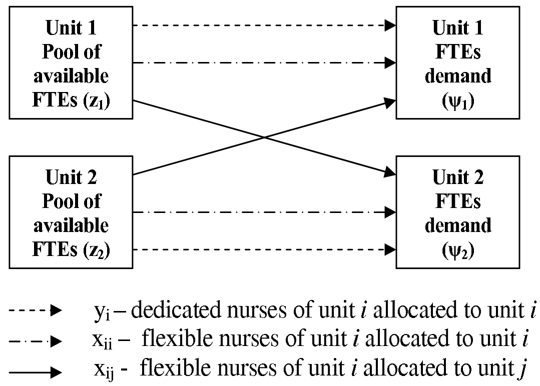

Figure 2: Network Structure

with standard hospital practice and literature (Inman, Blumenfeld, and Ko 2005), we assume that flexible nurses are first allocated to their home unit and only when the demand in their home unit is met can excess nurses be floated to the other unit. If the demand is still not met, then contract nurses are called in at higher cost. Demand for nursing hours is always met either by regular nurses or by contract nurses and shortage is never allowed to occur. The structure of the availability of nurses and the demand satisfied through allocations is shown in figure 2.

The objective is to determine the optimal amount of cross-training for each unit in order to minimize the total cost, which is the sum of contract cost and training cost, subject to meeting all the nursing hour requirements. The amount of cross-training is operationalized as the proportion of nurses who are cross-trained in each unit. The total number of nurses available is known a priori for both units. Demand in each unit is a function of the number of patients in each unit and their acuity. Conversion of the number of patients in a unit into required nursing hours based on their acuity is not considered here. We assume that required nursing hours (demand) is stochastic and exogenous to the model. This problem is formulated using two-stage stochastic programming with recourse in the second stage. Allocation of nurses in the second stage is the recourse for meeting the stochastic demand at the end of the first stage.

3.1 Notation

Ψ_i : Nursing hour requirements (demand) for unit i, which follows a general, continuous distribution

ψ_i	:	Realizations of the stochastic demand
z_i	:	Nurses available in unit i
u_i	:	Amount of cross-training in unit i
y_i	:	Nurses in home unit i who are dedicated and are allocated to home unit i
x_{ii}	:	Nurses in home unit i who are flexible (cross-trained) and are allocated to home unit i
x_{ij}	:	Nurses in home unit i who are flexible (cross-trained) and are floated to unit j
e_{ij}	:	Productivity of nurses who are floated from unit i to unit j
s_i	:	Contract wages for hiring contract nurses to meet the demand in unit i
t_i	:	Training cost for cross-trained nurses in unit i
k	:	Length of second stage

The demand Ψ_i in each unit is measured in terms of FTEs. The demand is stochastic and Ψ_1 follows a general, continuous distribution with cdf $\Phi_1(\cdot)$ and Ψ_2 follows a general, continuous distribution with cdf $\Phi_2(\cdot)$.

The total nurses available for allocation (z_i) in unit i is given a priori and cannot be varied in the model. Our model does not account for hiring, layoffs and absenteeism of regular nurses. It does not explicitly consider on-the-job regular training cost, but this cost can be added to the base wages for regular nurses and the model analysis will still hold. The base wages for regular nurses is a sunk cost for the hospital and so is not represented in the model. Of the total z_i, only $u_i \cdot z_i$ are cross-trained in each unit; u_i is the proportion of unit i nurses who are cross-trained.

Flexible nurses who are floated are not as productive as the nurses dedicated to that unit. The productivity parameter e_{ij} is used to capture this effect. Productivity is measured as the ratio of time taken by a dedicated nurse to do a task to the time taken by a flexible nurse floated to that unit to do the same task. The productivity parameter varies from 0 to 1. 0 implies that cross-trained (flexible) nurses cannot perform any duties in the floated unit and 1 implies that they are as productive as the home unit (dedicated) nurses. If x_{ij} nurses are allocated from unit i to unit j, then the available nurse resource in unit j will be $e_{ij} \cdot x_{ij}$ nurses. Productivity of dedicated nurses (y_i) and flexible nurses allocated to home unit (x_{ii}) are assumed to be 1.

When the demand for nursing hours is not met by flexible and dedicated nurses, contract nurses are called in to meet the demand at a higher cost. This contract cost is represented by s_i in the model. The productivity of contract nurses is assumed to be 1 since they are multi-skilled and experienced in a variety of tasks. If their productivity is less than 1, the model analysis will remain the same provided the contract cost, s_i, is inflated by the productivity of the contract nurse. Fixed cross-training costs per nurse, per time period, are represented by $\frac{t_i}{k}$ where k is the length of the second stage planning period. These training costs are incurred at the beginning of the planning horizon, before demands are realized.

3.2 Model Formulation

The model is formulated using two-stage stochastic programming with recourse (Birge and Louveaux 1997). In the first stage, the training costs for both the units are incurred and then in the second stage demand is realized and nurses are allocated, with excess demand covered by contract nurses.

Stage 1

$$\text{Min}_{u_i} \quad \sum_i (t_i \cdot u_i \cdot z_i) + k \cdot E_{\Psi_i}[Q(u_i, \psi_i)] \tag{1}$$

subject to

$$0 \le u_i \le 1, \ \forall\, i = 1, 2 \tag{2}$$

The objective function (1) is the sum of training cost and expected contract cost determined from stage 2, subject to the constraint (2) that the amount of cross-training varies between 0 and 1. 0 represents no cross-training and 1 represents complete cross-training (all nurses in that unit are cross-trained).

Stage 2 $\quad \mathbf{Q(u_1, u_2, \psi_1, \psi_2)}$:

$$\text{Min}_{x_1, x_2, x_{12}, x_{21}} \quad (\psi_1 - x_1 - e_{21} \cdot x_{21})s_1 + (\psi_2 - x_2 - e_{12} \cdot x_{12})s_2$$

subject to

$$x_1 + e_{21} \cdot x_{21} \leq \psi_1 \tag{3}$$

$$x_2 + e_{12} \cdot x_{12} \leq \psi_2 \tag{4}$$

$$x_{12} \leq u_1 \cdot z_1 \tag{5}$$

$$x_{21} \leq u_2 \cdot z_2 \tag{6}$$

$$x_1 + x_{12} \leq z_1 \tag{7}$$

$$x_2 + x_{21} \leq z_2 \tag{8}$$

$$x_{12}(\psi_1 - x_1) = 0 \tag{9}$$

$$x_{21}(\psi_2 - x_2) = 0 \tag{10}$$

$$x_1, x_2, x_{12}, x_{21} \geq 0 \tag{11}$$

In the above formulation, $x_1 = y_1 + x_{11}$ and $x_2 = y_2 + x_2$. In stage 2, the objective function (3) determines the contract cost for both units. Given a demand value ψ_i, we allocate y_i dedicated nurses to home unit i. Among the cross-trained nurses, we allocate x_{ii} to their home unit and altogether x_i nurses serve the home unit and the rest x_{ij} float to unit j.

Constraint (3) and (4) does not allocate more than the demand. Constraint (5) to (8) are the constraints for allocation of nurses to home unit and float unit. $(u_i \cdot z_i)$ are the number of nurses who are cross-trained. Constraints (9) and (10) does not allow flexible nurses to float to a unit when there are sufficient nurses in that unit to meet the demand. Constraint(11) is the non-negativity constraint.

4 Solving the Two-Stage Stochastic Programming Problem

In this section, we determine the solutions to the second stage problem based on the allocation strategy described in §3 and use the second stage optimal solutions to solve the first stage optimization problem.

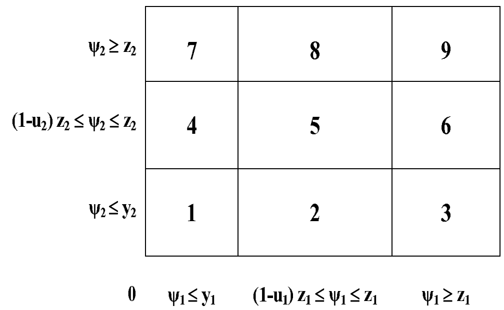

Figure 3: Regions for Second Stage Solution

4.1 Second stage - Allocation

From the formulation of the second stage linear program, we can see that the second stage decision variables are convex in its objective function. The solution is characterized based on whether demand can be (Cattani, Ferrer, and Gilland 2003):

1. satisfied with the non cross-trained nurse, $\psi_i \leq (1 - u_i)z_i$
2. satisfied with the home unit nurses, $\psi_i \leq z_i$
3. satisfied only with floated or contract nurses, $\psi_i \geq z_i$

The regions are explained below and are also represented in figure 3.

Region 1 : $(\psi_1 \leq (1 - u_1)z_1,\ \psi_2 \leq (1 - u_2)z_2)$:

$y_1^* = \psi_1,\ y_2^* = \psi_2,\ x_{ii}^* = 0,\ x_{ij}^* = 0, \forall\ i, j = 1, 2.$

In this region, demand in both units is satisfied with the nurses in home unit who are not cross-trained (dedicated nurses).

Region 2 : $((1 - u_1)z_1 \leq \psi_1 \leq z_1,\ \psi_2 \leq (1 - u_2)z_2)$:

$y_1^* = (1 - u_1)z_1,\ x_{11}^* = \psi_1 - (1 - u_1)z_1,\ x_{21}^* = 0$

$y_2^* = \psi_2,\ x_{22}^* = 0,\ x_{12}^* = 0$

In this region, unit 1's demand is satisfied using home unit nurses not cross-trained and some of the home unit nurses who are cross-trained while unit 2 demand is satisfied using home unit nurses who are not cross-trained.

Region 3 : $(\psi_1 \geq z_1,\, \psi_2 \leq (1-u_2)z_2)$:

$y_1^* = (1-u_1)z_1,\, x_{11}^* = u_1 z_1,\, x_{21}^* = min(u_2 \cdot z_2, (\psi_1 - z_1)/e_{21})$

$y_2^* = \psi_2,\, x_{22}^* = 0,\, x_{12}^* = 0$

In this region, demand in unit 1 may not be satisfied using all unit 1 nurses and cross-trained unit 2 nurses. Any excess demand is met by contract nurses. In unit 2, demand is met using nurses in home unit who are not cross-trained.

Region 4 : $(\psi_1 \leq (1-u_1)z_1,\, (1-u_2)z_2 \leq \psi_2 \leq z_2)$:

$y_1^* = \psi_1,\, x_{11}^* = 0,\, x_{21}^* = 0$

$y_2^* = (1-u_2)z_2,\, x_{22}^* = \psi_2 - (1-u_2)z_2,\, x_{12}^* = 0$

In this region, demand in unit 1 is satisfied using home unit 1 nurses who are not cross-trained and in unit 2 the demand is met using home unit 2 nurses both cross-trained and not cross-trained.

Region 5 : $((1-u_1) \cdot z_1 \leq \psi_1 \leq z_1,\, (1-u_2)z_2 \leq \psi_2 \leq z_2)$:

$y_1^* = (1-u_1)z_1,\, x_{11}^* = \psi_1 - (1-u_1)z_1,\, x_{21}^* = 0$

$y_2^* = (1-u_2)z_2,\, x_{22}^* = \psi_2 - (1-u_2)z_2,\, x_{12}^* = 0$

In this region, demand in both units are met using cross-trained nurses and not cross-trained nurses in their respective home units.

Region : 6 $(\psi_1 \geq z_1,\, (1-u_2)z_2 \leq \psi_2 \leq z_2)$:

$y_1^* = (1-u_1)z_1,\, x_{11}^* = u_1 z_1,\, x_{21}^* = min(z_2 - \psi_2, (\psi_1 - z_1)/e_{21})$

$y_2^* = (1-u_2)z_2,\, x_{22}^* = \psi_2 - (1-u_2)z_2,\, x_{12}^* = 0$

In this region, demand in unit 1 may not be satisfied with nurses from unit 1 and nurses floated from unit 2. Any excess demand is met using contract nurses. Demand in unit 2 is met with nurses from home unit 2 who are cross-trained and nurses who are not cross-trained. The nurses are allowed to float only after they satisfy their home unit demand first.

Region 7 : $(\psi_1 \leq (1-u_1)z_1,\, \psi_2 \geq z_2)$:

$y_1^* = \psi_1,\, x_{11}^* = 0,\, x_{21}^* = 0$

$y_2^* = (1-u_2)z_2,\, x_{22}^* = u_2 z_2,\, x_{12}^* = min(u_1 \cdot z_1, (\psi_2 - z_2)/e_{12})$

In this region, unit 2 demand may not be satisfied from unit 2 nurses and nurses from unit 1 who are cross-trained. Any demand in unit 2 is met using contract nurses. Demand in unit 1 is satisfied using nurses in the home unit who are not cross-trained.

Region 8 : ($(1-u_1)z_1 \leq \psi_1 \leq z_1$, $\psi_2 \geq z_2$):

$y_1^* = (1-u_1)z_1$, $x_{11}^* = \psi_1 - (1-u_1)z_1$, $x_{21}^* = 0$

$y_2^* = (1-u_2)z_2$, $x_{22}^* = u_2 z_2$, $x_{12}^* = min(z_1 - \psi_1, (\psi_2 - z_2)/e_{12})$

In this region, demand in unit 2 may not be satisfied with nurses from unit 2 nurses and unit 1 nurses who are cross-trained. In unit 1, the demand is first satisfied using nurses in home unit who are not cross-trained and then nurses who are cross-trained. Only after demand in unit 1 is satisfied, the nurses who are cross-trained in unit 1 are floated to unit 2. Any excess demand in unit 2 is met using contract nurses

Region 9 : ($\psi_1 \geq z_1$, $\psi_2 \geq z_2$):

$y_1^* = (1-u_1)z_1$, $x_{11}^* = u_1 z_1$, $x_{21}^* = 0$

$y_2^* = (1-u_2)z_2$, $x_{22}^* = u_2 z_2$, $x_{12}^* = 0$

In this region, nurses in unit 1 and unit 2 are not sufficient to meet the demand, so contract nurses have to be hired in both units.

Expectation of the solution in second stage :

The next step is to determine the expected value for the second stage problem. The solution from the nine regions are put into the objective function and the expectation over Ψ_1 and Ψ_2 are taken. Since the contract nurses are required only in region 3, 6, 7, 8 and 9, we see that the terms in the objective function (3) reduces and the resulting expectation terms are shown below.

$E_{\Psi_1,\Psi_2}[Q(u_1, u_2, \psi_1, \psi_2)]$:

Region 3 :

$$\int_0^{(1-u_2)z_2} \int_{z_1}^{\infty} \quad [\psi_1 - (1-u_1)z_1 - u_1 \cdot z_1 - e_{21} \cdot min(u_2 \cdot z_2, \frac{\psi_1 - z_1}{e_{21}})]s_1 \quad d\Phi_1(\psi_1)d\Phi_2(\psi_2) \quad (12)$$

Region 6 :

$$\int_{(1-u_2)z_2}^{z_2} \int_{z_1}^{\infty} \quad [\psi_1 - (1-u_1)z_1 - u_1 \cdot z_1 - e_{21} \cdot min(z_2 - \psi_2, \frac{\psi_1 - z_1}{e_{21}})]s_1 \quad d\Phi_1(\psi_1)d\Phi_2(\psi_2) \quad (13)$$

Region 7 :

$$\int_{z_2}^{\infty}\int_{0}^{(1-u_1)z_1} [\psi_2-(1-u_2)z_2-u_2\cdot z_2-e_{12}\cdot min(u_1\cdot z_1,\frac{\psi_2-z_2}{e_{12}})]s_2 \quad d\Phi_1(\psi_1)d\Phi_2(\psi_2) \quad (14)$$

Region 8 :

$$\int_{z_2}^{\infty}\int_{(1-u_1)z_1}^{z_1} [\psi_2-(1-u_2)z_2-u_2\cdot z_2-e_{12}\cdot min(z_1-\psi_1,\frac{\psi_2-z_2}{e_{12}})]s_2 \quad d\Phi_1(\psi_1)d\Phi_2(\psi_2) \quad (15)$$

Region 9 :

$$\int_{z_2}^{\infty}\int_{z_1}^{\infty}[\psi_1-(1-u_1)z_1-u_1\cdot z_1]s_1 d\Phi_1(\psi_1)d\Phi_2(\psi_2)+\int_{z_2}^{\infty}\int_{z_1}^{\infty}[\psi_2-(1-u_2)z_2-u_2\cdot z_2]s_2 d\Phi_1(\psi_1)d\Phi_2(\psi_2) \quad (16)$$

Splitting integrals in each of the regions 3, 6, 7 and 8 with a minimization operator we obtain following solutions:

Region 3 :

$min(u_2\cdot z_2,\frac{\psi_1-z_1}{e21})$: If $u_2\cdot z_2>\frac{\psi_1-z_1}{e_{21}}$ then $z_1+e_{21}\cdot u_2\cdot z_2>\psi_1$.

So, equation 12 becomes

$$\int_{0}^{(1-u_2)z_2}\int_{z_1}^{z_1+e_{21}\cdot u_2\cdot z_2} [\psi_1-z_1-e_{21}(\frac{\psi_1-z_1}{e_{21}})]s_1 \quad d\Phi_1(\psi_1)d\Phi_2(\psi_2)+$$
$$\int_{0}^{(1-u_2)z_2}\int_{z_1+e_{21}\cdot u_2\cdot z_2}^{\infty} [\psi_1-z_1-e_{21}\cdot u_2\cdot z_2]s_1 \quad d\Phi_1(\psi_1)d\Phi_2(\psi_2) \quad (17)$$

Region 6 :

$min(z_2-\psi_2,\frac{\psi_1-z_1}{e_{21}})$: If $z_2-\psi_2<\frac{\psi_1-z_1}{e_{21}}$ then $e_{21}(z_2-\psi_2)+z_1>\psi_1$.

So, equation 13 becomes

$$\int_{(1-u_2)z_2}^{z_2}\int_{z_1}^{e_{21}(z_2-\psi_2)} [\psi_1-z_1-e_{21}\frac{\psi_1-z_1}{e_{21}}]s_1 \quad d\Phi_1(\psi_1)d\Phi_2(\psi_2)+$$
$$\int_{(1-u_2)z_2}^{z_2}\int_{e_{21}(z_2-\psi_2)}^{\infty} [\psi_1-z_1-e_{21}(z_2-\psi_2)]s_1 \quad d\Phi_1(\psi_1)d\Phi_2(\psi_2) \quad (18)$$

Region 7 :

$min(u_1\cdot z_1,\frac{\psi_2-z_2}{e_{12}})$: If $u_1\cdot z_1<\frac{\psi_2-z_2}{e_{12}}$ then $z_2+e_{12}\cdot u_1\cdot z_1>\psi_2$.

So, equation 14 becomes

$$\int_{0}^{(1-u_1)z_1}\int_{z_2}^{z_2+e_{12}\cdot u_1\cdot z_1} [\psi_2-z_2-e_{12}\frac{\psi_2-z_2}{e_{12}}]s_2 \quad d\Phi_2(\psi_2)d\Phi_1(\psi_1)+$$
$$\int_{0}^{(1-u_1)z_1}\int_{z_2+e_{12}\cdot u_1\cdot z_1}^{\infty} [\psi_2-z_2-e_{12}\cdot u_1\cdot z_1]s_2 \quad d\Phi_2(\psi_2)d\Phi_1(\psi_1) \quad (19)$$

Region 8 :

$min(z_1 - \psi_1, \frac{\psi_2 - z_2}{e_{12}})$: If $z_1 - \psi_1 < \frac{\psi_2 - z_2}{e_{12}}$ then $e_{12}(z_1 - \psi_1) + z_2 > \psi_2$.

So, equation 15 becomes

$$\int_{(1-u_1)z_1}^{z_1} \int_{z_2}^{e_{12}(z_1-\psi_1)} [\psi_2 - z_2 - e_{12}\frac{\psi_2 - z_2}{e_{12}}]s_2 \quad d\Phi_2(\psi_2)d\Phi_1(\psi_1) + \int_{(1-u_1)z_1}^{z_1} \int_{e_{12}(z_1-\psi_1)}^{\infty} [\psi_2 - z_2 - e_{12}(z_1 - \psi_1)]s_2 \quad d\Phi_2(\psi_2)d\Phi_1(\psi_1) \tag{20}$$

Therefore,

$E_{\Psi_1,\Psi_2}[Q(u_1, u_2, \psi_1, \psi_2)] =$

$$\begin{aligned} &\int_0^{(1-u_2)z_2} \int_{(z_1+e_{21}\cdot u_2\cdot z_2)}^{\infty} [\psi_1 - z_1 - e_{21}\cdot u_2\cdot z_2]s_1 \quad d\Phi_1(\psi_1)d\Phi_2(\psi_2) \\ &+ \int_{(1-u_2)z_2}^{z_2} \int_{(z_1+e_{21}(z_2-\psi_2))}^{\infty} [\psi_1 - z_1 - e_{21}(z_2 - \psi_2)]s_1 \quad d\Phi_1(\psi_1)d\Phi_2(\psi_2) \\ &+ \int_0^{(1-u_1)z_1} \int_{(z_2+e_{12}\cdot u_1\cdot z_1)}^{\infty} [\psi_2 - z_2 - e_{12}\cdot u_2\cdot z_1]s_2 \quad d\Phi_2(\psi_2)d\Phi_1(\psi_1) \\ &+ \int_{(1-u_1)z_1}^{z_1} \int_{(z_2+e_{12}(z_1-\psi_1))}^{\infty} [\psi_2 - z_2 - e_{12}(z_1 - \psi_1)]s_2 \quad d\Phi_2(\psi_2)d\Phi_1(\psi_1) \\ &+ \int_{z_2}^{\infty} \int_{z_1}^{\infty} [\psi_1 - z_1]s_1 \quad d\Phi_1(\psi_1)d\Phi_2(\psi_2) \\ &+ \int_{z_2}^{\infty} \int_{z_1}^{\infty} [\psi_2 - z_2]s_2 \quad d\Phi_1(\psi_1)d\Phi_2(\psi_2) \end{aligned} \tag{21}$$

4.2 First stage - Optimal cross-training

The expectation terms from the second stage are put into the first stage objective function (1).

$$Min_{\ u1,\ u2} \quad t_1 \cdot u_1 \cdot z_1 + t_2 \cdot u_2 \cdot z_2 + k \cdot E_{\Psi_1,\Psi_2}[Q(u_1, u_2, \psi_1, \psi_2)] \tag{22}$$

subject to

$$0 \leq u_1 \leq 1 \tag{23}$$

$$0 \leq u_2 \leq 1 \tag{24}$$

Analysis of the Hessian matrix of the Lagrangian indicates that this constrained minimization problem is convex. First order conditions are used to determine optimal values, u_1^* and u_2^*. (See Appendix for analysis of Hessian and FOCs.) The following two equations give the closed form expression for the optimum amount of cross-training for unit 1 and unit 2 when the demand follows a general, continuous distribution.

$$\mathbf{\Phi_1[(1-u_1^*)z_1]\{1-\Phi_2[e_{12}\cdot u_1^*\cdot z_1+z_2]\}=\frac{t_1}{k\cdot e_{12}\cdot s_2}} \tag{25}$$

$$\mathbf{\Phi_2[(1-u_2^*)z_2]\{1-\Phi_1[e_{21}\cdot u_2^*\cdot z_2+z_1]\}=\frac{t_2}{k\cdot e_{21}\cdot s_1}} \tag{26}$$

Looking at equations (25) and (26), we can infer some of the implications of parameters such as contract cost, training cost and productivity. We see that when training cost per period ($\frac{t_i}{k}$) increases, the optimal amount of cross-training (u_i^*) decreases, as expected. An increase in the contract cost (s_j) results in an increase in the optimal amount of cross-training in unit i (u_i^*). These relationships and some more results are analyzed using a numerical example in the following section.

Other observations are :

1. Optimal amount of cross-training in unit i (u_i^*) does not depend on the contract cost of unit i (s_i).

2. Optimal amount of cross-training in unit i (u_i^*) does not depend on the training cost per period of unit j ($\frac{t_j}{k}$).

3. Optimal amount of cross-training in unit i (u_i^*) does not depend on the productivity of nurses floated from unit j to unit i (e_{ji}).

5 Numerical Analysis

Assuming Ψ_1 and Ψ_2 to be uniformly distributed between $[a_1, b_1]$ and $[a_2, b_2]$ we get the following equations.

For u_1^*:

$$\frac{(1-u_1)z_1 - a_1}{b_1 - a_1} \cdot \{1 - \frac{e_{12} \cdot u_1 \cdot z_1 + z_2}{b_2 - a_2}\} = \frac{t_1}{k \cdot e_{12} \cdot s_2} \tag{27}$$

Similarly, for u_2^*:

$$\frac{(1-u_2)z_2 - a_2}{b_2 - a_2} \cdot \{1 - \frac{e_{21} \cdot u_2 \cdot z_2 + z_1}{b_1 - a_1}\} = \frac{t_2}{k \cdot e_{21} \cdot s_1} \tag{28}$$

Solving (27) we get a quadratic equation (29) in u_1^*,

$$e_{12} \cdot z_1^2 \cdot u_1^2 - \{(b_2 - z_2)z_1 - e_{12} \cdot z_1(z_1 - a_1)\} \cdot u_1 + \{(z_1 - a_1) \cdot (b_2 - z_2) - \frac{t_1}{k \cdot e_{12} \cdot s_2} \cdot (b_1 - a_1) \cdot (b_2 - a_2)\} = 0 \tag{29}$$

Solving (28) we get a quadratic equation (30) in u_2^*,

$$e_{21} \cdot z_2^2 \cdot u_2^2 - \{(b_1 - z_1)z_2 - e_{21} \cdot z_2(z_2 - a_2)\} \cdot u_2 + \{(z_2 - a_2) \cdot (b_1 - z_1) - \frac{t_2}{k \cdot e_{21} \cdot s_1} \cdot (b_1 - a_1) \cdot (b_2 - a_2)\} = 0 \tag{30}$$

Let $\overline{\psi_1}$ and $\overline{\psi_2}$ denote the mean of the uniform distribution for the random variables Ψ_1 and Ψ_2. In other words, $\overline{\psi_1} = \frac{a_1+b_1}{2}$ and $\overline{\psi_2} = \frac{a_2+b_2}{2}$.

The numerical analysis includes five cases as shown in Table 1 depending on whether nurses available in unit i (z_i) is more or less than the mean demand in that unit ($\overline{\psi_i}$). Mean demand ($\overline{\psi_i}$) is taken as 40 for the numerical analysis. The values for z_i for different cases is given in Table 1.

Cases	$\mathbf{z_1}$	$\mathbf{z_2}$
$z_1 > \overline{\psi_1}$, $z_2 < \overline{\psi_2}$	50	30
$z_1 = \overline{\psi_1}$, $z_2 = \overline{\psi_2}$	40	40
$z_1 > \overline{\psi_1}$, $z_2 > \overline{\psi_2}$	50	50
$z_1 < \overline{\psi_1}$, $z_2 > \overline{\psi_2}$	30	50
$z_1 < \overline{\psi_1}$, $z_2 < \overline{\psi_2}$	30	30

Table 1: Alternative cases used in numerical analysis

Table 2 indicates the parameters and their values used in the numerical analysis. Productivity parameters vary from 0.05 to 0.95 in steps of 0.05. The training cost per period varies from $0 to

Parameters	**Level 1**	**Level 2**	**Level 3**
e_{12}	0.05 - 0.95	steps of 0.05	-
e_{21}	0.05 - 0.95	steps of 0.05	-
t_1/k	\$0 - \$75	steps of 5	-
t_2/k	\$0 - \$75	steps of 5	-
s_1	\$40	\$60	\$80
s_2	\$40	\$60	\$80
CV_1	0.2	0.5	0.7
CV_2	0.2	0.5	0.7

Table 2: Parameter levels for numerical analysis

\$75 in steps of \$5. Analysis is done for three levels of contract cost \$40, \$60 and \$80. Coefficient of variation (CV) is defined as the ratio of standard deviation to mean. Since mean is represented by $\overline{\psi_i}$, we get $CV_i = \frac{\sigma}{\overline{\psi_i}}$. If coefficient of variation is CV_i for unit i and $\overline{\psi_i}$ follows uniform distribution, then $[a_i, b_i] = [\overline{\psi_i} \cdot (1 - (CV_i/2)), \overline{\psi_i} \cdot (1 + (CV_i/2))]$.

Case 1 : $\mathbf{z_1} > \overline{\psi_1}$ and $\mathbf{z_2} < \overline{\psi_2}$

In this case, unit 1 has surplus nurses most of the time and unit 2 has insufficient nurses most of the time. Assuming CV$= 0.2$ for demand in unit 1 and unit 2, we plot the amount of cross-training (u_1^*) versus productivity (e_{12}) for varying training cost per period t_1/k for unit 1 in Figure: 4. Similar graphs are drawn for coefficient of variation 0.5 and 0.7 for demand as shown in the Figure 5 and 6.

For identical demand variation in unit 1 and unit 2, we can conclude the following from Figures 4, 5, 6.

1. In general, higher productivity of cross-trained nurses fosters more cross-training by effectively reducing the cost of covering demand with cross-trained nurses. When the cost of cross-training is relatively low, however, there is a productivity level beyond which further increases

in productivity reduce the level of cross-training. Above this threshold, the productivity of cross-trained nurses allow the benefits of flexibility to be obtained with fewer cross-trained nurses.

2. When training cost is high, the optimal amount of cross-training is zero for lower values of productivity, but increases with increasing values of productivity.

3. The general shortage of nurses in unit 2 causes no cross-training in unit 2 to be optimal.

When demand variation in unit 1 and unit 2 increases simultaneously or increases in unit 2, we can conclude the following from Figures 7 and 9 for low training cost and 8 and 10 for high training cost.

1. For low cross-training cost per period:

   - at lower productivity level, optimal amount of cross-training decreases with increase in demand variation

   - at higher productivity level, optimal amount of cross-training increases with increase in demand variation

2. For high cross-training cost per period and at a low productivity level increase in demand variation causes decrease in optimal amount of cross-training.

In all of the above analysis, the contract cost per period is assumed to be $60 per period. In Figure 11, the contract cost is varied to determine the effect of the contract cost on the optimal amount of cross-training. Increase in contract cost in unit j causes increase in cross-training in unit i.

Case 2 : $\mathbf{z_1} = \overline{\psi_1}$ and $\mathbf{z_2} = \overline{\psi_2}$

In this case, nurses available in both units equals the mean demand in their units. There is no cross-training when cross-training cost per period is high. Figure 12 shows that at lower values of training cost, the optimal amount of cross-training increases with higher values of productivity and is zero for lower values of productivity. There is no threshold value for productivity as seen in case

1. The optimal amount of cross-training increases with in increase in demand variability as seen in Figure 13.

Case 3 : $\mathbf{z_1 > \overline{\psi_1}}$ and $\mathbf{z_2 > \overline{\psi_2}}$

In this case, the nurses available in each unit is above the average demand. When training cost per period for unit i is low, the optimal amount of cross-training for unit i decreases with increase in coefficient of variation for unit i.

Case 4 : $\mathbf{z_1 < \overline{\psi_1}}$ and $\mathbf{z_2 < \overline{\psi_2}}$

In this case, the nurses available in each unit is less than the mean demand in the units. Since available nurses are less than the mean demand in both units there is little or no cross-training at all.

6 Conclusion

The model and analysis in this paper show that productivity of flexible nurses has a significant effect on the optimal amount of cross-training. We have derived a closed form expression that determines the optimal amount of cross-training to minimize the sum of cross-training cost and expected shortage cost. The analysis shows that at a given level of productivity for flexible nurses, the optimal amount of cross-training in unit i decreases with an increase in cross-training cost in unit i, and increases with an increase in shortage cost in unit j, as expected.

When cost of cross-training is high, an increase in productivity leads to an increase in the amount of cross-training. When the cost of cross-training is relatively low, however, there is a productivity level beyond which further increases in productivity reduce the amount of cross-training. Above this threshold, the productivity of cross-trained nurses allow the benefits of flexibility to be obtained with fewer cross-trained nurses.

For low cross-training productivity, more demand variability leads to less cross-training. For high cross-training productivity, the effect of demand variability on the amount of cross-training depends on cross-training cost. When cross-training cost is high, more variability continues to cause less cross-training. For low cross-training cost, however, increasing demand variability leads to more cross-training.

This paper has integrated the planning (amount of cross-training) and scheduling (allocation) phases of nurse staffing while utilizing flexible nurses in hospital. To date, the literature has focussed independently on each of these two issues. In our model, we try to integrate both planning and scheduling phases of nurse staffing across a planning horizon. The paper has provided insights on the interaction effects of productivity of flexible nurses, cross-training costs and contract costs on the optimum amount of cross-training.

Extending the model to more than two units introduces the possibility of different patterns of cross-training, so it becomes necessary to determine the pattern that best fits the situation. Another possible extension is to have varying costs for flexible nurses when they are floated from one unit to another. We have considered a model where an external resource (the contract nurse) is hired, to attain 100% service level. It is possible to consider a pool of highly flexible resources (similar to contract nurse) who are permanent employees, but are paid higher than regular nurses and lower than contract nurses. It would be interesting to determine the optimal number of such highly flexible nurses that should be hired at a given wage rate.

7 Appendix

The Lagrangian function for the first stage is given by

$$\Lambda(u_1, u_2, \mu) = t_1 \cdot u_1 \cdot z_1 + t_2 \cdot u_2 \cdot z_2 + k \cdot E_{\Psi_1, \Psi_2} Q(u_1, u_2, \psi_1, \psi_2) - \mu_1 \cdot u_1 - \mu_2 \cdot u_2 + \mu_3 \cdot (u_1 - 1) + \mu_4 \cdot (u_2 - 1) \tag{31}$$

$$\frac{\partial^2 \Lambda}{\partial u_1, u_1} = e_{12} \cdot k \cdot s_2 \cdot z_1^2 \cdot (\{1 - \Phi_2[e_{12} \cdot u_1 \cdot z_1 + z_2]\} \cdot \Phi_1'[(1-u_1)z_1] + e_{12} \cdot \Phi_1[(1-u_1)z_1] \cdot \Phi_2'[e_{12} \cdot u_1 \cdot z_1 + z_2]) \tag{32}$$

$$\frac{\partial^2 \Lambda}{\partial u_2, u_2} = e_{21} \cdot k \cdot s_1 \cdot z_2^2 \cdot (\{1 - \Phi_1[e_{21} \cdot u_2 \cdot z_2 + z_1]\} \cdot \Phi_2'[(1-u_2)z_2] + e_{21} \cdot \Phi_2[(1-u_2)z_2] \cdot \Phi_1'[e_{21} \cdot u_2 \cdot z_1 + z_2]) \tag{33}$$

$$\frac{\partial^2 \Lambda}{\partial u_1, u_2} = 0 \tag{34}$$

$$\frac{\partial^2 \Lambda}{\partial u_2, u_1} = 0 \tag{35}$$

The Hessian matrix with second order conditions with respect to u_1 and u_2 for the above formulation shows that it is positive definite (Blume and Simon 1994) and so the objective function (22) is convex in u_1 and u_2.

The first order condition for first stage Lagrange function (31) is given below :

$$\frac{\partial \Lambda}{\partial u_1} = \mu_3 - \mu_1 + t_1 \cdot z_1 - k \cdot e_{12} \cdot s_2 \cdot \Phi_1[(1-u_1)z_1] \cdot \{1 - \Phi_2[e_{12} \cdot u_1 \cdot z_1 + z_2]\} = 0 \quad (36)$$

$$\frac{\partial \Lambda}{\partial u_2} = \mu_4 - \mu_2 + t_2 \cdot z_2 - k \cdot e_{21} \cdot s_1 \cdot \Phi_2[(1-u_2)z_2] \cdot \{1 - \Phi_1[e_{21} \cdot u_2 \cdot z_2 + z_1]\} = 0 \quad (37)$$

$$\mu_1 \cdot u_1 = 0 \quad (38)$$

$$\mu_2 \cdot u_2 = 0 \quad (39)$$

$$\mu_3 \cdot (1 - u_1) = 0 \quad (40)$$

$$\mu_4 \cdot (1 - u_2) = 0 \quad (41)$$

Solving for the lagrange multipliers we get the following cases:

Case 1 : ($\mu_1 > 0$ and $\mu_2 > 0$)

From the constraints (38) and (39), we see that $u_1^* = 0$ and $u_2^* = 0$, consequently from constraint (40) and (41) we get $\mu_3 = 0$ and $\mu_4 = 0$. So constraint (38) and (39) is binding.

Case 2 : ($\mu_1 > 0$ and $\mu_4 > 0$)

Implies $u_1^* = 0$ and consequently $\mu_3 = 0$. Since $\mu_4 > 0$, $u_2^* = 1$ from constraint (41) and $\mu_2 = 0$. Substituting $u_2^* = 1$ in constraint (37), we get $t_2 \cdot z_2 = -\mu_4$. This is not possible since all the values are positive and so this case is infeasible.

Case 3 : ($\mu_3 > 0$ and $\mu_2 > 0$)

Implies $u_2^* = 0$ and consequently $\mu_4 = 0$. But $\mu_1 = 0$ and so $u_1^* = 1$. Substituting in constraint (36) we see that $t_1 \cdot z_1 = -\mu_3$. So this case is infeasible.

Case 4 : ($\mu_1 = 0$ and $\mu_3 = 0$, $\mu_2 = 0$ and $\mu_4 = 0$)

The solution is $u_1^* = [0, 1)$ and $u_2^* = [0, 1)$ and satisfies the equations (36) and (37).

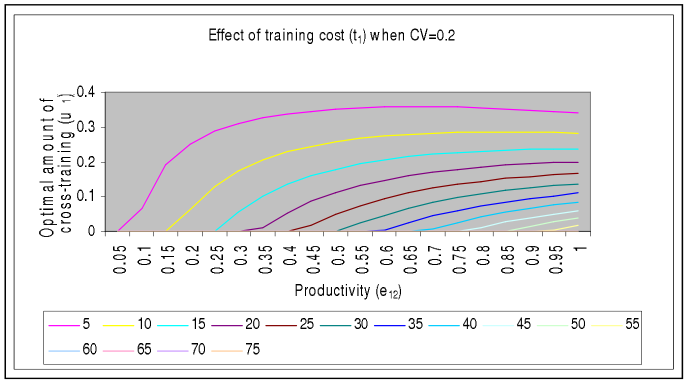

Figure 4: Effect of training cost on optimal amount of cross-training with CV=0.2

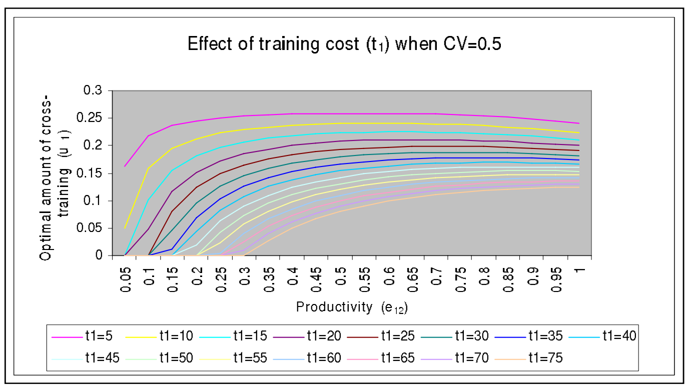

Figure 5: Effect of training cost on optimal amount of cross-training with CV=0.5

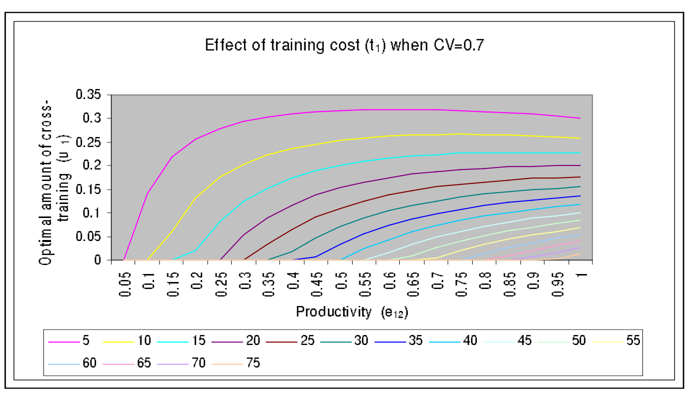

Figure 6: Effect of training cost on optimal amount of cross-training with CV=0.7

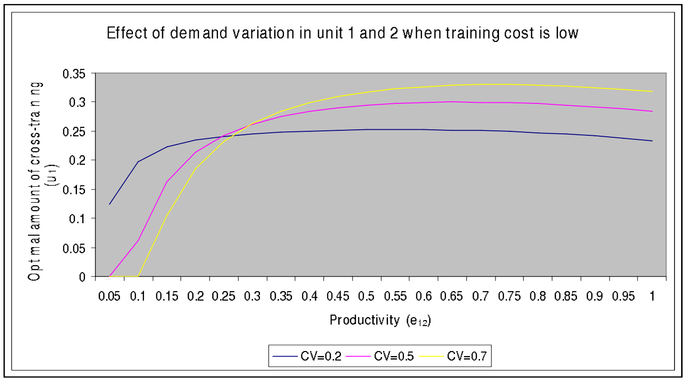

Figure 7: Effect of demand variation in unit 1 and 2 on optimal amount of cross-training at low training cost

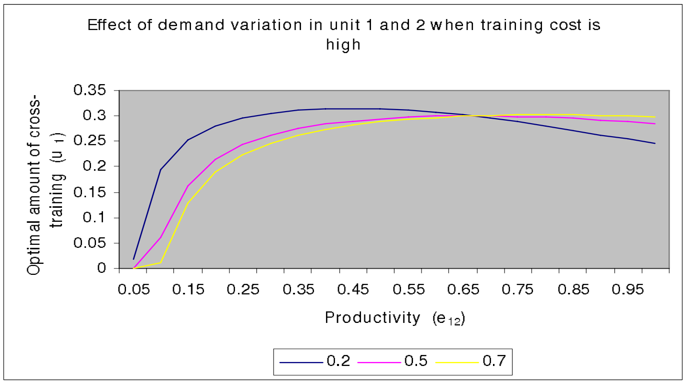

Figure 8: Effect of demand variation in unit 1 and 2 on optimal amount of cross-training at high training cost

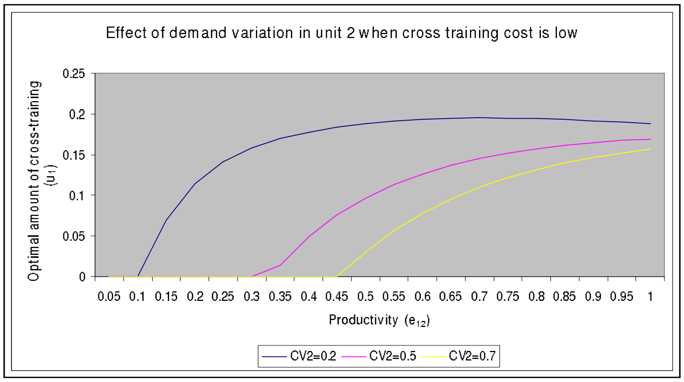

Figure 9: Effect of demand variation in unit 2 on optimal amount of cross-training at low training cost

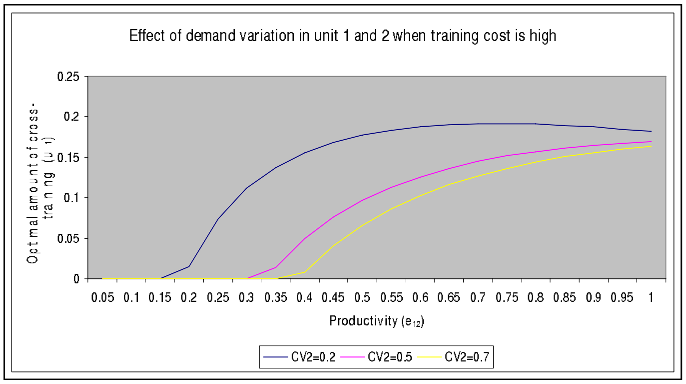

Figure 10: Effect of demand variation in unit 2 on optimal amount of cross-training at high training cost

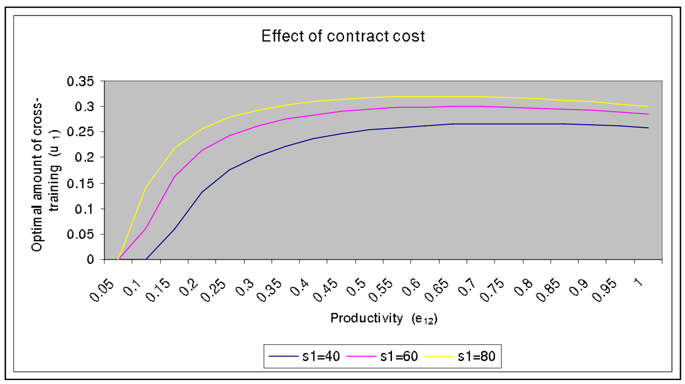

Figure 11: Effect of contract cost on optimal amount of cross-training

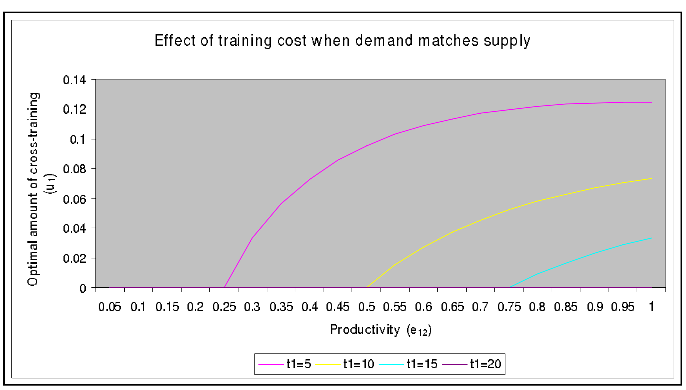

Figure 12: Effect of training cost on optimal amount of cross-training when demand matches supply

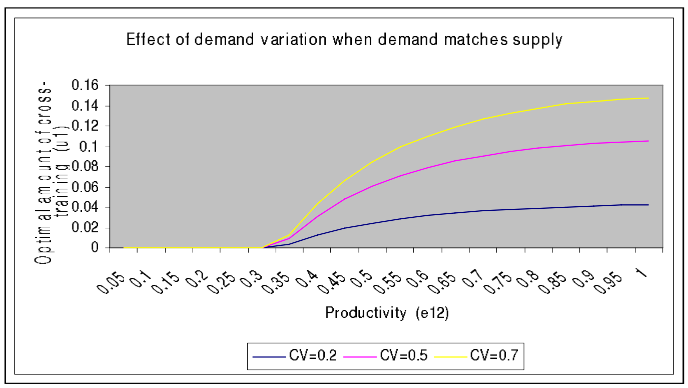

Figure 13: Effect of demand variation on optimal amount of cross-training when demand matches supply

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