The Classical Model of Consumption and Production

The **Classical Model** represents a primitively basic model of an economy. It is a “no-exchange economy”, but a “pure production-economy”: one person divides time into work and leisure, and supplies work hours in order to produce what they alone consume. That one person is therefore the sole firm. Prices are irrelevant: there is no one to trade with or buys things from. This is therefore also referred to as the” **Robinson Crusoe Model**”, or for those not familiar with that story, you can call it the “castaway model”.

Ingredients: 1 consumer, 1 firm, one unit of time T (e.g. T = 16 waking hours in a day).

There is no past or future: the consumer cannot save, invest, have debt. The population does not grow[1].

Optimal Policy (i.e. the **equilibrium**): $\{w^*, c^*, L^*, I^*, \pi^*\}$, where w = wage, c = consumption, L = labor hours, l = T-L leisure hours, $I = w \times L$ = income, and π = profit. Although the consumer consumers everything produced, she *is* the firm and therefore *receives all profit* from *the firm.*

Recall that “**wage**” is as **pay-off for work-per-hour**: if you work 5 hours and can collect 10 coconuts/hour, then the wage is 10/hour, and income is 50 coconuts.

[1] As opposed to a “Blue Lagoon” economy, or “Gilligan’s Island” economy, where the population *could* expand.

1. THE CONSUMER (labor-leisure choice)

The consumer receives utility from consumption c and leisure l, but work permits consumption. Since there is only one consumer, who is the firm, she receives all profits from production. Her total income I based on wage labor wL and profit π is

$$I = wL + \pi$$

Since there is no future nor past, all income is consumed. The time and consumption constraints are:

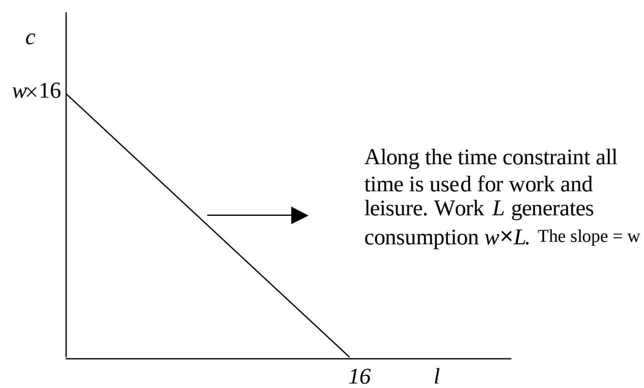
$$T = L + l$$
$$c = wL + \pi$$

For the sake simplicity, since the consumer takes profits π as given, let's assume they are zero for now:

$$\text{Let } \pi = 0$$

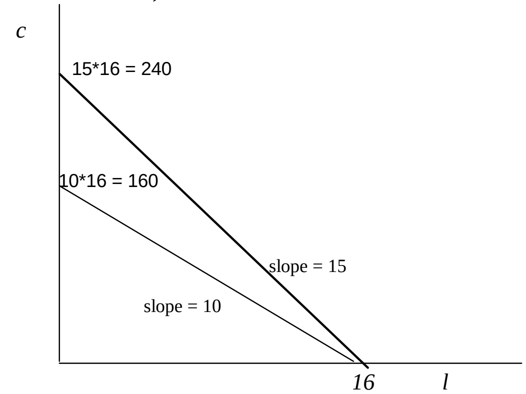
If you are not working, you are enjoying the time off. If all time goes to leisure $l = T$ then $c = 0$.

If all time goes to labor then $c = w \times T$. Suppose $T = 16$, then



As wage increases, consumption possibilities rise. Nothing can change the time allowance except, perhaps, hallucinogenics, which we rule out for obvious reasons: T remains fixed.

If $T = 16$ and $w = 10$ increases to $w = 15$, then



1.1 OPTIMAL LABOR-LEISURE CHOICE

The consumer chooses an "*optimal bundle*", or "*policy*", $\{c^*, l^*\}$ that maximizes utility $U(c,l)$ subject to the time constraint:

$$\boxed{\max_{c,l} U(c,l) \quad \text{subject to } c = wL + \pi \text{ and } L = T - l}$$

where wage w and profits π are taken as given. Again, **for simplicity assume profit is zero**

$$\pi = 0$$

The optimal bundle must satisfy two conditions, an optimality condition and the time/consumption constraints:

$$\text{Optimality Condition}: \frac{MU_c}{MU_l} = \frac{1}{w}$$

$$\text{Time/Consumption}: c = w(T - l)$$

1.2 EXAMPLE (Cobb-Douglas)

Consider Cobb-Douglas preferences:

$$U(c,l) = c^{1/4} l^{3/4}$$

and let w = 20/hour, π = 0, and T = 16 hours. The marginal utilities are

$$MU_c = \frac{\partial}{\partial c} U(c,l) = \frac{1}{4} c^{-3/4} l^{3/4} = \frac{1}{4}\left(\frac{l}{c}\right)^{3/4}$$

$$MU_l = \frac{\partial}{\partial l} U(c,l) = \frac{3}{4} c^{1/4} l^{-1/4} = \frac{3}{4}\left(\frac{c}{l}\right)^{1/4}$$

hence

$$\frac{MU_c}{MU_l} = \frac{\frac{1}{4}\left(\frac{l}{c}\right)^{3/4}}{\frac{3}{4}\left(\frac{c}{l}\right)^{1/4}} = \frac{1}{3}\left(\frac{l}{c}\right)^{3/4}\left(\frac{l}{c}\right)^{1/4} = \frac{l}{3c}$$

so the optimal bundle must satisfy

$$\text{Optimality Condition}: \frac{l}{3c} = \frac{1}{w}$$

$$\text{Time/Consumption}: c = w(T - l)$$

The solution is of the two equations in two unknowns is

$$3c = lw$$
$$3w(T - l) = lw$$
$$3T - 3l = l$$
$$\frac{3}{4}T = l \quad \Rightarrow \quad L = \frac{1}{4}T \quad \Rightarrow \quad c = w\frac{1}{4}T$$

The optimal amount of labor supply is simply a fixed portion of time based entirely on the preference parameter for consumption 1/4. In the Cobb-Douglas case it is insensitive to wage, making this utility model unrealistic but simple to use.

Plugging in the values that we know, we get

The rest of the solution is

$$l^* = 16 - L^* = 16 - 4 = 12$$
$$c^* = wL^* = 20 \times 4 = 80 = I^*$$

Therefore $\{c^*, l^*\} = \{80, 12\}$.

If we plot the time and consumption constraint, the optimal choices satisfy.

1.3 EXAMPLE (general Cobb-Douglas)

The general Cobb-Douglas utility function is

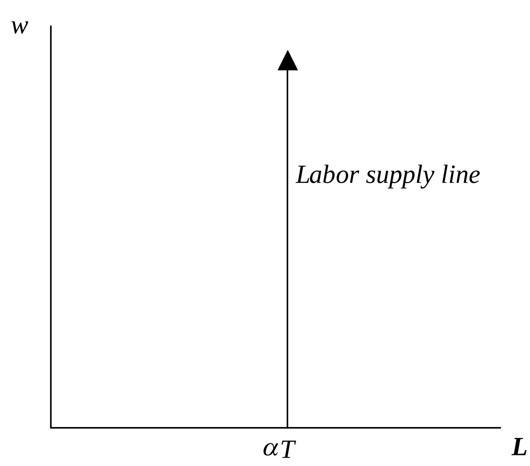
$$U(c, l) = c^{\alpha} l^{\beta} \quad 0 \le \alpha, \beta \le 1 \quad \alpha + \beta = 1$$

The solution is (*verify this yourself*)

$$L^* = \alpha T, \quad l^* = \beta T, \quad c^* = \alpha w T$$

If α = 1/2, β = 1/2, w = 20 , π = 0, and T = 15 then c = .5×20×16 = 160 and l = .5×16 = 8. Notice the consumption preference parameter 1/2 is twice as large as Example 1.2, and consumption is now twice as large. The leisure parameter is smaller, and now the person works more with less leisure.

Cob-Douglas preferences implies the individual works the same number of hours, irrespective of the wage. This means labor supply is vertical:



1.4 EXAMPLE (tractable utility function)

A nice utility function that implies for a more realistic labor supply and a simple competitive equilibrium (see Part 3, below) is the following.

The utility is

$$U(c,l) = \mathrm{c} - \frac{1}{2}(T-l)^2$$

It is "**linear in consumption**". The marginal utilities and MU's are

$$MU_{\mathrm{c}} = 1 \qquad MU_{\mathrm{l}} = T - l \quad \Rightarrow \quad \frac{MU_{\mathrm{c}}}{MU_{\mathrm{l}}} = \frac{1}{T-l}$$

Notice marginal utility with respect to consumption is positive (more is better), but constant: this utility function assumes the person does not experience diminishing marginal returns.

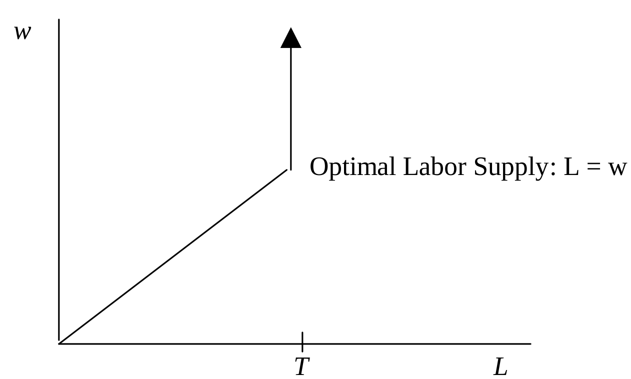
Notice marginal utility respect to leisure is positive since $T \geq l$, and decreasing as leisure increases.

$$\text{Optimality Condition}: \frac{1}{T\text{-}l} = \frac{1}{w}$$
$$\text{Time/Consumption}: c = w(T-l) + \pi$$

The solution here is painfully simple: we only need to use the optimality condition and recall $T - l = L$:

$$\text{Optimality Condition}: \frac{1}{T\text{-}l} = \frac{1}{w} \Rightarrow w = T - l = L$$

So, optimal labor supply is $L = w$, a simple linear increasing function in wages. More wage = more labor. Of course, labor hours cannot be greater than T:



2. THE FIRM (profit maximization)

You already know this story from the lecture notes on consumer and **producer theory**. Since *price is irrelevant*[2] the firm's profit maximization problem looks like

$$\boxed{\pi(L) = A \times F(K^*, L) - wL}$$

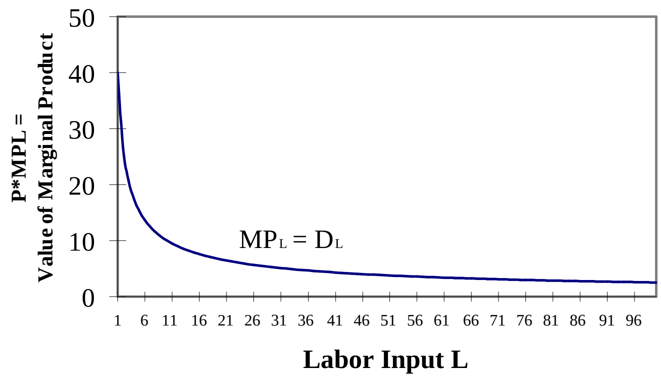
where K^* is the fixed short-run level of capital.

The first order condition is

$$\frac{\partial}{\partial L}\pi(L) = 0 \quad \Rightarrow \quad A \times \frac{\partial}{\partial L} F(K^*, L) - w = 0$$
$$\Rightarrow \quad MP_L = w$$

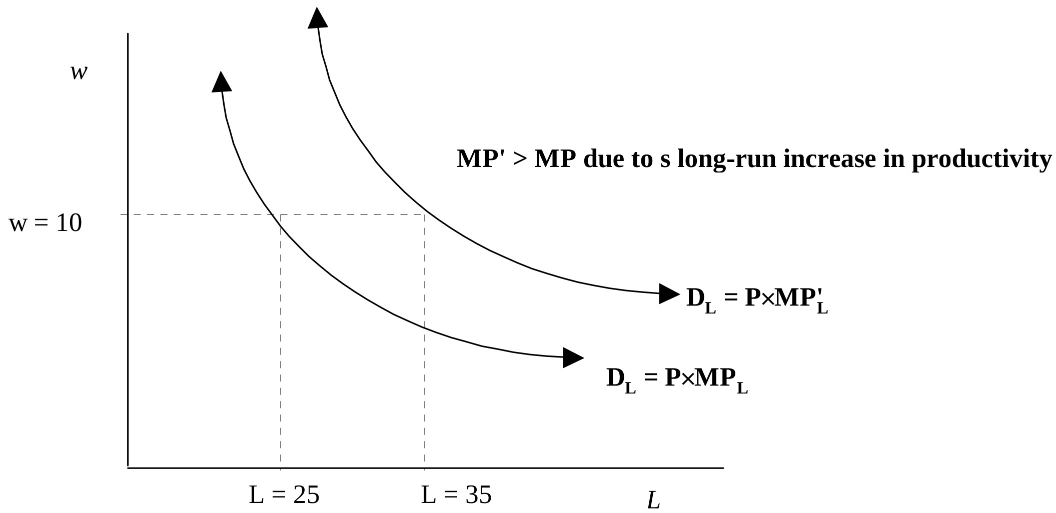
Notice $\mathbf{MP_L}$ gives us a Demand for Labor $\mathbf{D_L}$ which itself tells us the firms Willingness-to-Pay: the firm is willing to pay no more than *w* for each unit of labor *L* based on labor's productivity:

Demand for Labor



If workers are more productive (e.g. human capital improves, or on-the-job training improves), then they can produce more faster, which translates to lower costs to the firm. Hence profit increases, output is therefore increased requiring more labor inputs: the demand for labor shifts up:

[2] "Price" simply has no meaning in a pure production economy (i.e. Tom Hanks alone with a volleyball). Notice we are only thinking of "consumption" in the abstract, so the firm will produce whatever it is that is consumed (e.g. Soylent Green: great 1970's movie about green chips produced for mass consumption). We effectively fix $P = 1$: the price of your entire day of consumption is your entire day of income: 1 day of *c* has a price of 1 day of work.



2.1 EXAMPLE

Fix capital at $K = 1$, and assume $w = 10$, $A = 100$, and

$$Y = A \times F(L) = 100 \times L^{2/5}$$

Then the profit maximization problem reduces to

$$MP_L = w$$

$$100 \times \frac{2}{5}\frac{1}{L^{3/5}} = 10 \quad \Rightarrow \quad 4^{5/3} = L \quad \Rightarrow \quad L = 10.07937$$

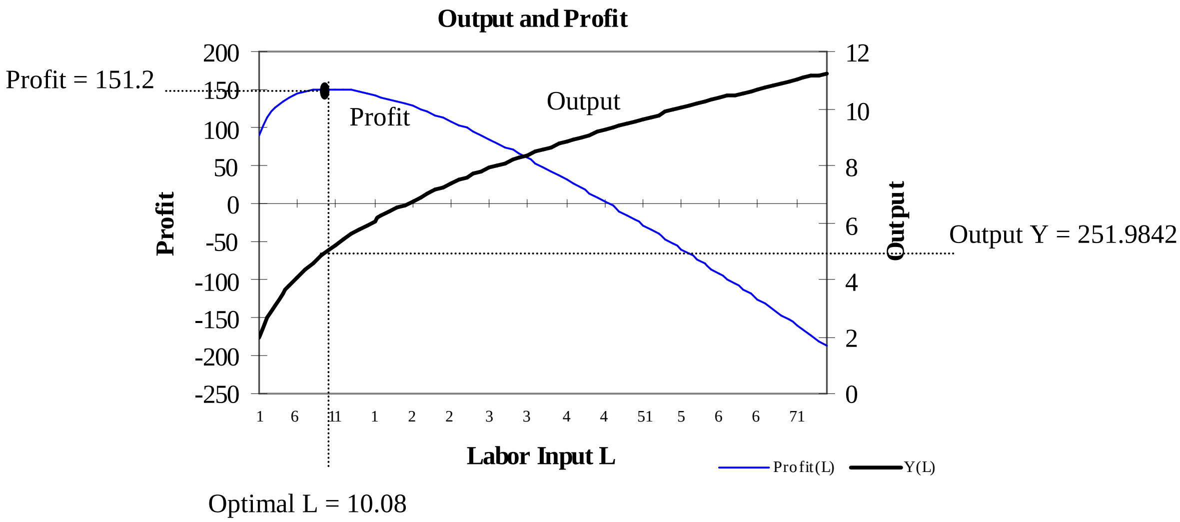
The firm's profit maximizing level of labor is 10.08 units (e.g. individuals/day, hours, etc., whatever the units are). The resulting level of output and profit are

$$Y = A \times F(L^*) = 100 \times 10.07937^{2/5} = 251.9842$$

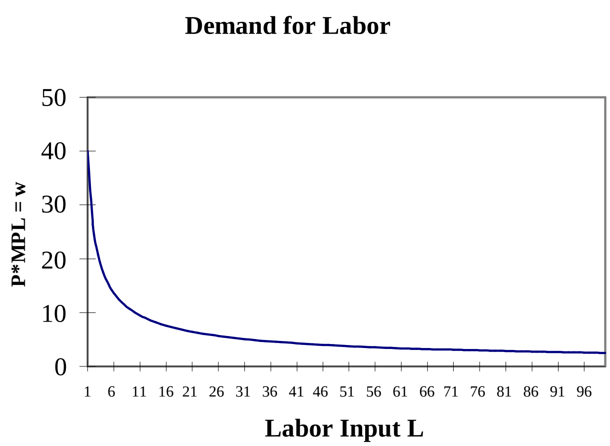
$$\pi = 251.9842 - 10 \times 10.07937 = 151.1898$$

Notice we can represent and plot profit as a function of labor:

$$\pi(L) = 100 \times L^{3/5} - 10 \times L$$



The demand equation for labor is simply characterized by the Value Marginal Product of Labor, which itself is the most a firm will pay in wages to the labor:



3. COMPETITIVE EQUILIBRIUM (optimal consumption, labor supply and production)

In the **equilibrium**, there exists a unique wage w^* that **clears the labor market**: at w^* the consumers quantity supplied of labor hours equals the firm' quantity demanded of labor hours. The result is an equilibrium number of labor hours L^*; the consumer's equilibrium consumption $c^* = w^* L^* + \pi^*$ and leisure time $T - L^*$; and the firm's equilibrium level of output Y^* and profit $\pi^* = Y - w^* L^*$.

Since $c^* = w^* L^* + \pi^*$ and $\pi^* = Y - w^* L^*$ the consumer simply consumes all output:

$$c^* = w^* L^* + \pi^* = w^* L^* + Y^* - w^* L^* = Y^*$$

This is not surprising: there is no past and future, so *all output* is *consumed*.

So, the equilibrium condition is

$$\boxed{w^* \text{ such that } L_d = L_s = L^* \text{ and } c^* = Y^*}$$

3.1 COBB-DOUGLAS PRODUCTION AND LINEAR-IN-CONSUMPTION UTILITY

Let's go back to previous consumer and firm examples, and stitch them together. Assume the consumer time allotment is $T = 16$, and has **linear-in-consumption** utility

$$U(c, l) = \mathrm{c} - \frac{1}{2}(16 - l)^2$$

and assume the firm's production function is of the **Cobb-Douglas** form

$$Y = A \times F(L) = 100 \times L^{2/5}$$

Recall the consumer's optimality condition implies her supply of labor is

$$L_s = w$$

Recall the firm's optimal conditions imply

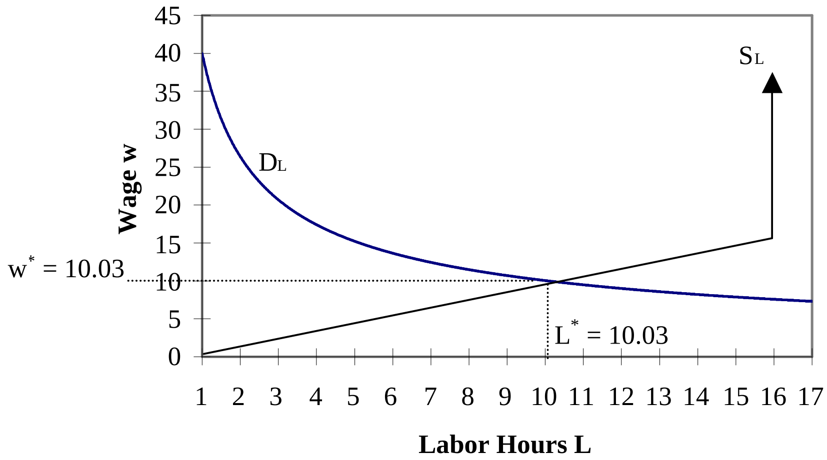
$$MP_L = w$$

$$100 \times \frac{2}{5}\frac{1}{L^{3/5}} = w \quad \Rightarrow \quad \left(\frac{40}{w}\right)^{5/3} = L_d^*$$

$$\Rightarrow w = \frac{40}{\left(L_d^*\right)^{3/5}}$$

Graphically

Market Equilibrium



The equilibrium wage w^* solves

$$L_d = L_s$$

$$w = \left(\frac{40}{w}\right)^{5/3}$$

$$w^{8/3} = 40^{5/3}$$

$$w = 40^{5/8} = 10.03$$

Therefore, at w = 10.03/hour the consumer supplies 10.03 hours, thereby optimizing utility, and the firm maximizes profit by employing 10.03 labor hours.

Output is

$$Y^* = 100 \times \left(L^*\right)^{2/5} = 100 \times (10.03)^{2/5} = 251.48$$

hence consumption is

$$c^* = Y^* = 251.48$$

The **output** and **profit functions** are

$$Y(L) = 100 \times L^{2/5}$$

$$\pi(L) = 100 \times L^{2/5} - 10.03 \times L$$

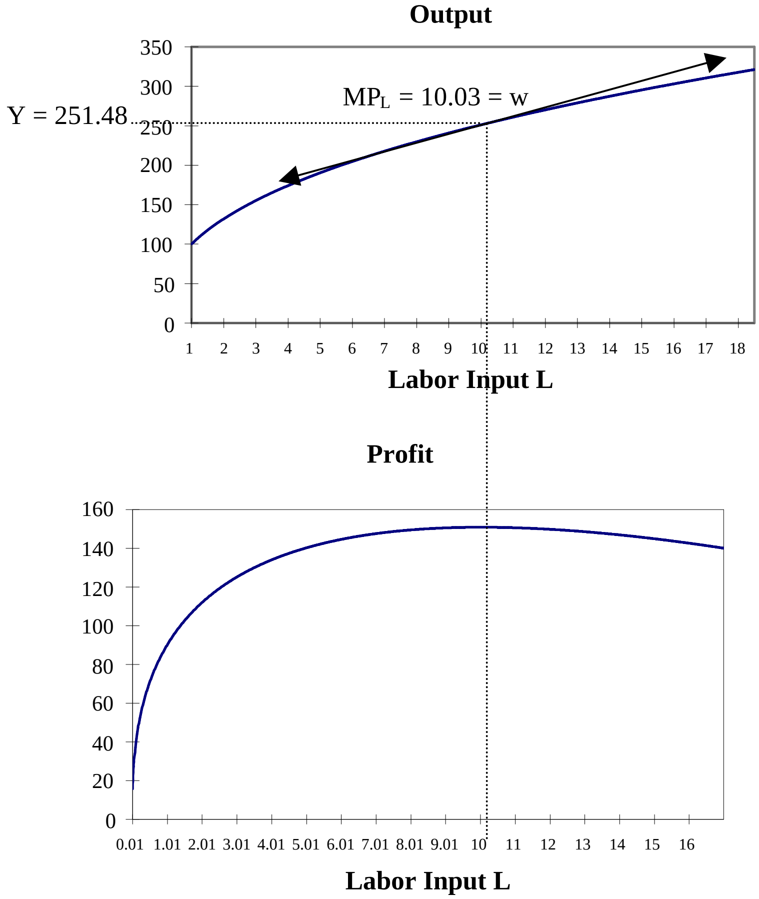
Notice profit is zero when

$$\pi(L) = 100 \times L^{2/5} - 10.03 \times L = 0$$

$$\Rightarrow \quad L = \left(\frac{100}{10.03}\right)^{5/3} = 46.18$$

But labor hours can be no greater than T = 16 hours, so profit is never negative in the present economy.

Graphically,



3.2 CHANGES IN FUNDAMENTALS

If the firm's **total factor productivity** decreases (e.g. medical services in New Orleans post-Katrina) then it takes more labor to accomplish the same level of prior output. This means that labor is less valuable and therefore the firm demands less of it at any wage. This depresses the equilibrium wage, the equilibrium quantity labor demanded and supplied fall, output falls, etc.

The result is displayed in the figure below.

Market Equilibrium